Phase of phase conjugation and its effect in the double phase-conjugate resonator

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Expressions for the phase of reflection from a photorefractive phase-conjugate mirror are obtained as a function of the intensity and phase of the pump and the probe beams. The phase is independent of these parameters in common photorefractive conditions in which the index grating is spatially shifted 90° with respect to the light-interference pattern. Multiple solutions exist for the phase and intensity of the reflection at large coupling strength. Oscillation conditions involving frequency detuning are obtained for the double phase-conjugate resonator (resonator formed with two phase-conjugate mirrors).

In the past decade there was considerable theoretical and experimental research in optical phase conjugation and its applications, such as aberration correction,[1] information processing,[2] and optical bistability.[3] Photorefractive crystals were shown to provide exceptionally high phase-conjugate reflectivity in four-wave mixing (4WM) experiments. Although there was significant progress in the coupled-wave theory describing the nonlinear interactions in photorefractive crystals and the magnitude of the phase-conjugate reflectivity was solved,[4–6] no attention was paid to the output phase of the phase-conjugate beam. In this paper we investigate the output phase of the phase conjugation as a function of various parameters. These studies provide a better understanding of 4WM in photorefractive media and are important, for example, in the analysis of double phase-conjugate resonators.[7–9]

In the following discussion we adopt the same notation as in Ref. 4 and 5. The 4WM geometry is shown in Fig. 1. For simplicity, we assume that the nonlinear medium is lossless and that the four interacting beams are plane waves. Let the electric-field amplitude associated with the jth beam be

$$E_j = A_j(\mathbf{r})\exp[i(\mathbf{k}_j \cdot \mathbf{r} - \omega t)] + \text{c. c.} \tag{1}$$

and

$$A_j(\mathbf{r}) = |A_j(\mathbf{r})|\exp[i\psi_j(\mathbf{r})], \qquad j = 1, \ldots 4, \tag{2}$$

where $\psi_j(\mathbf{r})$ is the phase factor associated with the complex amplitude $A_j(\mathbf{r})$. By using the coupled-wave theory for thick holograms,[10] taking the slowly varying field approximation, and assuming that only one grating (the transmission grating in this case) dominates, the applicable coupled-wave equations can be written as

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = \frac{-\gamma}{I_0}(A_1A_4{}^* + A_2{}^*A_3)A_4, \tag{3a}$$

$$\frac{\mathrm{d}A_2{}^*}{\mathrm{d}z} = \frac{-\gamma}{I_0}(A_1A_4{}^* + A_2{}^*A_3)A_3{}^*, \tag{3b}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}z} = \frac{\gamma}{I_0}(A_1A_4{}^* + A_2{}^*A_3)A_2, \tag{3c}$$

$$\frac{\mathrm{d}A_4{}^*}{\mathrm{d}z} = \frac{\gamma}{I_0}(A_1A_4{}^* + A_2{}^*A_3)A_1{}^*, \tag{3d}$$

where γ is a complex coupling constant, $\gamma = |\gamma|e^{i\phi}$, which is a material parameter of the nonlinear medium.[11] I_0 is the sum of the intensities of the four beams:

$$I_0 = I_1 + I_2 + I_3 + I_4, \tag{4}$$

where

$$I_j = |A_j|^2.$$

In the undepleted pump approximation in which $|A_1|^2$, $|A_2|^2 \gg |A_3|^2, |A_4|^2$, Eqs. (3) reduce to

$$\frac{\mathrm{d}A_3}{\mathrm{d}z} = \frac{\gamma}{I_0}[|A_2|^2A_3 + (A_1A_2)A_4{}^*], \tag{5a}$$

$$\frac{\mathrm{d}A_4{}^*}{\mathrm{d}z} = \frac{\gamma}{I_0}[|A_1|^2A_4{}^* + (A_1{}^*A_2{}^*)A_3]. \tag{5b}$$

With boundary conditions $A_3(l) = 0$ and $A_4{}^*(0)$, the solutions to Eqs. (5) are

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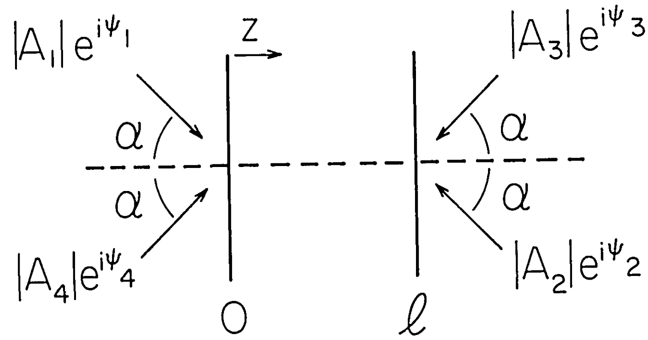

Fig. 1. Schematic diagram of nonlinear four-wave mixing in a nonlinear medium. A_1 and A_2 are the pumping beams, A_3 is the phase-conjugate output, and A_4 is the probe beam.

$$A_3(z) = A_4{}^*(0)\,\frac{A_1/A_2{}^*}{r^{-1}e^{-\gamma l}+1}\{\exp[\gamma(z-l)]-1\}, \tag{6a}$$

$$A_4{}^*(z) = A_4{}^*(0)\,\frac{1}{r^{-1}e^{-\gamma l}+1}\{r^{-1}\exp[\gamma(z-l)]+1\}, \tag{6b}$$

where r is the pump-beam intensity ratio

$$r = \frac{A_2A_2{}^*}{A_1A_1{}^*} = \frac{I_2}{I_1}\,. \tag{7}$$

Since the phase of $(A_1/A_2{}^*)$ in Eq. (6a) is $(\psi_1+\psi_2)$, which is constant along z, the phase of $A_3(0)$,$\psi_3(0)$ is

$$\psi_3(0) = \psi_1(0) + \psi_2(l) - \psi_4(0) + \mathrm{Im}\,\ln\left[\frac{e^{-\gamma l}-1}{r^{-1}e^{-\gamma l}+1}\right]. \tag{8}$$

In particular, $\psi_3(0) = \psi_1(0) + \psi_2(l) - \psi_4(0)$ if γ is real, which corresponds to a $\pi/2$ phase shift between the refractive-index grating and the light-interference fringes. This happens in photorefractive crystals when the index grating is formed by charge diffusion. In general, $\psi_3(0)$ is a function of the complex coupling constant and the pump ratio.

Without the undepleted pump approximation, $\psi_3(0)$ can be determined as follows:

$$\psi_3(0) = \mathrm{Im}\,\ln\left[\frac{A_1{}^*(0)}{A_2(0)}\,\frac{A_3(0)}{A_4{}^*(0)}\right] + \psi_1(0) + \psi_2(0) - \psi_4(0). \tag{9}$$

The given boundary conditions are $A_1(0)$, $A_2(l)$, $A_3(l) = 0$, $A_4(0)$. Therefore $\psi_1(0)$, $\psi_2(l)$, and $\psi_4(0)$ are known. $[A_1{}^*(0)/A_2(0)]$ and $[A_3(0)/A_4{}^*(0)]$ are derived in Ref. (5),

$$\frac{A_1(z)}{A_2{}^*(z)} = -\left\{\frac{[\Delta-(\Delta^2+4|c|^2)^{1/2}]D\,e^{-\mu z} - [\Delta+(\Delta^2+4|c|^2)^{1/2}]D^{-1}\,e^{\mu z}}{2c^*(D\,e^{-\mu z} - D^{-1}\,e^{\mu z})}\right\}, \tag{10}$$

$$\frac{A_3(z)}{A_4{}^*(z)} = \left\{\frac{[\Delta-(\Delta^2+4|c|^2)^{1/2}]E\,e^{-\mu z} - [\Delta+(\Delta^2+4|c|^2)^{1/2}]E^{-1}\,e^{\mu z}}{2c^*(E\,e^{-\mu z} - E^{-1}\,e^{\mu z})}\right\}, \tag{11}$$

where

$$\Delta = I_2 + I_3 - I_1 - I_4,$$

$$\mu = \frac{\gamma(\Delta^2+4|c|^2)^{1/2}}{2I_0},$$

$$D = \left[\frac{\Delta + (\Delta^2+4|c|^2)^{1/2} + 2|c|^2/I_2(l)}{\Delta - (\Delta^2+4|c|^2)^{1/2} + 2|c|^2/I_2(l)}\right]^{1/2} e^{\mu l},$$

$$E = \left[\frac{\Delta + (\Delta^2+4|c|^2)^{1/2}}{\Delta - (\Delta^2+4|c|^2)^{1/2}}\right]^{1/2} e^{\mu l}.$$

$|c|^2$ is given by the equation

$$[\,|c|^2 - I_1(0)I_2(l)]\,|\Delta T + (\Delta^2 + 4\,|c|^2)^{1/2}|^2 + 4|c|^2|T|^2I_4(0)I_2(l) + 2|c|^2I_4(0)(\Delta^2+4|c|^2)^{1/2}(T+T^*) = 0,$$

and $T = \tanh(\mu l)$.

$\psi_2(0)$ in Eq. (9) can be calculated as follows: If we rewrite Eq. (3b) into

$$\frac{\mathrm{d}(\ln A_2{}^*)}{\mathrm{d}z} = \frac{-\gamma}{I_0}\left(\frac{A_1}{A_2{}^*}\frac{A_3{}^*}{A_4}I_4 + I_3\right),$$

then integrate, we get

$$\psi_2(0) = \psi_2(l) + \int_l^0\left\{\mathrm{Re}\left(\frac{\gamma}{I_0}\right)\mathrm{Im}\left[\frac{A_1(z)}{A_2{}^*(z)}\frac{A_3{}^*(z)}{A_4(z)}I_4(z)\right] + \left\{I_3(z) + \mathrm{Re}\left[\frac{A_1(z)}{A_2{}^*(z)}\frac{A_3{}^*(z)}{A_4(z)}I_4(z)\right]\right\}\mathrm{Im}\left(\frac{\gamma}{I_0}\right)\right\}\mathrm{d}z, \tag{12}$$

where Re and Im are the real and imaginary parts, respectively. The integrand in Eq. (12) is known from Eqs. (10) and (11); therefore $\psi_3(0)$ is completely known. Again, from Eqs. (10) and (11), if γ is real,

$$\mathrm{Im}\left[\frac{A_1(z)}{A_2{}^*(z)}\frac{A_3{}^*(z)}{A_4(z)}\right] = 0.$$

Therefore

$$\psi_3(0) = \psi_1(0) + \psi_2(l) - \psi_4(0), \tag{13}$$

and the phase of the conjugate reflection is independent of the intensities of the interacting beams. The integral in Eq. (12) can be evaluated numerically. Figure 2 shows some numerical curves of $\psi_3(0)$ against the ln probe ratio, $\ln q = \ln\{I_4(0)/[I_1(0) + I_2(l)]\}$, for various pump-beam ratios r and phases of the complex coupling constant ϕ. $|\gamma l|$ is chosen to be 6 for all curves. Some of the curves show multiple values in $\psi_3(0)$ that are associated with the multiple values in the phase-conjugate intensity reflectivity R.[12] Multivalued behavior in $\psi_3(0)$ and R will occur only when $|\gamma l|$ is large. We also observed that the vertical series of curves is similar to the horizontal series of curves. In other words, the effect of changing the pump ratio on the system is similar to that of changing the phase shift between the light-interference pattern and the refractive-index pattern. This is reasonable because the phase shift between the light-interference pattern and the index grating causes energy coupling from one beam to the other beam. As a result, the pump ratio of the

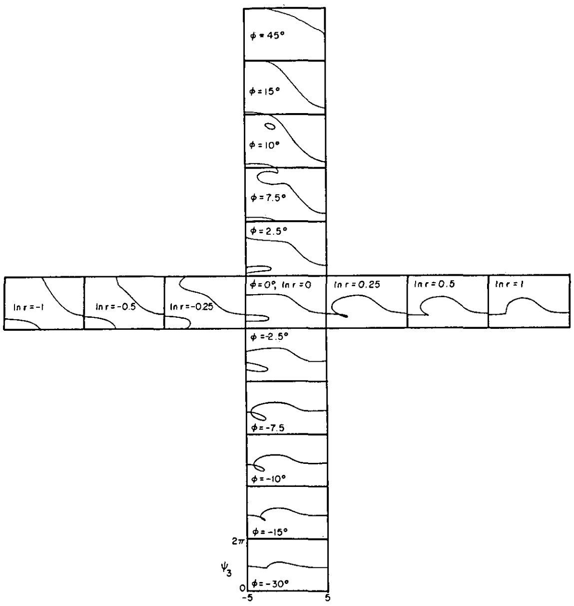

Fig. 2. Numerical curves of ψ_3 versus ln q for various ln r and ϕ. $|\gamma l|$ was chosen to be 6. For each of the curves, the range of ψ_3 is from 0 to 2π and from −5 to 5 for ln q.

system changes accordingly. This kind of similarity relationship is difficult to observe from the set of coupled-wave equations, Eqs. (3), but is clearly shown in Fig. 2.

To demonstrate an application of the above analysis, let us consider oscillation between a pair of photorefractive phase-conjugate mirrors, which are pumped at the same frequency ω (see Fig. 3). In the absence of a photovoltaic effect and no externally applied electric field, the index grating is formed by charge diffusion, and if the oscillation is degenerate, then γ is real. From Eq. (13) the net accumulated round-trip phase change is $\Delta\psi = (\psi_{21} + \psi_{22}) - (\psi_{11} + \psi_{12})$, where ψ_{ij} is the input phase of the jth pumping beam in the ith phase-conjugate mirror. In general $\Delta\psi$ is not equal to $2\,m\pi$, where m is an integer. This should force the oscillation to be nondegenerate.

Experimentally, we have observed this nondegenerate oscillation.[8] We can now analyze the situation in the light of the earlier sections of this paper. Without loss of generality, we take the frequency of the field propagating from left to right as $\omega - \delta$ and $\omega + \delta$ for the field traveling in the opposite direction, Fig. 3. This frequency offset between the probe beam and the pumping beams will cause the refractive-index grating responsible for beam coupling to move in space in synchronism with the light-interference pattern. The finite response time of the medium induces a phase lag between the interference pattern and the index grating. As a result, the coupling constant becomes complex, and it is given by

$$\gamma = \frac{i\gamma_0}{1 + i\delta\tau} \tag{14}$$

according to current theories of the photorefractive effect.[13] The constant γ_0 and τ are characteristics of the crystal and its orientation with respect to the various beams. τ, a characteristic time for formation of gratings in the crystal, is approximately inversely proportional to the total pumping intensity,[14] whereas in $BaTiO_3$ and $Sr_{1-x}Ba_xNb_2O_6$, τ is of the order of few seconds for milliwatt-per-square-millimeter beams.[14,15] The coupling constant γ_0 is almost independent of the total pumping intensity. With the nondegenerate oscillation, the net round-trip phase change becomes

$$\Delta\psi = (\psi_{g2} - \psi_{g1}) + (\psi_{21} + \psi_{22}) - (\psi_{11} + \psi_{12}) + \frac{2\delta l}{c},$$

where l is the distance between the two phase-conjugate mirrors. ψ_g is the phase shift in the phase-conjugate mirror output when the grating is moving. It also depends on δ, τ, and the intensities of the oscillation beam and the pumping beams. $\psi_g(\delta, \tau, I_j; j = 1 \ldots 4) = \psi_3(0) + \psi_4(0) - \psi_1(0) - \psi_2(l)$ can be calculated from Eqs. (8) and (12). The oscillation condition becomes

$$\Delta\psi = 2m\pi, \tag{15}$$

where m is the integer. Since δ is limited to the reciprocal response time of the crystal, which is of the order of seconds, the term $2\delta l/c$ is negligible.

In summary, we have obtained solutions to the phase of a phase-conjugate beam in nonlinear 4WM. There exist multiple solutions at the large nonlinear coupling constant, e.g., $|\gamma l| = 6$. The similarity between the change in the pump ratio r and the change in the phase angle ϕ of the complex coupling constant is demonstrated by numerically plotting $\psi_3(0)$ against the ln probe ratio. An oscillation condition for the double phase-conjugate resonator is also obtained. When the index pattern is shifted 90° with respect to the light-interference pattern, the phase of the reflectivity is independent of the intensities of the interacting beams.

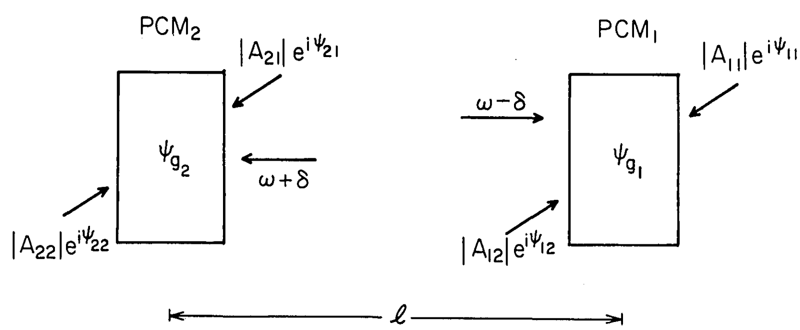

Fig. 3. Schematic diagram of a double phase-conjugate resonator.

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