Neutrino Mass Matrix and Hierarchy

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Abstract. We build a model to describe neutrinos based on strict hierarchy, incorporating as much as possible, the latest known data, for Δ_{sol} and Δ_{atm} , and for the mixing angles determined from neutrino oscillation experiments, including that from KamLAND. Since the hierarchy assumption is a statement about mass ratios, it lets us obtain all three neutrino masses. We obtain a mass matrix, M_ν and a mixing matrix, U , where both M_ν and U are given in terms of powers of Λ , the analog of the Cabibbo angle λ in the Wolfenstein representation, and two parameters, ρ and κ , each of order one. The expansion parameter, Λ , is defined by $\Lambda^2 = m_2/m_3 = \sqrt{(\Delta_{sol}/\Delta_{atm})} \approx 0.16$, and ρ expresses our ignorance of the lightest neutrino mass m_1 , ($m_1 = \rho\Lambda^4 m_3$), while κ scales s_{13} to the experimental upper limit, $s_{13} = \kappa\Lambda^2 \approx 0.16\kappa$. These matrices are similar in structure to those for the quark and lepton families, but with Λ about 1.6 times larger than the λ for the quarks and charged leptons. The upper limit for the effective neutrino mass in double β -decay experiments is $4 \times 10^{-3} eV$ if $s_{13} = 0$ and $6 \times 10^{-3} eV$ if s_{13} is maximal. The model, which is fairly unique, given the hierarchy assumption and the data, is compared to supersymmetric extension and texture zero models of mass generation.

1 INTRODUCTION

The hierarchical model has been very successful in describing the mass patterns and mixing matrices for quarks and charged leptons [1]. Both the mass patterns and mixing angles are dominated by an expansion parameter, which for each family is given by $\lambda = \sqrt{(m_2/m_3)}$. Furthermore the λ 's for the three families are roughly equal, $0.22 < \lambda < 0.25$. Here, we will try to see whether neutrinos can be brought simply into the standard fold. We have a fair handle on the mixing matrix, but as far as the masses are concerned, we only know the two mass-squared differences, Δ_{sol} and Δ_{atm} . This allows the mass ratio (m_2/m_3) to range from about 1 (degeneracy) to small, ~ 0.1 , (hierarchy). To determine all three masses one more equation is needed and it is provided by the hierarchy assumption.

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The three neutrino mass eigenvalues, m_1, m_2, m_3 give a diagonal mass matrix. This matrix can be undiagonalised by the mixing matrix, U , and the results classified [2] according to possible mass assignments consistent with Δ_{sol} and Δ_{atm} . In this paper, we try to build a model based on strict hierarchy, incorporating the known data as much as possible. Since the hierarchy assumption is a statement about mass ratios, it lets us obtain all three masses from the two Δ 's. This leads to a mass matrix and mixing matrix, almost entirely in terms of powers of Λ , the analog of the Cabibbo angle λ in the Wolfenstein representation. These matrices are similar in structure to the quark and lepton families, but with Λ about 1.6 times larger than λ for the other families. The effective mass, $\langle m \rangle$, measured in $\beta\beta_{0\nu}$ decay [3], is then obtained. The model, which is fairly unique given the hierarchy assumption and the data, is compared to models of mass generation [4, 5].

2 DETERMINATION OF THE NEUTRINO MASS MATRIX AND MIXING MATRIX

The mixing matrix, U , [6] which rotates mass (Majorana) eigenstates $\Psi_{1,2,3}$ into flavor eigenstates $\Psi_{\nu_e, \nu_\mu, \nu_\tau}$ is parameterized as usual

$$U = \begin{bmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{23} + c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & -c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & c_{12}s_{23} + s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix} \quad (1)$$

Letting $\delta = 0$ (no CP violation), and assuming maximal mixing for the atmospheric oscillation [7], i.e., $s_{23} = c_{23} = 1/\sqrt{2}$, we have for U :

$$U = \begin{bmatrix} Cc_{13} & -Sc_{13} & s_{13} \\ \frac{1}{\sqrt{2}}S + \frac{1}{\sqrt{2}}Cs_{13} & \frac{1}{\sqrt{2}}C - \frac{1}{\sqrt{2}}Ss_{13} & -\frac{1}{\sqrt{2}}c_{13} \\ \frac{1}{\sqrt{2}}S - \frac{1}{\sqrt{2}}Cs_{13} & \frac{1}{\sqrt{2}}C + \frac{1}{\sqrt{2}}Ss_{13} & \frac{1}{\sqrt{2}}c_{13} \end{bmatrix} \quad (2)$$

The angle θ_{13} is known to be small [8, 9, 10], with an upper limit $s_{13} < 0.13$ and at present no lower limit. We start with the two rotations known not to vanish, θ_{12} and θ_{23} . With $\theta_{13} = 0$, we have:

$$U = \begin{bmatrix} C & -S & 0 \\ S/\sqrt{2} & C/\sqrt{2} & -1/\sqrt{2} \\ S/\sqrt{2} & C/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \quad (3)$$

where $S = \sin(\theta_{12})$, $C = \cos(\theta_{12})$ and we have set the rotation angle $\theta_{23} = \pi/4$ (maximal mixing) and the angle $\theta_{13} = 0$, no CP violation.

To lowest order in S (expanding C in Eq. (3) in terms of S), U is given by

$$U_1 = \begin{bmatrix} 1 & -S & 0 \\ S/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ S/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \quad (4)$$

which, except for the extreme θ_{23} mixing, is much like the quark mixing matrix. In fact, if we consider U_1 to be the result of two successive rotations, $U_1 = v_1 v_0$, we get

$$v_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \quad (4a)$$

and

$$v_1 = \begin{bmatrix} 1 & -S/\sqrt{2} & -S/\sqrt{2} \\ S/\sqrt{2} & 1 & 0 \\ S/\sqrt{2} & 0 & 1 \end{bmatrix} \quad (4b)$$

This suggests that the appropriate expansion parameter for U is given by:

$$\varepsilon = S/\sqrt{2} = \sin(\theta_{12})/\sqrt{2}. \quad (5)$$

Recent KamLAND experiments [11] and analyses of this data [12, 13, 14], extracting θ_{12} , show that it is roughly 34° , i.e., $\tan^2(\theta_{12}) \approx 0.45$, consistent with earlier SNO experiments [15].

Turning to the diagonal mass matrix, we define the conventional hierarchical mass pattern by:

$$m_3 : m_2 : m_1 = 1 : \Lambda^2 : \rho\Lambda^4. \quad (6)$$

($\rho = 1$ would correspond to strict hierarchy). The three eigenvalues are then $m_3, m_2 = \Lambda^2 m_3$ and $m_1 = \rho\Lambda^4 m_3$.

The hierarchy expansion is in terms of the traditional hierarchy parameter, Λ .

$$\Lambda = \sqrt{(m_2/m_3)}. \quad (7)$$

For quarks and charged leptons, the mass matrices, parametrized by λ , and the mixing matrices, given in terms of θ_{ij} are related. The observed mixing angles of the mixing matrices are given as powers of λ , the Cabibbo angle, as seen in the Wolfenstein [16] representation of the V_{CKM} . We will try to determine the analogous relationship for neutrinos from the data. We note that both expansion parameters, ε and Λ , can be evaluated, independently, from experimental data. With $\tan^2(\theta_{12}) = T^2 \approx 0.45$, we have:

$$S^2 \approx 0.31 \quad (8)$$

and therefore

$$\varepsilon^2 = S^2/2 \approx 0.16 \quad (9)$$

On the other hand $\Lambda^2 = m_2/m_3$, (the expansion parameter for the mass matrix) can be evaluated in the hierarchical expansion, using

$$\Delta_{sol} = m_2^2 - m_1^2 \approx 7.1 \times 10^{-5} eV^2 \quad (10)$$

and

$$\Delta_{atm} = m_3^2 - m_2^2 \approx 2.7 \times 10^{-3} eV^2 \quad (11)$$

which forms the ratio

$$\begin{aligned} \sqrt{(\Delta_{sol}/\Delta_{atm})} &= \sqrt{[(m_2^2 - m_1^2)/(m_3^2 - m_2^2)]} \\ &= \sqrt{[(\Lambda^4 m_3^2 - \rho^2 \Lambda^8 m_3^2)/(m_3^2 - \Lambda^4 m_3^2)]}. \end{aligned} \quad (12)$$

The value for Δ_{sol} is obtained in the analyses of KamLAND data [12, 13]; the value for Δ_{atm} is obtained from analyses by S. Pascoli, et al., [14] and G.L. Fogli, et al., [17]. Expanding in Λ^2 we obtain

$$\sqrt{(\Delta_{sol}/\Delta_{atm})} = \Lambda^2 + (1/2)\Lambda^6(1 - \rho^2). \quad (13)$$

Thus to order Λ^4 we have

$$\Lambda^2 = \sqrt{(\Delta_{sol}/\Delta_{atm})} = 0.16, \quad (14)$$

so that

$$\varepsilon^2 = \Lambda^2 = 0.16 \quad (15)$$

or

$$\sin(\theta_{12})/\sqrt{2} \approx \Lambda \quad (16)$$

Phenomenologically, at least, there is a close relationship between θ_{12} and Λ .

We will use equation 16 as an equality. We can now express both the mass matrix, M , as well as the rotation matrix U , in terms of one parameter, defined in equation 7 as $\Lambda = \sqrt{(m_2/m_3)}$, in analogy with the Cabibbo angle for quarks and charged leptons.

The mixing matrix, U , is now

$$U = \begin{bmatrix} \sqrt{(1 - 2\Lambda^2)} & -\sqrt{2}\Lambda & 0 \\ \Lambda & \sqrt{(1/2)(1 - 2\Lambda^2)} & -1/\sqrt{2} \\ \Lambda & \sqrt{(1/2)(1 - 2\Lambda^2)} & -1/\sqrt{2} \end{bmatrix} \quad (17)$$

While $\Lambda = 0.4$ is a rather large number, the expansions made in equation 17 are of the square roots and thus, effectively, the expansion parameter is $\Lambda^2 = 0.16$.

The mass matrix M_ν is given by

$$M_\nu = U M U^{-1} \quad (18)$$

with

$$M = m_3 \begin{bmatrix} \Lambda^4 \rho & 0 & 0 \\ 0 & \Lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

To order Λ^4 , M_ν , is given by

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\rho + 2) & -\Lambda^3 & -\Lambda^3 \\ -\Lambda^3 & -\Lambda^4 + (1/2)\Lambda^2 + 1/2 & -\Lambda^4 + (1/2)\Lambda^2 - 1/2 \\ -\Lambda^3 & -\Lambda^4 + (1/2)\Lambda^2 - 1/2 & -\Lambda^4 + (1/2)\Lambda^2 + 1/2 \end{bmatrix} \quad (20)$$

where, using eq. (7), $\Lambda = \sqrt{(m_2/m_3)} = \sqrt{.16} = 0.4$ and

$$m_3 = \sqrt{(\Delta_{atm} + \Delta_{sol})} = 5.2 \times 10^{-2} eV. \quad (21)$$

While the analytic expressions are to the orders in Λ indicated, the data, unfortunately, are not. Thus, all results will be given to at most two significant figures.

We now can determine the masses, m_1, m_2 and m_3 to order Λ^4 :

$$m_3 = \sqrt{(\Delta_{atm} + \Delta_{sol})} = 5.2 \times 10^{-2} eV \quad (22)$$

$$m_2 = \Lambda^2 m_3 = \sqrt{(\Delta_{sol}/\Delta_{atm})} m_3 = 8.3 \times 10^{-3} eV \quad (23)$$

and

$$m_1 = \rho \Lambda^4 m_3 = \rho (\Delta_{sol}/\Delta_{atm}) m_3 = 1.3 \rho \times 10^{-3} eV \quad (24)$$

We may let ρ range from, say, 2 to -2 and still consider $|m_1|$ to be of order Λ^4 or smaller. These masses follow directly from the hierarchy assumption and the experimental values: $\Delta_{sol} = m_2^2 - m_1^2 \approx 7.1 \times 10^{-5} eV^2$ and $\Delta_{atm} = m_3^2 - m_2^2 \approx 2.7 \times 10^{-3} eV^2$ and are independent of the mixing matrix U . The effective neutrino mass $\langle m \rangle$ measured in $\beta\beta_{0\nu}$ decay, [3] however, does depend on the neutrino mass matrix, M_ν and is given by $M_{\nu_{ee}}$, the 1,1 matrix element. From equations (17) and (19) we have, to order Λ^6

$$\langle m \rangle = M_{\nu_{ee}} = m_3 \Lambda^4 [\rho(1 - 2\Lambda^2) + 2] = (0.7\rho + 2) \times 1.3 \times 10^{-3} eV \quad (25)$$

Taking $\rho = 2$, ($m_1 = 2.6 \times 10^{-3} eV$) as an extreme case, we obtain as an upper limit (with $s_{13} = 0$):

$$\langle m \rangle \leq 4 \times 10^{-3} eV \quad (26)$$

On the other hand, m_1 and therefore ρ , may be negative. For $\rho = -2$, ($m_1 = -2.6 \times 10^{-3} eV$), we have

$$\langle m \rangle = M_{\nu_{ee}} = \Lambda^6 m_3 = 8 \times 10^{-4} eV. \quad (27)$$

Thus, the limits on $\langle m \rangle$ in this model are, approximately,

$$10^{-3} < \langle m \rangle < 4 \times 10^{-3} eV \quad (28)$$

3 INCLUSION OF $s_{13} = \sin(\theta_{13})$

While the present data do not demand a non-vanishing θ_{13} , several models of hierarchy generation do [2, 18]. We want to investigate the effect of a finite s_{13} .

Since we know that $s_{13} < 0.13$ [8, 9, 10] and there is at present no lower limit, we will scale s_{13} :

$$s_{13} = \kappa \Lambda^2 \quad (29)$$

where $1 \geq |\kappa| \geq 0$.

Substituting for s_{13} in eq. (2) and forming the mass matrix M_ν , we obtain to order Λ^4 :

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\kappa^2 + \rho + 2) & -\Lambda^3 - \frac{1}{\sqrt{2}}\Lambda^2\kappa & -\Lambda^3 + \frac{1}{\sqrt{2}}\Lambda^2\kappa \\ -\Lambda^3 - \frac{1}{\sqrt{2}}\Lambda^2\kappa & -\frac{1}{2}\Lambda^4(\kappa^2 + 1) + \frac{1}{2}\Lambda^2 + \frac{1}{2} & \frac{1}{2}\Lambda^4(\kappa^2 + 1) + \frac{1}{2}\Lambda^2 - \frac{1}{2} \\ -\Lambda^3 + \frac{1}{\sqrt{2}}\Lambda^2\kappa & \frac{1}{2}\Lambda^4(\kappa^2 - 1) + \frac{1}{2}\Lambda^2 - \frac{1}{2} & -\frac{1}{2}\Lambda^4(\kappa^2 + 1) + \frac{1}{2}\Lambda^2 + \frac{1}{2} \end{bmatrix} \quad (30)$$

We now have two parameters, ρ and κ , where ρ is defined by $m_1 = \rho\Lambda^4 m_3$ and $s_{13} = \kappa\Lambda^2$, $|\kappa| \leq 1$. There are three special regimes for κ which are interesting.

1. $\kappa = 0$, $s_{13} = 0$ This is the case which was discussed earlier. We give here the leading elements of M_ν which depend on κ .

$$M_{\nu_{ee}} = \langle m \rangle = \Lambda^4(\rho + 2)m_3 \quad (31)$$

$$M_{\nu_{e\mu}} = M_{\nu_{e3}} = -\Lambda^3 m_3 \quad (32)$$

2. $\kappa \approx 1$ (upper limit), $s_{13} = \Lambda^2$

$$M_{\nu_{ee}} = \langle m \rangle = \Lambda^4(\rho + 2 + \kappa^2)m_3 = \Lambda^4(\rho + 3)m_3 \quad (33)$$

$$M_{\nu_{e\mu}} = -M_{\nu_{e\tau}} = \pm(1/\sqrt{2})\Lambda^2 m_3 \quad (34)$$

and the most interesting possibility,

3. $\kappa = \kappa'\Lambda$ i.e., $s_{13} = \kappa'\Lambda^3$. For case (3) the mass matrix to order Λ^4 is:

$$M_{\nu_4} = m_3 \begin{bmatrix} \Lambda^4(\rho + 2) & -\Lambda^3\left(1 + \frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^3\left(1 - \frac{\kappa'}{\sqrt{2}}\right) \\ -\Lambda^3\left(1 + \frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} \\ -\Lambda^3\left(1 - \frac{\kappa'}{\sqrt{2}}\right) & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} \end{bmatrix} \quad (35)$$

and

$$M_{\nu_{e\mu}} = -\Lambda^3[1 + (1/\sqrt{2})\kappa']m_3 \quad (36)$$

$$M_{\nu_{e\tau}} = -\Lambda^3[1 - (1/\sqrt{2})\kappa']m_3 \quad (37)$$

Case (3) is the only case which allows a zero in an off diagonal element. Texture zeros have been considered as a possible source of hierarchies and mixing angles [4,

5, 19, 20]. From Eq. (35), we see that only $\kappa = \kappa'\Lambda$ provides the possibility of having two texture zeroes.

Taking $\rho = -2$ and $\kappa' = \pm\sqrt{2}$ will make $M_{\nu_{ee}}$ and either $M_{\nu_{e\mu}}$ or $M_{\nu_{e\tau}}$ vanish to order Λ^4 . With $\kappa' = -\sqrt{2}$, M_{ν_4} becomes

$$M_{\nu_4} = m_3 \begin{bmatrix} 0 & 0 & -2\Lambda^3 \\ 0 & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} \\ -2\Lambda^3 & -\Lambda^4 + \Lambda^2/2 - \frac{1}{2} & -\Lambda^4 + \Lambda^2/2 + \frac{1}{2} \end{bmatrix} \quad (38)$$

In terms of masses we substitute $m_2 = \Lambda^2 m_3$, $m_1 = \rho \Lambda^4 m_3 = -2\Lambda^4 m_3$ and get

$$M_{\nu_4} = \begin{bmatrix} 0 & 0 & -\sqrt{-2m_1 m_2} \\ 0 & \frac{1}{2}(m_1 + m_2 + m_3) & \frac{1}{2}(m_1 + m_2 - m_3) \\ -\sqrt{-2m_1 m_2} & \frac{1}{2}(m_1 + m_2 - m_3) & \frac{1}{2}(m_1 + m_2 + m_3) \end{bmatrix} \quad (39)$$

Eq. (39) is identical to the matrix derived for the hierarchical case by B. R. Desai et. al.[20], who systematically categorize the neutrino mass matrices, consistent with experimental constraints, with two texture zeros. Note that for this model $\langle m \rangle = M_{\nu_{ee}}$ vanishes to order Λ^4 . In order to compare with a recent model for hierarchy generation,[4] we continue with $\rho = -2$, but do not specify κ' in $s_{13} = \kappa'\Lambda^3$. In that case, keeping only the leading order in Λ in each matrix element of M_{ν_6} , we obtain :

$$M_{\nu_6} = m_3 \begin{bmatrix} \Lambda^6(4 + \kappa'^2) & -\Lambda^3(1 + \frac{1}{\sqrt{2}}\kappa') & -\Lambda^3(1 - \frac{1}{\sqrt{2}}\kappa') \\ -\Lambda^3(1 + \frac{1}{\sqrt{2}}\kappa') & \frac{1}{2} & -\frac{1}{2} \\ -\Lambda^3(1 - \frac{1}{\sqrt{2}}\kappa') & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (40)$$

The leading orders of Λ in each matrix element are the orders indicated in the work of Ramond, et al.,[4] (and 'tuned' by Fishbane and Kaus [21]). This model suggests, within a super symmetric extension of the standard model, that the existence of mass hierarchies within fermionic sectors imply at least one additional $U(1)$ family symmetry one of which must be anomalous, with a cancellation of its anomaly through the Green-Schwarz mechanism then implying relations across fermionic sectors. This has the additional property of predicting Λ , which should be the same for all families. However, the data for neutrinos, Δ_{atm} and Δ_{sol} , suggest that $\Lambda \approx 0.4$, while for the other family sectors, we have the traditional $\lambda \approx 0.25$.

4 SUMMARY

We have shown that the assumed hierarchy pattern and the present data imply that a mixing matrix, U , and mass matrix, M_ν may be expressed in terms of powers of the expansion parameter Λ and two parameters ρ and κ of order

one. The parameter ρ expresses our ignorance of the lightest neutrino mass, m_1 , where $m_1 = \rho\Lambda^4 m_3 \approx 1.3\rho \times 10^{-3}eV$ and κ scales s_{13} to the experimental upper limit, $s_{13} = \kappa\Lambda^2 \approx 0.16\kappa$. The simplicity of U and M_ν comes from the observed relationship, $S^2/2 = \Lambda^2 = m_2/m_3$. The expansion parameter, Λ , where $\Lambda^2 = m_2/m_3 = \sqrt{(\Delta_{sol}/\Delta_{atm})} \approx 0.16$ is identical in spirit, though not in value, to the Wolfenstein parameter [16], λ , in the quark V_{CKM} and is measured by solar and atmospheric oscillation experiments. The upper limit for the effective neutrino mass in double β -decay experiments is $4 \times 10^{-3}eV$ if $s_{13} = 0$ and $6 \times 10^{-3}eV$ if s_{13} is maximal.

The models of hierarchical mass generation that we compared to, supersymmetric extension [4] and texture zeroes [4, 5, 19, 20] each demand that $M_{\nu_{ee}}$ be of order Λ^6 or even vanish. This implies $\rho = -2, m_1 \approx -2.6 \times 10^{-3}eV$, in Eq. (30), and thus $\langle m \rangle \approx 10^{-3}eV$ or smaller. Both of these models require the $M_{\nu_{e\mu}}$ and $M_{\nu_{e\tau}}$ terms to be of order Λ^3 or smaller. Therefore s_{13} in Eq. (30) has to be of order Λ^3 , i.e., $s_{13} = \kappa'\Lambda^3 = 0.06\kappa'$, where κ' is of order unity or smaller. More specifically, for a texture zero in $M_{\nu_{e\mu}}$ or $M_{\nu_{e\tau}}$ one must have $\kappa' = \pm\sqrt{2}$ or $s_{13} \approx 0.09$. All these demands are well below the present upper experimental upper limits of $\langle m \rangle$ and s_{13} .

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6 REFERENCES

- [1] H. Fritzsch and Z. Xing, Prog. Part. Nucl. Phys., **45** (2000) 1, and Ref. 51 therein; P. Fishbane and P. Kaus, J. Phys., **G26** (2000) 295.
- [2] E. Ma, arXiv: hep-ph/0208097.
- [3] S. Pascoli and S.T. Petcov, Phys. Lett., **B544** (2002) 239.
- [4] See, for example, F. S. Ling and P. Ramond, Phys. Lett., **B543** (2002) 29; and earlier work, Irges, S. Lavignac and P. Ramond, Phys. Rev., **D58** (1998) 035003.
- [5] S. F. King, arXiv: hep-ph/0208266.

- [6] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys., **28** (1962) 870; Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett., **86** (2001) 5651-5656.
- [7] S. Fukuda et. al., Phys. Rev. Lett., **85** (2000) 3999 and references therein.
- [8] M. Appolonio et al., Phys. Lett., **B 466** (1999) 415 (CHOOZ Collaboration).
- [9] F. Boehm et al., Phys. Rev., **D64** (2001) 112001 (PaloVerde Collaboration).
- [10] G.L. Fogli, et al., arXiv: hep-ph/021247.
- [11] K. Eguchi, et al., Phys. Rev. Lett., **90** (2003) 021802.
- [12] J.N. Bahcall, M.C. Gonzales-Garcia and C. Pena-Garay, arXiv: hep-ph/0212147.
- [13] V. Barger and D. Marfatia, arXiv: hep-ph/0212126.
- [14] S. Pascoli, S.T. Petcov and W. Rodejohann, arXiv: hep-ph/0212113.
- [15] Q. R. Ahmed et al., Phys. Rev. Lett., **89** (2002) 011301, 011302.
- [16] L. Wolfenstein, Phys. Rev. Lett., **51** (1983) 1945.
- [17] G.L. Fogli, et al., arXiv: hep-ph/0208026.
- [18] C. Giunti and M. Tanimoto, Phys. Rev., **D66** (2002) 053013.
- [19] P. Frampton, S. Glashow, and D. Marfatia, Phys. Lett., **B536** (2002) 79.
- [20] B. R. Desai, D. P. Roy, and A. R. Vaucher, arXiv: hep-ph/0209035.
- [21] P. Fishbane and P. Kaus, J. Phys., **G25** (1999) 1629.