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"Optimal Linear Parameter-Varying Control Design for a Pressurized Water Reactor"

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Optimal Linear Parameter-Varying Control Design for a Pressurized Water Reactor

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Abstract

The applicability of employing parameter-dependent control to a nuclear pressurized water reactor is investigated. The synthesis technique produces a controller which achieves specified performance against the worst-case time variation of a measurable parameter which enters the plant in a linear fractional manner. The plant can thus have widely varying dynamics over the operating range. The results indicate this control technique is comparable to linear control when small operating ranges are considered.

1 Introduction

In France and certain other countries the major contribution to electricity production is provided by nuclear power. When this is the case, the nuclear power plant must provide electricity as it is needed, and the plant becomes a time-varying system with dynamics changing slowly as the power changes. Nonetheless, large transients can occur, for example, when the plant shuts down. Most nuclear power plants are pressurized water reactors (PWR). The dynamics of a PWR change enough over its operating range that a linear controller cannot guarantee performance over the entire range, especially when operating conditions change suddenly. Indeed, previous work [BenBod94] showed that a fixed linear controller such as an $\mathcal{H}_\infty$ controller cannot maintain performance in the presence of parametric uncertainty corresponding to the change in the operating conditions of the actual plant.

In designing controllers for plants which operate over a wide dynamic range, a common technique is to schedule various fixed-point designs. Unfortunately, there are no known methods for scheduling such controllers which provide an *a priori* guarantee on the resulting performance or stability of the closed-loop system. Additionally, large and often unacceptable transients can occur when switching between controllers. Recent advances in optimal control theory provide a design technique which avoids these difficulties by producing an optimal parameter-dependent controller; i.e., the controller is already scheduled depending on parameter values which are not known beforehand [Pac94], [ApkGah94]. The controller is optimized to provide performance against the worst-case time variation of the parameters. Such a controller is called a linear parameter-varying (LPV) controller.

A possible approach to control a PWR is to design a parameter-dependent controller with the output power as the parameter. One advantage such a controller would have over a standard gain-scheduled controller is that performance and stability could be guaranteed over the operating range of the plant, and large transients in switching are avoided. An additional advantage of LPV synthesis is that the controller is designed in one step, rather than designing several controllers and then scheduling them. The potential drawback of LPV synthesis is that an LPV optimal controller is optimized against a worst-case time variation of parameters, and this time-variation may be so unrealistic that the controller has no performance. Our goal in this paper is to determine if LPV synthesis can produce controllers which have reasonable performance, and possibly produce a better control design for the PWR.

Section 2 is devoted to the problem statement; in section 3, the model of the PWR as it pertains to LPV design is discussed. Section 4 overviews the synthesis theory, and the structure for controller design on the PWR; the behavior of these controllers is examined in section 5. Our conclusions and some directions for future work end the paper.

2 Problem Statement

The main objective in controlling a PWR is to provide the commanded power while respecting certain physical constraints. Consider the application depicted in Figure 1. The pressurized water in the primary circuit transmits the heat generated by the nuclear reaction to the steam generator. In the steam generator, water of the secondary circuit turns into hot steam, which drives a turbo-alternator to generate electricity. The rate of the reaction is regulated by the control rods. The rods capture neutrons, slowing down the nuclear reaction; withdrawing them increases the reaction. The PWR has two independent sets of rods which are used as controls.

The PWR has an inner control loop which holds the pressure in the primary circuit constant. Thus for a steam flow increase in the secondary circuit, the temperature in the primary circuit will decrease. From a control standpoint, the required power corresponds to a specific steam flow that may be viewed as a measurable disturbance. Hence, one natural control objective is to track a temperature reference derived from the steam flow. Because of the way in which the control rods enter the reactor, the rate of reaction is always higher at the bottom of the reactor. The *axial offset* is defined as the difference in power generated between the top and bottom of the PWR. Safety specifications require minimizing the axial offset; this also increases the lifetime of the fuel and reduces op-

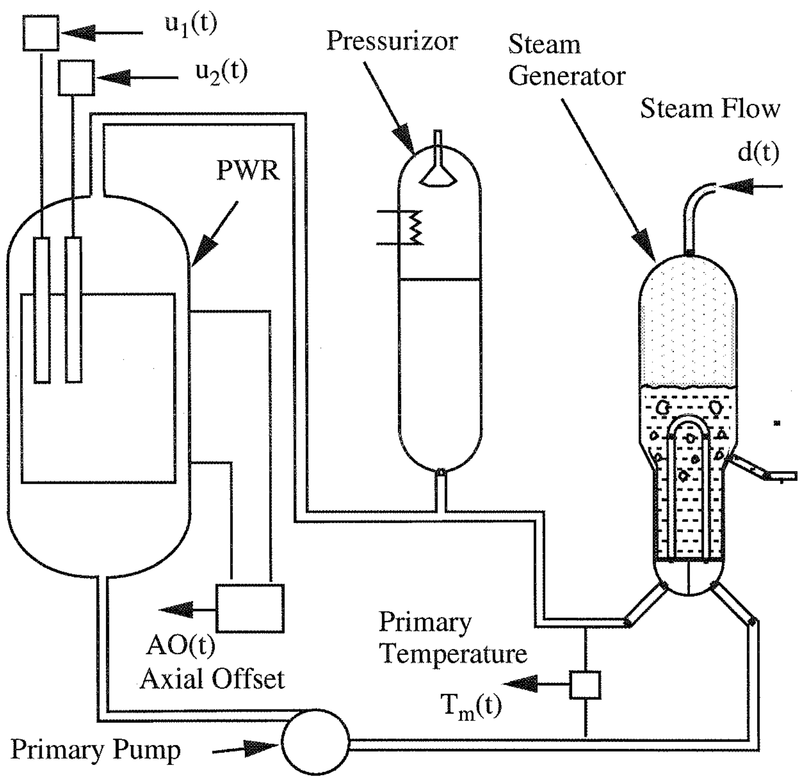

Figure 1: Primary Circuit and Steam Generator.

erating costs. To achieve such objectives two control inputs are available, the rates of motion of the control rods, denoted u_1 and u_2. The positions of the control rods are denoted v_1 and v_2, respectively. The positions are, of course, measurable. Due to the physics of the reactor, u_2 has more authority than u_1 at low power, and using it results in a smaller axial offset. At high power, however, u_2 has almost no authority, so all control must come from u_1.

Previously, the authors designed an $\mathcal{H}_\infty$ controller using a linear model of the plant identified at 50% of the nominal power, P_n [BenBod94]. In addition to actuator dynamics and modelling error, the uncertainty description covered the variations in dynamics of the plant depending on power with time-invariant uncertainty. This controller performs satisfactorily up to $0.9P_n$, but not at $0.99P_n$. Figure 2 shows the step-responses of the closed-loop systems. T_m is the mean temperature of the plant, and T_{ref} (dashed lines) is the reference signal. P_I is the primary power, and d (dashed lines) is the steam flow input. Input-output signals corresponding to the nominal model are plotted in solid lines while those resulting from the perturbed models corresponding to $0.9P_n$ and $0.99P_n$ are displayed in dark and light shaded lines, respectively. The control signals are plotted in dark (u_1) and light (u_2) shaded lines. The above suggests that a linear controller is not enough to ensure performance over the entire operating range.

3 Modeling

The synthesis technique for designing a parameter-dependent controller requires a model which has accurate dynamics over the operating range of interest. A general time-varying system is shown pictorially in Figure 3, where $x(k), e(k), y(k), d(k)$, and $u(k)$ are the state, error, measurement, disturbance and input vectors, respectively. We assume the time-variation of the plant can be represented as a linear-fractional transformation (LFT) of a parameter and a constant matrix. Thus $P(k)$ is given by

$$P(k) = P_{22} + P_{21}\Delta(k)\left(I - P_{11}\Delta(k)\right)^{-1} P_{12} \tag{1}$$

where

$$\Delta(k) = \begin{bmatrix} \delta_1(k) I_{n_1} & & \\ & \ddots & \\ & & \delta_m(k) I_{n_m} \end{bmatrix} \tag{2}$$

with $|\delta_i(k)| \leq 1$ for all k. The δ_i are assumed to be measurable. Any rational time-varying system can be represented in this framework, and many others can be arbitrarily closely approximated. This type of system is known as a parameter-dependent LFT system.

Previous work on the PWR ([BenIrv93]) identified three sixth-order linear models with uncertainty which characterize the behavior around $0.5P_n$, $0.9P_n$, and $0.99P_n$. These models were identified using a realistic non-linear simulator of the PWR developed by Electricité de France (EDF). The uncertainty description describes unmodelled dynamics

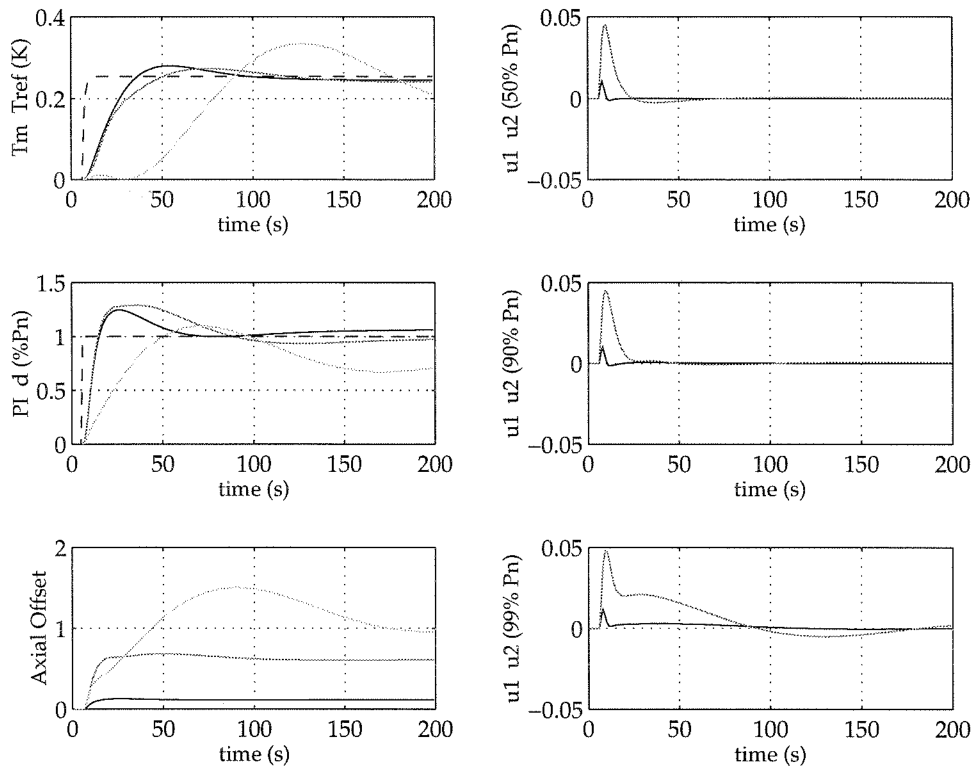

Figure 2: Closed-loop responses of an $\mathcal{H}_\infty$ controller with linearized models of the PWR at $0.5P_n$ (solid), $0.9P_n$ (dark), and $0.99P_n$ (light).

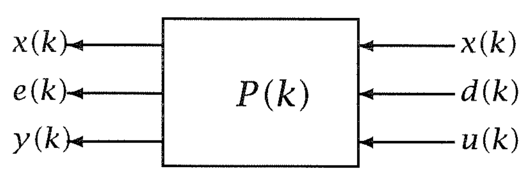

Figure 3: Time-Varying System.

and actuator uncertainty. The models can be reduced quite accurately to first-order models using frequency-weighted balanced truncation.

To derive the parameter dependence, each term of the three first-order models is compared; those which vary are fitted with a rational function of δ, $-1 \le \delta \le 1$, using a least-squares technique. For the PWR, a first order LFT of the form $d + c\delta(1 - a\delta)^{-1}b$ fit the parameters extremely well, as shown in Figure 4. In this figure, $0.5P_n$ corresponds to $\delta = -1$, $0.9P_n$ corresponds to $\delta = 0.6$, and $0.99P_n$ to $\delta = 0.998$ (these are the asterisks in the figure). The resulting model with δ-dependence, $P(\delta)$, becomes

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|ccc} a(\delta) & b_1 & b_{v_1}(\delta) & b_{v_2}(\delta) \\ \hline c_1 & d_{11} & d_{12} & d_{Tm_2}(\delta) \\ c_2 & d_{12} & d_{22} & \kappa b_{v_2}(\delta) \\ c_{AO}(\delta) & 0 & d_{AO_1}(\delta) & d_{33} \end{array}\right] \tag{3}$$

The inputs for this model are the steam disturbance d, v_1, and v_2; the outputs are the mean temperature T_m, the power P, and the axial offset AO, respectively (see Figure 1).

From Figure 4, notice the time constant $a(\delta)$ is inversely proportional to the operating power, and changes by a factor of 2 over the operating range. Also, the variation of b_{v_2} and d_{P_2} differs only by a constant, κ, which is used to reduce the size of the final Δ-block. More importantly, the effectiveness of u_2 decreases as the power increases, and is almost zero at full power. The gain in the axial offset channel increases as power increases, making it more difficult to control at high power. In particular, the effect of u_1 on the axial offset (d_{AO_1}) increases, while the effect of u_2 is decreasing. This makes it practically impossible to require any performance on axial offset at high power.

4 Synthesis

In this section a brief overview of the synthesis theory is presented. Since our intent is to convey only a general understanding of the theory, we will be somewhat loose in our notation. A complete and rigorous explanation of the synthesis technique can be found in [Pac94].

From the previous section, the plant has the structure given in Figure 5. The controller we will design for this plant will also be parameter-dependent, depending on the same measurable δ_i's as the plant; it will thus have the form shown in Figure 6. P can be augmented to collect all the time-varying parameters and states together; K can then be treated as a simple matrix. This is depicted in Figure 7, where R is the augmented form of P, and K is a matrix. The problem thus appears as a robust control problem with a special structure on the plant and parameters. The design objective is to find a controller K such that the interconnection is stable and the $\ell_2 \to \ell_2$ induced norm from d to e is small for all allowable parameter variations $\Delta(k)$ (see Equation 2). This is simply a small-gain condition. Since the small-gain theorem can be quite conservative, we can reduce the conservatism by introducing scaling matrices from a set $\mathcal{D}$ which commutes with the set of parameter variations.

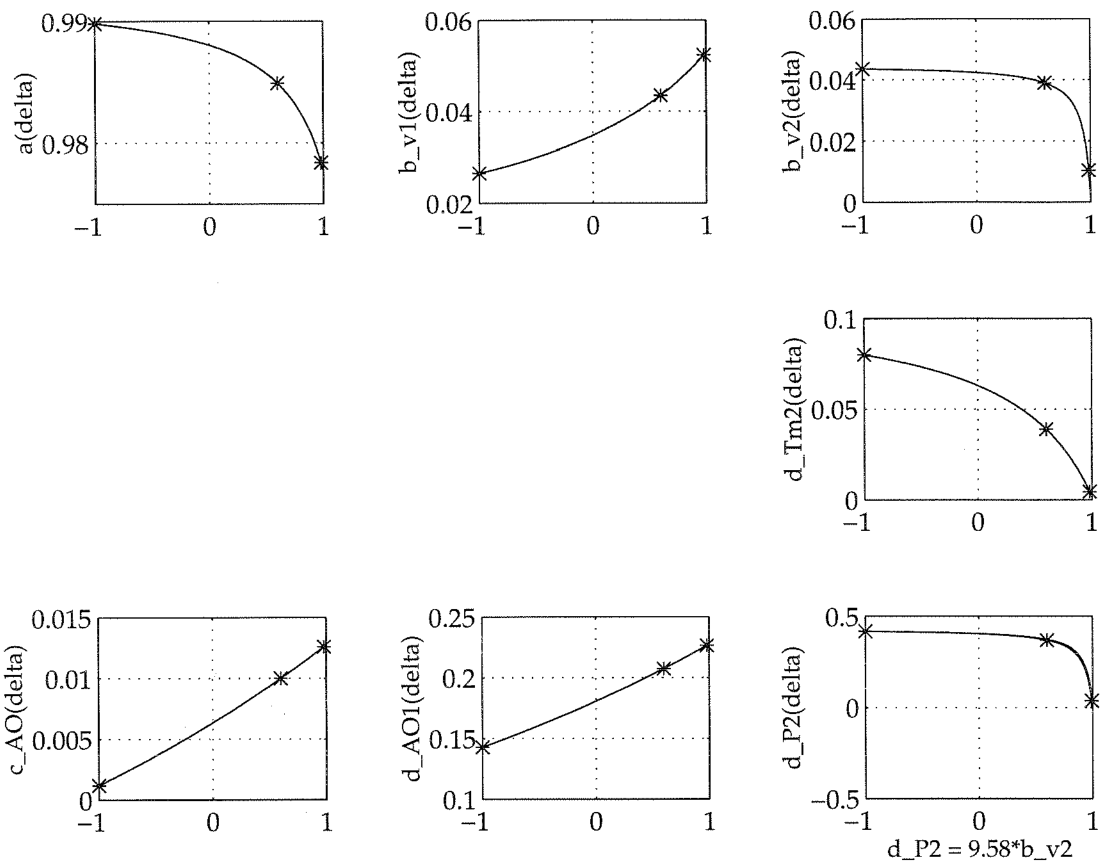

Figure 4: Parameter Variations versus δ for the model of Equation 3. A '*' shows an actual value, and the line shows the LFT fit.

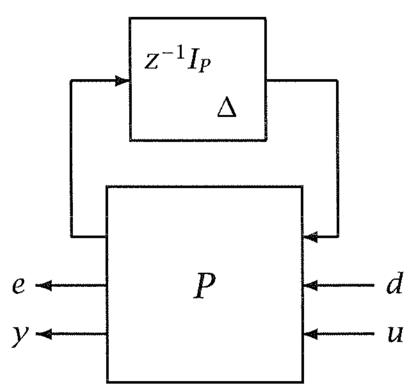

Figure 5: Parameter-Dependent Plant. The $z^{-1}I_P$ term represents the states of P, and the Δ represents the time variation of Equation 2.

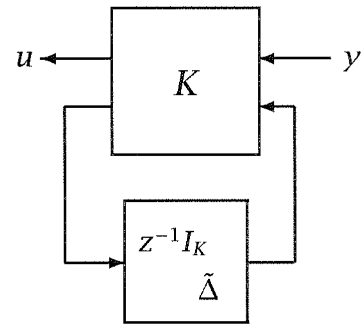

Figure 6: Parameter-Dependent Controller; $z^{-1}I_K$ represents the states of the controller and $\tilde{\Delta}$ the time variations.

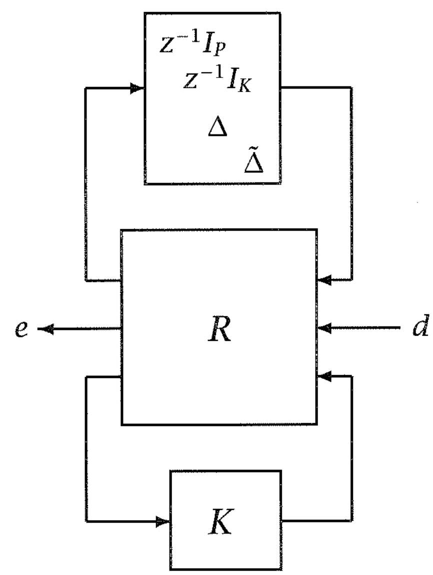

Figure 7: Parameter-Dependent Closed-Loop System.

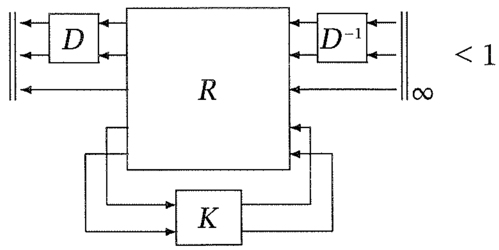

Figure 8: Diagram of Theorem 1.

The resulting condition is then the state-space upper bound (SSUB) of [PacDoy93]. Introducing the notation $\mathcal{F}_l(R,Q) = R_{11} + R_{12}Q(I - R_{22}Q)^{-1}R_{21}$ for a block partitioned 2×2 matrix R with $\det(I - R_{22}Q) \neq 0$, this condition becomes (compare Lemma 3.1 of [Pac94] and Theorem 10.4 of [PacDoy93]):

Theorem 1 *Let R be given as above, along with an uncertainty structure Δ. If there is a $D \in \mathcal{D}$ and a stabilizing, finite-dimensional, time-invariant K such that*

$$\left\| \begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix} \mathcal{F}_l(R,K) \begin{bmatrix} D^{-1} & 0 \\ 0 & I \end{bmatrix} \right\|_\infty < 1 \tag{4}$$

then there is a γ, $0 \leq \gamma < 1$, such that for all parameter sequences $\delta_i(k)$ with $\|\delta_i\|_\infty \leq 1$, the system in Figure 7 is internally exponentially stable, and for zero initial conditions, if $d \in \ell_2$, then $\|e\|_2 \leq \gamma \|d\|_2$.

Pictorially, this theorem is shown in Figure 8. The important fact about Theorem 1 is that the synthesis of D and K to meet the objective can be cast as a computationally tractable convex optimization problem; nevertheless, to show this is beyond the scope of this paper: it can be found in [Pac94]. The synthesis procedure is a γ-iteration, just as $\mathcal{H}_\infty$ synthesis is.

A few points are important in understanding the ramifications of employing the SSUB. Most importantly, this technique designs a controller optimal with respect to a time-varying perturbation with memory (the sequence $\Delta(k)$ of Equation 2). The relationship between such an operator and a parameter useful in gain-scheduling is tenuous, at best. Depending on the problem, this technique could conceivably yield controllers so conservative as to have no performance. Nonetheless, if a controller with acceptable performance can be designed with this technique, then it will achieve performance for all variations of the operating point. As a corollary of this, a time-varying operator with memory in general does not have a spectrum, so there is no way to "filter" it to achieve a closer relationship to an operating parameter. Moreover, it is interesting to contrast this technique with μ-synthesis, where instead of the SSUB, the frequency-domain upper bound is usually employed; this difference reflects the different assumptions about the type of perturbations.

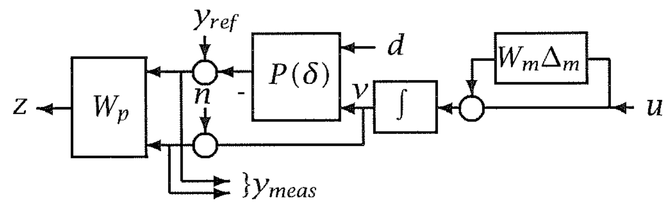

Figure 9: Synthesis structure for the PWR.

Controller Design

Once the parameterized model $P(\delta)$ is obtained, the controller design becomes similar to an $\mathcal{H}_\infty$ design. Both LPV and $\mathcal{H}_\infty$ synthesis produce controllers which reject disturbances. A tracking problem, such as the PWR, can be cast in this framework by rejecting the low frequency components of the error between the plant output and the reference. The tracking will become faster as higher frequencies are rejected. The synthesis structure used is shown in Figure 9, with uncertainty and performance weights included.

W_m is a multiplicative uncertainty which covers unmodelled dynamics and modelling errors as the plant changes operating point. The performance weight W_p is a diagonal matrix

$$W_p = \begin{pmatrix} W_T & & & \\ & W_P & & \\ & & W_{AO} & \\ & & & W_{pos} \end{pmatrix} \tag{5}$$

which weights the performance on temperature, power, axial offset, and vertical position of the control rods. To insure low steady-state error in tracking, W_T resembles an integrator. These weights can depend on δ as well, so, ideally, high performance could be required at one operating point and lower performance at another. Nevertheless, since a δ in the weight adds another time-varying perturbation with memory, it may be that performance is degraded instead.

Three controllers were designed. The first is called "LPV #1" and is an LPV controller with weights similar to the weights employed for the $\mathcal{H}_\infty$ controllers of Figure 2. The second is called "LPV #2" and uses the same weights as LPV #1, except W_{AO} was allowed to depend on δ. W_{AO} was designed to require high performance in axial offset for $\delta \approx -1$, but less performance as $\delta \to 1$.

5 Results

The LPV controllers will be compared with $\mathcal{H}_\infty$ controllers designed for the plant models at $0.5P_n$ and $0.99P_n$. The controller designed at $0.5P_n$ will be called "H50" and the one at

$0.99P_n$ "H99."

Figure 10 shows the step responses of the closed-loop systems consisting of each of the controllers and a linearization of the plant at $0.99P_n$. Step responses are shown because we are interested in the low frequency rejection properties of the closed-loop system. In the first column of plots, the dashed lines are the reference signals; the light shaded lines are the responses with the first LPV controller; the solid lines are with the second LPV controller; the dark shaded lines are with H99. The second column of plots shows u_1 and u_2 for each of the controllers; u_1 is the solid line and u_2 the shaded one. Figure 11 is identical to Figure 10, except the responses are with respect to a linearization of the plant at $0.5P_n$.

Because the control rods are almost withdrawn from the reactor at high power, the plant is more difficult to control. Referring to Figure 10, LPV #2 tracks the temperature with less overshoot than either H99 or LPV #1. There is less use of the control rods in LPV #2 as well. The axial offset for all three controllers is about the same. LPV #2 is the slowest at tracking the temperature reference, while LPV #1 is the fastest, at the expense of a large overshoot in the primary power and axial offset. At this operating point, we consider LPV #2 the best of these controllers.

Some of this behavior is preserved in Figure 11, but the model is quite different here. Again, LPV #1 is the fastest and LPV #2 the slowest. The major difference is that u_2 has more control authority at this power, so controllers do better to use it more than u_1, since this results in lower axial offset. H50 does use it more, and the axial offset is considerably lower. At this operating point, we consider H50 the best controller.

At low power, u_2 is the dominant control, but as the power increases u_1 should be used more and more to better meet the control objectives. The LPV controllers do not change strategy between these operating points. Notice that the control plots for LPV #2 are almost identical, up to a scale change in magnitude. This is probably a result of the worst-case nature of LPV controllers. Since achieving worst-case performance does not require a change of strategy, and may in fact forbid one, the controllers do not change their use of the inputs.

6 Conclusion

In this paper we investigated using an LPV controller to control a PWR. Our results indicate that in a setting where only small changes in operating conditions are made, LPV control is comparable in performance to linear control, and does better at high power. This is an interesting result because the dynamics of the PWR change slowly, but the LPV synthesis is a worst-case time-varying design. The prejudice against applying this technique is that worst-case time variations are "fast", and thus controllers with low performance would result.

An advantage of the LPV designs over conventional methods of gain scheduling is they design a controller of fixed order that works reasonably for all plants in the operating regime. Their major drawback for the PWR is they do not switch control strategy

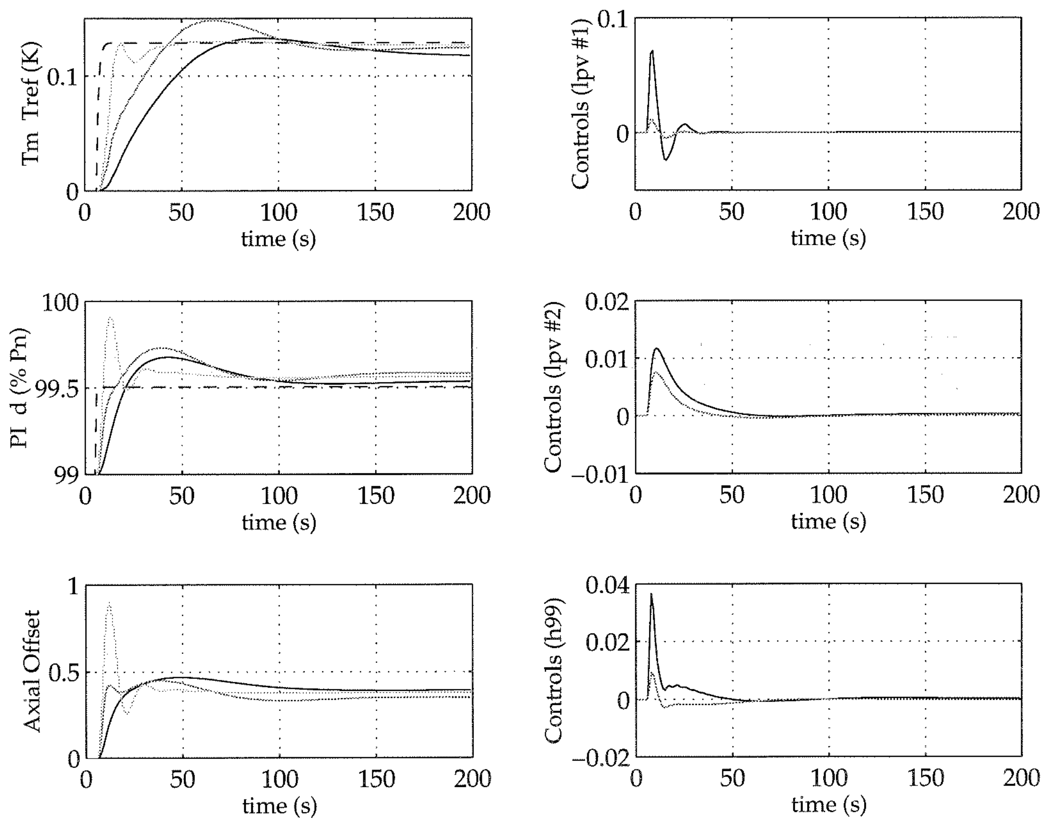

Figure 10: Comparison of Three Controllers at $0.99P_n$.

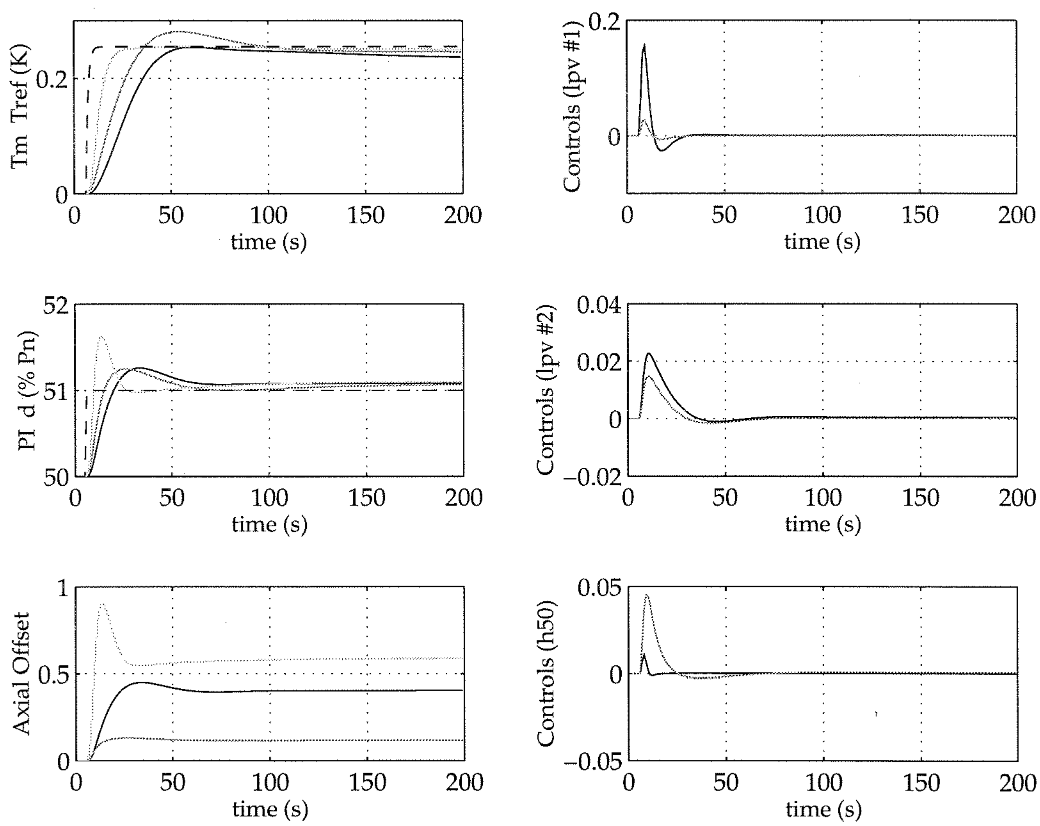

Figure 11: Comparison of Three Controllers at $0.5P_n$.

between low and high power. This is perhaps attributable to the worst-case nature of the designs, but methods are now being investigated which attempt to alleviate this problem without losing the performance guarantees. Another result of our work is that parameter variations can be placed in the weights with beneficial effect; we are still exploring this.

The most important continuation of this work is to test the LPV controllers and various gain-scheduled controllers on a good nonlinear simulation of the PWR. The questions of transient behavior can then be properly addressed. An excellent nonlinear simulation of the PWR exists at EDF, but was not available for use at the time of this writing.

A further question to explore is whether the size of the time-varying parameter block (in the case of $P(\delta)$, 6) is a significant factor limiting the performance. $P(\delta)$ could be reduced and controllers designed for the reduced-order plant. Do they work and achieve significantly better performance? One way of checking whether the plant is reducible in the size of the Δ-block is to treat the state as an input and output, and the Δ-block as the state, then look at the Hankel singular values of the system. For the PWR they are: 2.5448, 0.1031, 0.0325, 0.0187, 0.0152, and 0.0035. This indicates the size of the Δ-block could probably be reduced by at least one. A more sophisticated method is found in [Beck94], where the state is included in the reduction, i.e., true multivariable model reduction. This technique provides reduction of the uncertainty description in a manner similar to balanced truncation model reduction. Preliminary results with this also indicate a reduction of one is quite reasonable.

Acknowledgements

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