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SHOCK PROPAGATION IN POLYDISPERSE BUBBLY FLOWS

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ABSTRACT

The effect of distributed bubble size on shock propagation in homogeneous bubbly liquids is computed using a continuum two-phase model. An ensemble-averaging technique is employed to derive the statistically averaged equations and a finite-volume method is used to solve the model equations. The bubble dynamics are incorporated using a Rayleigh-Plesset-type equation which includes the effects of heat transfer, liquid viscosity and compressibility. For the case of monodisperse bubbles, it is known that relaxation oscillations occur behind the shock due to the bubble dynamics. The present computations for the case of polydisperse bubbles show that bubble size distributions lead to additional damping of the shock dynamics. If the distribution is sufficiently broad, the statistical effect dominates over the physical damping associated with the single bubble dynamics. This smooths out the oscillatory shock structure.

INTRODUCTION

A fundamental understanding of the dynamics of bubbly flows is of great importance in engineering (e.g. underwater explosions, turbomachinery, hydraulic equipment). Shock dynamics in bubbly flows have been extensively studied for many years [1–8]. Most of the previous studies have focused on shock propagation in monodisperse bubbly liquids (i.e. all the bubbles initially have the same size.). However, in flows of practical interest, the nuclei size is broadly distributed; thus the size distributions need to be included for more realistic modeling. We treat the liquid and disperse phases as a continuum mixture and solve statistically averaged equations to determine the shock structure.

First, we present the model assumptions, closure conditions and bubble-dynamic modeling. Then, we solve one-dimensional shock propagation in dilute bubbly liquids with bubble size distributions and describe the effects of the size distributions upon the shock dynamics.

METHOD AND RESULTS

We use an ensemble-averaging technique [9,10] to derive the averaged governing equations. This model assumes that (a) the bubbles are spherical, (b) mutual interactions among the bubbles are negligible, (c) wavelengths in the mixture are large compared to the bubble radius and (d) the bubbles advect with the ambient liquid velocity (no slip). Assumption (a) implies that fission and coalescence are not permitted, so that the bubble number is conserved in time. Assumptions (b) to (d) are valid in the dilute limit (low void fraction, $\alpha \to 0$), which is used for the model closure. Under these assumptions, we write the conservation equations as

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial \rho_m u_m}{\partial x} = 0, \tag{1}$$

$$\frac{\partial \rho_m u_m}{\partial t} + \frac{\partial}{\partial x}\left(\rho_m u_m^2 + p_l\right) = \frac{\partial \tilde{p}}{\partial x}, \tag{2}$$

$$\frac{\partial n}{\partial t} + \frac{\partial n u_m}{\partial x} = 0, \tag{3}$$

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where ρ_m is mixture density ($\approx (1-\alpha)\rho_l$, ρ_l is liquid density), u_m is velocity, p_l is liquid pressure described by the Tait equation of state, n is bubble number per unit volume of the mixture, and $\tilde{p}$ represents pressure fluctuations due to the phase interactions [11]:

$$\tilde{p} = \alpha\left(p_l - \frac{\overline{R^3 p_{bw}}}{\overline{R^3}} - \rho_m \frac{\overline{R^3\dot{R}^2}}{\overline{R^3}}\right). \tag{4}$$

Here, R is bubble radius and p_{bw} is bubble wall pressure. The bars denote moments with respect to the (normalized) bubble size distributions, $f(R_0)$, where R_0 is equilibrium radius.

The Gilmore equation [12] is used to evaluate the spherical bubble dynamics. Heat conduction and vapor flux at the bubble wall are estimated using the reduced-order model of Preston *et. al.* [13]. The bubble-dynamic equations can be written in a conservation form using equation (3):

$$\frac{\partial n\varphi}{\partial t} + \frac{\partial n\varphi u_m}{\partial x} = n\dot{\varphi}, \tag{5}$$

where φ represents bubble-dynamic variables (e.g. bubble radius, bubble wall velocity).

The system of equations thus consists of the ensemble-avereaged equations (1) to (3) and the bubble-dynamic equations (5). A third-order TVD Runge-Kutta scheme [14] marches the system forward in time. The spatial descretization is handled by a fifth-order finite-volume WENO scheme [15] coupled with an HLLC approximate Riemann solver [16]. Linear wave propagation is computed using this method and the dispersion relation agrees well with the theory of Commander *et. al.* [17].

Figure 1 shows liquid pressure distribution for steady shock propagation in an air/water mixture. The initial bubble size is assumed to be lognormally distributed around $R_0^{\text{ref}} = 10\,\mu$m with standard deviation σ. We examine the effects of bubble size distributions by changing the value of σ. Note that other distribution functions can be used in this model. At $t = 0$, Hugoniot relations for a steady shock corresponding to $p_l/p_{l0} = 2$ are imposed by a diaphragm at $x = 0$. For the monodisperse case ($\sigma = 0$), the relaxation oscillations appear behind the leading shock as expected. It can been seen that in the polydisperse case, the bubble size distributions damp the relaxation oscillations. If the distribution is sufficiently broad ($\sigma = 0.7$), the oscillatory structure is completely smoothed out. This new "apparent damping" mechanism results from phase cancellations amongst the different-sized bubbles [18]. We quantify the "statistical damping" effect by comparing to the single-bubble-dynamic damping and show that the bubble statistics play a major role in the shock dynamics for the cases with a broad size distribution. Additional parameter studies will include shock strength and initial void fraction.

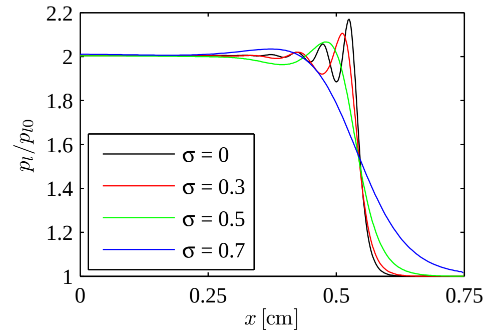

Figure 1. Liquid pressure distribution at $t = 26.9\,\mu$s for shock propagation in an air/water mixture of $R_0^{\text{ref}} = 10\,\mu$m and $\alpha = 0.5\,\%$ at STP.

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