Supplementary Material for
"Gapless excitations in strongly fluctuating superconducting wires"

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MICROSCOPIC PHASE ACTION

In order to describe correlations of the order parameter in a superconducting wire we examine its microscopic action obtained from the BCS Hamiltonian by a Hubbard-Stratonovich transformation followed by an expansion around the saddle point [1, 2]. In the low temperature limit, this yields [1, 2]:

$$S \;=\; \nu_0 A\Delta_0^2 \int_0^L dx \int_0^{1/T} d\tau \left\{ \frac{\rho^2}{2}\left[\ln\left(\rho^2\right)-1\right] + 2\xi_0^2\rho^2\left[\phi'^2 + \frac{1}{v_\phi^2}\dot{\phi}^2\right] + \xi_0^2\left[\rho'^2 + \frac{1}{v_\rho^2}\dot{\rho}^2\right]\right\},$$

where L and A are the wire's length and cross section, respectively, $\xi_0^2 = \pi D/8\Delta_0$, $v_\rho = \sqrt{(3\pi/2)D\Delta_0}$ the amplitude velocity, $v_\phi = \sqrt{\pi D\Delta_0(2AV_c\nu_0+1)} \propto v_\rho\sqrt{N_\perp}$ the phase velocity, V_c the Fourier transform of the short range Coulomb interaction, $N_\perp = p_F^2 A/\pi^2$ is the number of one dimensional channels in the wire, ν_0 the density of states, D the electronic diffusion constant in the normal state, and the SC order parameter is parameterized as $\Delta = \Delta_0\rho e^{i\phi}$, with Δ_0, the mean field solution. Rescaling the imaginary time by $y = v_\rho\tau$, the low energy excitations of the system are phase fluctuations whose action follow:

$$S[\phi] \;=\; K/2\int dxdy\left\{(\partial_x\phi)^2 + (\partial_y\phi)^2/N_\perp\right\}. \tag{1}$$

where the phase stiffness is

$$K \;=\; \frac{4\nu_0 A\Delta_0^2\xi_0^2}{v_\rho} \approx \frac{R_Q}{2R_\xi}. \tag{2}$$

The system described by this model undergoes a Kosterlitz Thouless phase transition between a quasi-ordered phase (superconductor) and a disordered phase where phase slip pairs unbind [3]. Correlations of the order parameter in the disordered phase decay exponentially:

$$\langle\Delta(x,\tau)\Delta^\dagger(0,0)\rangle \;=\; \Delta_0^2 e^{-x/\xi_{KT}}e^{-\tau/\tau_{KT}}, \tag{3}$$

over a typical length ξ_{KT}, and time τ_{KT}. This corresponds to

$$\langle\Delta\Delta^\dagger\rangle_{q,\Omega} \;=\; \frac{\Delta_0^2\xi_{KT}\tau_{KT}}{(1+q^2\xi_{KT}^2)(1+\Omega^2\tau_{KT}^2)}. \tag{4}$$

LEADING ORDER CORRECTION TO THE TUNNELING DENSITY OF STATES OF A FLUCTUATING SUPERCONDUCTOR

The tDOS is given by

$$\nu_\epsilon = -\frac{1}{\pi}\mathrm{Im}G^R(r,r,\epsilon) = -\frac{1}{\pi}\mathrm{Im}\int\frac{d^3p}{(2\pi)^3}G^R(p,\epsilon), \tag{5}$$

where $G^R(r,r,\epsilon)$ is the retarded Green's function which can be expressed to second order in the pairing amplitude:

$$G(p,\omega_n) = G_0(p,\omega_n) + T\sum_{q,\Omega} G_0(p,\omega_n)\Lambda(q,\omega_n,\omega_n+\Omega)G_0(p+q,\omega_n+\Omega)\Lambda(q,\omega_n+\Omega,\omega_n)G_0(p,\omega_n)\langle\Delta\Delta^\dagger\rangle_{q,\Omega}. \quad (6)$$

Here:

$$\begin{aligned} G_0(k+q,\omega)^{-1} &= i(\omega) + \frac{i}{2\tau}\text{sign}(\omega) - \xi \\ \Lambda(\omega,\omega+\Omega,q) &= \frac{1}{2\tau}\frac{\Theta(\omega(\omega+\Omega))}{|2\omega+\Omega| + Dq^2 + 1/\tau_\phi}, \end{aligned}$$

and correlations of the order parameter are given by Eq. (4). The density of states is then given by $\delta\nu(\epsilon) = \frac{\nu(\epsilon)-\nu_0}{\nu_0} = -\frac{1}{\pi}\text{Im}\ I^R(\epsilon)$, where $I^R(\epsilon) = I(i\omega_n \to \epsilon + i\delta)$ is the analytic continuation of

$$I(\omega_n) = 2\pi i\,\text{sign}(\omega_n)T\sum_{q,\Omega}\frac{\Theta(\omega_n(\omega_n+\Omega))\langle\Delta\Delta^\dagger\rangle_{q,\Omega}}{(|2\omega_n+\Omega| + Dq^2 + \tau_\phi^{-1})^2}. \quad (7)$$

Using Eq. (4) to describe the phase fluctuations in a phase-slip proliferated wire, in the low energy limit $\tau_\phi \ll \tau_{KT}$ we may approximate Eq. (7) as

$$\begin{aligned} I(\omega_n) &\approx 2\pi i\,\text{sign}(\omega_n)\frac{\Delta_0^2}{(|2\omega_n| + \tau_\phi^{-1})^2}T\sum_{\Omega}\frac{\Theta(\omega_n(\omega_n+\Omega))\tau_{KT}}{1+\Omega^2\tau_{KT}^2}\int\frac{dq}{2\pi}\frac{\xi_{KT}}{1+q^2\xi_{KT}^2} \\ &= \frac{\pi i\,\text{sign}(\omega_n)\Delta_0^2}{(|2\omega_n| + \tau_\phi^{-1})^2}\left\{\frac{i}{4\pi}\left[\Psi\left(\frac{1}{2}+\frac{\omega_n}{2\pi T}+\frac{i}{2\pi T\tau_{KT}}\right) - \Psi\left(\frac{1}{2}+\frac{\omega_n}{2\pi T}-\frac{i}{2\pi T\tau_{KT}}\right)\right] + \frac{1}{2}\coth\frac{1}{2T\tau_{KT}}\right\}, \quad (8) \end{aligned}$$

where $\Psi(z)$ is the digamma function.

LEADING ORDER CORRECTION TO THE SELF ENERGY

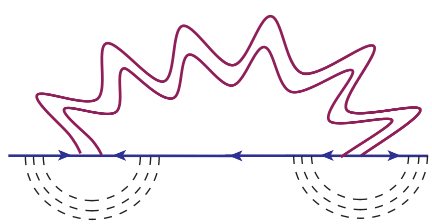

FIG. 1: The leading order correction to the self energy, given by Eq. (9). The solid line is the bare electronic Green's function, G_0, the double wavy is the renormalized pairing interaction, $\langle\Delta\Delta^\dagger\rangle$, and the dashed lines are the impurity scattering.

The leading order correction to the self energy, shown in Fig. 1 is given by $G^{-1} = G_0^{-1} - \Sigma$ with:

$$\Sigma = \sum_q T\sum_\Omega \bar{G}(k+q,\omega+\Omega)\langle\Delta\Delta^\dagger\rangle_{q,\Omega}\Lambda^2(\omega,\omega+\Omega,q). \quad (9)$$

The integral over fermionic momentum is dominated by $\xi \approx 1/\tau$. Since, $\omega\tau, \Omega\tau, Dq^2\tau \ll 1$, we can approximate $\bar{G}(k+q,\omega+\Omega) \approx \bar{G}(k,\omega)$. This gives

$$\begin{aligned} \Sigma &\approx \bar{G}(k,\omega)\sum_q T\sum_\Omega\frac{\Theta(\omega(\omega+\Omega))}{4\tau^2(|2\omega+\Omega| + Dq^2 + 1/\tau_\phi)^2}\langle\Delta\Delta^\dagger\rangle_{q,\Omega} \\ &\equiv \bar{G}(k,\omega)A(\omega). \quad (10) \end{aligned}$$

Using this expression for the self energy we can write the Green's function as:

$$G(k,\omega)^{-1} = i(\omega) + \frac{i}{2\tau}\mathrm{sign}(\omega) - \xi_k - \Sigma(\omega) \tag{11}$$

$$= i\tilde{\omega} - \xi - \frac{1}{i\tilde{\omega}+\xi}A(\omega), \tag{12}$$

where $\tilde{\omega} = \omega + \frac{1}{2\tau}\mathrm{sign}(\omega)$. The density of states is given by:

$$\nu(i\omega) = -\frac{1}{\pi}\int dk G(k,\omega) = \frac{1}{\pi}\nu_0 \int d\xi \frac{i\tilde{\omega}+\xi}{\tilde{\omega}^2+\xi^2+A(\omega)}$$

$$= \nu_0 \frac{i\tilde{\omega}}{\sqrt{\tilde{\omega}^2 + A(\omega)}} \tag{13}$$

where the odd integral over ξ vanishes. In the limit of $\omega\tau \ll 1$ we have:

$$\nu(i\omega) = \nu_0 \frac{i\mathrm{sign}(\omega)}{\sqrt{1+4\tau^2 A(\omega)}} \approx \nu_0 \frac{i\mathrm{sign}(\omega)}{\sqrt{4\tau^2 A(\omega)}} \tag{14}$$

where the last approximation is valid beyond the perturbative limit where $4\tau^2 A(\omega) \gg 1$.

In order to evaluate $4\tau^2 A(\omega)$, we note that $4\tau^2 A(\omega) = \frac{I(\omega)}{2\pi i \mathrm{sign}(\omega)}$ where $I(\omega)$ is given by Eq. (7). Using Eq. (8) in the limit $T\tau_{KT} \ll 1$ we find:

$$4\tau^2 A(\omega) = \frac{\Delta_0^2}{2}\frac{1}{(|2\omega| + 1/\tau_\phi)^2}\left\{\frac{i}{4\pi}\left[\Psi(\frac{1}{2}+\frac{\omega}{2\pi T}+\frac{i}{2\pi T\tau_{KT}}) - \Psi(\frac{1}{2}+\frac{\omega}{2\pi T}-\frac{i}{2\pi T\tau_{KT}})\right] + \frac{1}{2}\coth(\frac{1}{2T\tau_{KT}})\right\}$$

$$= \frac{\Delta_0^2}{2}\frac{1}{(|2\omega| + 1/\tau_\phi)^2}\left\{\frac{i}{4\pi}\left[i\pi + 2i\pi T\tau_{KT} - 2i\omega\tau_{KT}\right] + 1/2\right\} \tag{15}$$

Here we have assumed $\omega \sim T \ll 1/\tau_{KT}$. Performing the analytic continuation $i\omega \to \epsilon + i\delta$ we find

$$4\tau^2 A(i\omega \to \epsilon + i\delta) = \frac{\Delta_0^2}{2}\frac{1}{(-2i\epsilon + 1/\tau_\phi)^2}\left\{\frac{i}{4\pi}\left[i\pi + 2i\pi T\tau_{KT} - 2\epsilon\tau_{KT}\right] + 1/2\right\}$$

$$= \frac{\Delta_0^2}{2}\frac{1}{(-2i\epsilon + 1/\tau_\phi)^2}\left(\frac{1}{4} - \frac{i}{2\pi}\epsilon\tau_{KT} - \frac{1}{2}T\tau_{KT}\right) \tag{16}$$

The density of states is given by

$$\nu(\epsilon) = \Im\nu(i\omega \to \epsilon + i\delta) = \Im\left[i\nu_0 \frac{2}{\Delta_0}\frac{(-2i\epsilon + 1/\tau_\phi)}{\sqrt{1/2 - \frac{i}{\pi}\epsilon\tau_{KT} - T\tau_{KT}}}\right]. \tag{17}$$

In the low temperature limit $T\tau_{KT}, T\tau_\phi \ll 1$, we can replace $\nu(T) = -\int d\epsilon \nu(\epsilon)\frac{df}{d\epsilon} \approx \nu(\epsilon = 0, T)$, leading to:

$$\frac{\nu(T)}{\nu_0} = \frac{2\sqrt{2}}{\Delta_0 \tau_\phi(T)}. \tag{18}$$

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