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Instability in magnetic materials with dynamical axion field

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It has been pointed out that the axion electrodynamics exhibits instability in the presence of a background electric field. We show that the instability leads to a complete screening of an applied electric field above a certain critical value and the excess energy is converted into a magnetic field. We clarify the physical origin of the screening effect and discuss its possible experimental realization in magnetic materials where magnetic fluctuations play the role of the dynamical axion field.

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Introduction.— The electrodynamics is a $U(1)$ gauge theory usually defined by the Maxwell action with the Lagrangian density

$$\mathcal{L}_{\rm EM} = \frac{1}{8\pi}(\epsilon \vec{E}^2 - \frac{1}{\mu}\vec{B}^2), \tag{1}$$

where $\vec{E}$ and $\vec{B}$ represent electric and magnetic fields, and the permittivity ϵ and permeability μ are both unity in vacuum. Gauge invariance allows an additional term in the Lagrangian density

$$\mathcal{L}_\theta = \frac{\alpha}{4\pi^2}\theta \vec{E}\cdot\vec{B}, \tag{2}$$

where α is the fine structure constant. Integrating over a closed space-time with periodic boundary conditions, we obtain the quantization $S_\theta = \int d^4x\, \mathcal{L}_\theta = \theta n$, where n is an integer. Namely, S_θ is a topological term. The quantization also implies that the bulk properties depends on θ only modulo 2π. While S_θ generically breaks the parity and time-reversal symmetry, both symmetries are intact at $\theta = 0$ and $\theta = \pi$.

The topological term was originally introduced in high-energy physics. In particular, the similar term can be defined for the quantum chromodynamics, which is a non-Abelian gauge theory for the strong interaction. Within the Standard Model of Particle Physics, there is a priori no reason to set θ to the time-reversal invariant values. If θ has a generic value, a strong violation of time-reversal or the CP symmetry should follow, conflicting with current experimental limitations on the CP violation. The solution proposed in Ref. [2] to address this problem introduces a dynamical pseudo-scalar field, called axion field, which couples to $\vec{E}\cdot\vec{B}$, so that its expectation value absorbes the parameter θ. Let us, for the moment, use the same symbol θ for the axion field. It was shown that the axion field would relax into the groundstate corresponding to $\theta = 0$, thereby dynamically recovering the time-reversal symmetry. This solves the "strong CP problem". At the same time, it leads to prediction of a new particle, the axion, corresponding to quantum of θ-field. While the axion is a possible component of dark matter, direct detection of the hypothetical particle so far remains elusive.

Condensed matter physics often provides realization of intriguing theoretical concepts, which originate in but are rather difficult to observe experimentally in high-energy physics. The quantum electrodynamics with the topological term (2) is such an example. It was pointed out in Ref. [1] that this system at the nontrivial, time-reversal invariant angle $\theta = \pi$ is an effective theory for topological insulators. The presence of the topological term leads to several interesting effects.

The topological angle θ is a *static* parameter for a topological insulator. Nevertheless, in Ref. [3] it was pointed out that, when there is an antiferromagnetic order in an insulator, the magnetic fluctuations can couple to electrons, playing the role of the dynamical axion field. Interesting effects due to the dynamical axion field were predicted in the presence of an applied magnetic field. Such a system is called "topological magnetic insulator" (TMI), and it gives a condensed-matter realization of the axion electrodynamics.

It should be noted, however, that the presence of the dynamical axion field does *not* require the system to be a topological insulator; a topologically *trivial* insulator could have the dynamical axion field if there is an appropriate coupling between magnetic fluctuations and electrons.

In this paper, we focus our attention to an applied electric field. In the context of the AdS/CFT corredspondence, it was pointed out in Ref. [4] that the Maxwell theory with the Chern-Simons term in $(4+1)$ space-time dimensions is unstable in the presence of a constant electric field, and a similar instability was also found in its dimensional reduction to $(3+1)$ dimensions: the massless axion electrodynamics [5, 6]. (See also the earlier work [7].) However, the eventual fate of the unstable gauge theory in the Minkowski space and its physical meaning have not been clearly understood.

In this paper, we study the instability of the axionic electrodynamics in $(3+1)$ dimensions with a massive axion field, and its possible realization in magnetic systems. In particular, we show that the instability leads to a complete screening of an applied electric field above a critical value, proportional to the axion mass. This also leads to a spontaneous generation of a magnetic flux from the material. We will discuss how this phenomenon can be detected experimentally.

Instability.— We consider the axionic electrodynamics defined by the Lagrangian density, in which the electromagnetic field are coupled to the axion field ϕ,

$$\begin{aligned}\mathcal{L} =&\mathcal{L}_{\rm EM} + \frac{\alpha}{4\pi^2}(\theta+\phi)\vec{E}\cdot\vec{B}+\\ &+g^2 J\left((\partial_t\phi)^2-\nu_i^2(\partial_i\phi)^2-m^2\phi^2\right),\end{aligned} \tag{3}$$

where J, $\nu_{i=x,y,x}$, and m are the stiffness, velocity, and mass of the axion [3]. The time reversal symmetry is broken unless $\theta=0$ or π. For application to magnetic materials, the axion velocity ν_i can be anisotropic and is much smaller than the speed of light in vacuum, which is set to unity.

Suppose we turn on the electric field E in the z-direction and consider fluctuations with momentum k in the x-direction. The axion mixes with a photon polarized in the y-direction, giving rise to the dispersion,

$$\begin{aligned}\omega^2 =&\frac{1}{2}\left((c'^2+\nu^2)k^2+m^2\right)\pm\\ &\pm\frac{1}{2}\sqrt{\left((c'^2-\nu^2)k^2-m^2\right)^2+4m^2c'^2k^2E^2/E_{\rm crit}^2},\end{aligned} \tag{4}$$

where

$$E_{\rm crit} = \frac{m}{\alpha}\sqrt{\frac{(2\pi)^3 g^2 J}{\mu}}, \tag{5}$$

$c' = 1/\sqrt{\epsilon\mu}$ is the speed of light in the medium, and $\nu=\nu_{i=2}$. In particular, if $E>E_{\rm crit}$, we find $\omega^2<0$ for the range of momentum,

$$0 \ < \ k \ < \ \frac{m}{\nu}\sqrt{\left(\frac{E}{E_{\rm crit}}\right)^2-1}. \tag{6}$$

Namely, the electric field larger than $E_{\rm crit}$ is unstable.

Consider a flat plate of the material described by (3) and sandwich it by (non-topological) insulators with permittivity ϵ_0. Apply a constant external electric field E_0 perpendicular to the interface between the material and the ordinary insulator. The boundary condition at the interface gives,

$$\epsilon E + \frac{\alpha}{\pi}(\theta+\phi)B = \epsilon_0 E_0, \tag{7}$$

where E and B are components of the electric and magnetic fields normal the interface. In addition, the conservation of magnetic flux enforces that B is continuous across the boundary. For now, we assume the Neumann boundary condition for the axion field ϕ, as it enables a simple analysis. Later, we will study the case with the Dirichlet boundary condition, which is relevant for physical realization.

According to Ref. [9], the end-point of the instability depends on spacetime geometry. In the anti-de Sitter space, which is of interest in applications to the AdS/CFT correspondence, the end-point is found to be spatially modulated. On the other hand, in the flat Minkowski space, which is of our interest in this paper, the end-point turns out to be spatially homogeneous, leading to screening of the electric field.

Assuming homogeneity of the fields, the equation of motion sets,

$$\phi = \frac{\alpha}{8\pi^2 g^2 J m^2}EB. \tag{8}$$

The boundary condition (7) then gives,

$$\epsilon E = \frac{\epsilon_0 E_0 - \alpha\theta B/\pi}{1+c'^2B^2/E_{\rm crit}^2}. \tag{9}$$

The energy-density $\mathcal{H}$ in the material can then be expressed in terms of B as,

$$\mathcal{H} = \frac{1}{8\pi\epsilon}\frac{(\epsilon_0 E_0-\alpha\theta B/\pi)^2}{1+c'^2B^2/E_{\rm crit}^2}+\frac{1}{8\pi\mu}B^2. \tag{10}$$

Minimizing the energy density determines B.

For example, if we turn off the axion by sending $m^2\to\infty$, the minimum energy configuration is,

$$E = \frac{\epsilon_0}{\epsilon+\alpha^2\theta^2\mu/\pi^2}E_0, \quad B = \epsilon_0\mu\frac{\alpha\theta/\pi}{\epsilon+\alpha^2\theta^2\mu/\pi^2}E_0 \tag{11}$$

as expected from the Witten effect [8]. We note that, although the bulk theory defined by eq. (3) has 2π-periodicity in θ, the results here are no longer periodic in θ since the boundary breaks the periodicity.

Let us first analyze the energy density (10) when $\theta=0$. When the applied electric field is lower than the critical value as in $\epsilon_0 E_0 < \epsilon E_{\rm crit}$, the only solution is,

$$E = \frac{\epsilon_0}{\epsilon}E_0, \quad B=0. \tag{12}$$

However, if the external electric field E_0 is raised above $\frac{\epsilon}{\epsilon_0}E_{\rm crit}$, we find states with lower energy given by,

$$E = E_{\rm crit}, \quad B = \pm\sqrt{\mu E_{\rm crit}(\epsilon_0 E_0-\epsilon E_{\rm crit})}. \tag{13}$$

Thus, there is a second order phase transition at $\epsilon_0 E_0 = \epsilon E_{\rm crit}$. Above this value, the electric field inside the material stays constant and the magnetic field is increased instead. We show an example of the energy density as a function of B above the critical value, in Fig. 1. This generalizes the result of [9], where it was shown that, when the axion mass $m=0$ and $E_{\rm crit}=0$, the electric field is totally screened.

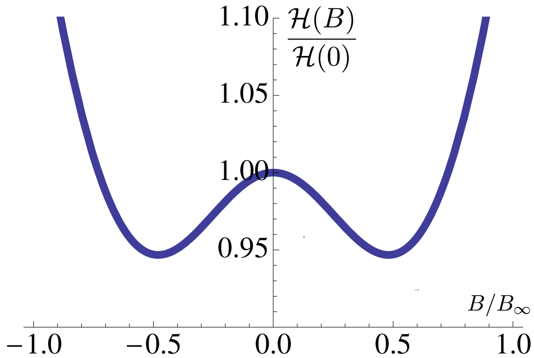

FIG. 1: The energy density (10) for $\theta = 0$ and $\epsilon_0 E_0 = 1.3\epsilon E_{\rm crit}$, which is slightly above the critical value $\epsilon_0 E_0 = \epsilon E_{\rm crit}$. The energy $\mathcal{H}(B)$ shows a double-well structure and the spontaneous breaking of the time reversal symmetry $B \to -B, \phi \to -\phi$. The magnetic field corresponding to the potential minima is spontaneously generated, resulting in the screening of the electric field to $E = E_{\rm crit}$. The magnetic field is normalized by the asymptotic value B_∞ given in eq. (14).

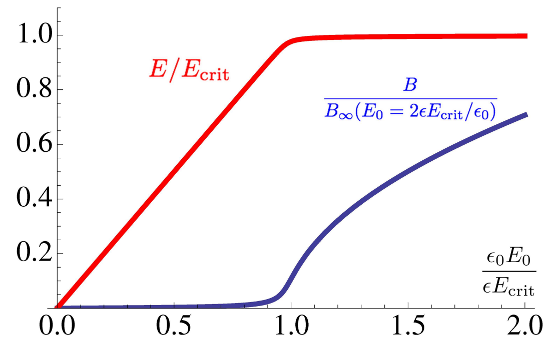

FIG. 2: The electric field E (red) inside the material and the induced magnetic field B (blue) as functions of the external field E_0, evaluated at $\theta = \pi$. B and E are determined by the global minimum of the energy density (10). The fields E_0, E and B are normalized by $\frac{\epsilon}{\epsilon_0}E_{\rm crit}$, $E_{\rm crit}$, and $B_\infty(E_0 = 2\frac{\epsilon}{\epsilon_0}E_{\rm crit})$, respectively. The behavior appears to indicate a second-order phase transition, although the transition is smoothed out by the effect of non-zero θ.

Now, let us turn to the case with $\theta \neq 0$. When $\epsilon_0 E_0 \ll \epsilon E_{\rm crit}$, there is a unique minimum energy configuration, which reduces to (11) in the limit of $E_0 \to 0$. For $\epsilon_0 E_0 \gg \epsilon E_{\rm crit}$, the energy density has two local minima in B. Since the time-reversal symmetry is broken explicitly for $\theta \neq 0$ ($\theta = \pi$ does not preserve the time revsersal-symmetry in the presence of the boundary), the two minima have different energies. In the limit of $E_0 \to \infty$, the more stable of the two behaves as

$$E \to E_{\rm crit}, \quad B \to B_\infty = \sqrt{\epsilon_0 \mu E_0 E_{\rm crit}}. \tag{14}$$

One can show that the configuration (11) for small E_0 is smoothly connected to (14) for large E_0. Namely, the phase transition at $\epsilon_0 E_0 = \epsilon E_{\rm crit}$ is smoothened out when $\theta \neq 0$. However, for realistic values of $\theta \sim \pi$, the smoothing effect is small and the solution is similar to the one at $\theta = 0$ with a second-order phase transition, except that the groundstate is chosen uniquely for any E_0.

We note that, in realistic systems, higher order terms of ϕ are expected in the effective theory, since the range of ordered magnetic moment, which corresponds to shift of ϕ, is bounded. However, although it will be important for $E \gg E_{\rm crit}$, this does not affect the existence of the transition because $\phi, B \sim 0$ near the transition.

For experimental realization of the screening effect discussed below, the Dirichlet boundary condition $\phi = \phi_0$ will also turn out to be relevant. In this case, the solution is no longer uniform in the z-direction. Nevertheless, away from the boundary, the solution asymptotically approaches to the stationary configuration, which is given by the solution obtained in the above for the Neumann boundary condition, with the replacement $\theta \to -\phi_0$. The screening of the electric field occurs in the transient region with a finite length.

If we ignore the coupling to the electromagnetic field, the axion field ϕ approaches to the stationary solution exponentially, with the decay length of ν/m. However, we should take into account mixing of the axion and the photon via the coupling ϕEB. This gives the axion field the effective mass $m_{\rm eff}$, which is given by

$$m_{\rm eff}^2 = m^2 + \frac{\alpha^2 B^2}{8\pi^3 g^2 J \epsilon}. \tag{15}$$

When $\phi_0 = -\theta = 0$ and the applied electric field is above the critical value $\epsilon_0 E_0 > \epsilon E_{\rm crit}$, the magnetic field is given by (13). When $\phi_0 \neq 0$, this is the asymptotic value for $E_0 \to \infty$. Substituting this value of B in the above, we find that the screening occurs within the lengthscale

$$\frac{\nu}{m_{\rm eff}} = \frac{\nu}{m}\sqrt{\frac{\epsilon E_{\rm crit}}{\epsilon_0 E_0}}. \tag{16}$$

Physically, the screening of the electric field occcurs because of the induced charge density $\propto \vec{\nabla}\phi \cdot \vec{B}$ in the axion electrodynamics [10]. Namely, the axion field ϕ is shifed inside the material creating the gradient $\vec{\nabla}\phi$ near the boundary. By generating the magnetic field $\vec{B}$, this induces a charge density at the boundary, screening the electric field inside the material.

From this physical picture based on screening, we can also understand why the second-order transition occurs when $\phi_0 = 0$. Under a given applied electric field, the sign of the charge needed to screen the electric field is uniquely determined. However, induced charge is proportional to $\partial_z \phi \cdot B$, which is invariant under the time reversal $\phi \to -\phi, B \to -B$. The symmetry is preserved for the boundary condition $\phi_0 = 0$, and there is no preferred sign of B. The system breaks the symmetry spontaneously to produce screening charge, for $E > E_{\rm crit}$.

On the other hand, $\phi_0 \neq 0$ introduces a gradient of ϕ near the boundary, choosing the preferred sign of B to produce screening charge. Thus the symmetry is broken explicitly and the phase transition is smeared.

Possible Experimental Realization in Magnetic Materials.— Let us discuss possible realization of the axionic instability in condensed matter systems. In Ref. [3], Bi_2Se_3 doped with $3d$ transition metal elements such as Fe (Bi_2Se_3-Fe hereafter) is discussed as a candidate for TMI, which is described by the axionic electrodynamics. Although the mechanism proposed in this paper is not restricted to any particular system, we shall examine possible physical realization using Bi_2Se_3-Fe as a reference. Because of the magnetic doping, a magnetic order M^-, which is ferromagnetic in the xy-plane and antiferromagnetic along the z-direction, may appear [3]. In the following, we assume that the electric field is applied along the z-axis.

The (relative) permittivity, axion mass, and axion coupling in Bi_2Se_3-Fe are estimated [3] to be $\epsilon \sim 100$, $m \sim 2$ meV, and $(2\pi)^3 g^2 J = \alpha(0.4\ \mathrm{T/meV})^2$. This yields a rather high value of $E_{\mathrm{crit}} = 2.4 \times 10^8$ V/m, which is above the breakdown field of typical semiconductors.

The critical field E_{crit} is reduced for smaller axion mass m. In the ordered phase, the axion mass is proportional to the spontaneous magnetic order M^- [3]. Thus, the axion mass may be reduced by tuning the system close to the critical point, so that the antiferromagnetic order becomes small. If, for example, $E_{\mathrm{crit}} \sim 10^7$ V/m and a TMI film of thickness $\sim 10^{-8}$ m is used, the voltage difference across the system is of the order of 0.1 V. The corresponding energy is well below the bandgap ~ 0.3 eV of Bi_2Se_3, justifying the low-energy description.

However, TMIs such as Bi_2Se_3-Fe are expected to have surface electronic states. For the undoped topological insulator (such as Bi_2Se_3), the surface states are described by massless Dirac fermions. In the doped TMI, the surface state has a gap m_5 due to the time-reversal symmetry breaking. In Bi_2Se_3-Fe, it was estimated [3] that $m_5 = 1$ meV and $m = 2$ meV; these are of the same order of the magnitude. However, in order to suppress the screening effect by the surface states and to enhance the effect by the dynamical axion in the bulk, it is desirable to keep the surface Dirac mass m_5 large while tuning the system near the criticality to reduce the axion mass m. This could be achieved by sandwitching the TMI by ferromagnets, which enforces the magnetic order near the surfaces [1] and determines the boundary value ϕ_0.

The axion mass m and the spin-wave velocity ν are expected to be of the order of U and $J_{ex}a$, respectively, where J_{ex} is the effective exchange interaction and a is the lattice constant. Thus, the screening length (16) is expected to be of the order of the lattice constant when U is sufficiently large. This implies that the present effect could be observed in thin film samples.

Experimentally, the instability discussed in this paper is observed as an increase of capacitance above the critical electric field. Another experimentally observable consequence is the generation of magnetic field as in eq. (14).

In the above scenario, the necessity to gap out the surface Dirac mode by sandwitching the TMI by ferromagnets may pose an additional complication. If we begin with a topologically *trivial* insulator, there is no surface Dirac mode. In fact, for the instability discussed here, it is not essential that the system is based on a nontrivial topological insulator. Even in a topologically trivial insulator, if there is an appropriate coupling with the magnetic order and the electrons, the magnetic fluctuations can play the role of the axion field, following the argument parallel to that of Ref. [3]. The difference is that, starting from a trivial insulator we expect $\theta \sim 0$, instead of $\theta \sim \pi$ in a TMI. However, as long as the dynamical axion ϕ is present in the low-energy effective theory (3), the mechanism proposed in this paper should work. The class of magnetic materials with a dynamical axion field may be called axionic insulators.

Cr_2O_3 is a topologically trivial insulator with the band gap of 3.4 eV, and a magnetic long-range order. It exhibits magneto-electric effects [11]; it is also suggested that the antiferromagnetic fluctuations in Cr_2O_3 play the role of the dynamical axion field [12]. The present mechanism could in principle be realized in such a system. The large band gap itself is advantageous for observation of the effect, as the material can withstand stronger electric field. However, the large band gap also implies a large coupling g, which is approximately proportional to the band gap. This results in a larger critical field because of eq. (5), cancelling the advantage.

The axion mass m is proportional to the on-site Coulomb repulsion U and the magnetic order [3]. The Coulomb repulsion U in Cr_2O_3 is estimated to be about 5 eV [13]; thus the axion mass m is expected to be of the same order, resulting in a too large value of E_{crit}. In order to observe the instability, we need to find a system with smaller axion mass. Again, this could be achieved by tuning the system near the magnetic criticality, for example by mixing with nonmagnetic ions.

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