Coupled resonators employing phase-conjugating and ordinary mirrors

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We calculate the oscillation conditions and the eigenfrequencies for phase-conjugating-resonator–normal-resonator coupled optical systems. With an eye toward applications to interferometry, we choose specific examples for which it is shown that the conditions for oscillation and the eigenfrequencies depend on the normal-resonator path length. The examples include both linear displacement and rotation sensing (Sagnac) resonant interferometers. Our results suggest that if the distortion-correcting and self-aligning properties of the phase-conjugating resonator are retained in the more complicated system, then these hybrid resonators may offer some advantages over their conventional counterparts.

Optical resonators with phase-conjugating mirrors (PCM's) utilizing four-wave mixing have demonstrated some fascinating properties, such as the correction of intracavity phase aberrations[1–3] and self-alignment.[2,4] However, the fundamental resonance frequency of the phase-conjugating resonators (PCR's) demonstrated so far are insensitive to resonator length.[1,4–7] This feature does not make them a good choice for displacement sensors. The primary motivation for this research stems from a desire to incorporate the useful characteristics of the PCR into the realm of conventional resonant interferometry.

To this end we study the consequences of coupling phase-conjugating and normal resonators. Specifically, we calculate the eigenfrequencies and the oscillation conditions for three such hybrid structures. These structures are shown in Fig. 1. The first (Fig. 1a) is the simplest configuration (in terms of the number of mirrors). Here, the PCR formed by the PCM and M_1 is coupled through the transmission of M_1 to the normal resonator formed by M_1 and M_2. This structure reveals an unusual mode-frequency spectrum. In Fig. 1b the PCR formed by the PCM and M_2 is coupled to the folded Fabry–Perot resonator formed by M_1, M_2, and M_3. In Fig. 1c the PCR formed by PCM_A and PCM_B is coupled to the Sagnac (or ring) resonator formed by M_1, M_2, M_3, and M_4. In each case we derive the eigenfrequencies and the oscillation conditions for the structure by replacing all the normal mirrors by a single complex mirror whose electric-field amplitude reflection and transmission coefficients are strongly dependent on the incident-wave frequency (Fig. 1d). Thus the problem is reduced to that of a single PCR having one complex mirror and one or two PCM's.

In the following discussion it is assumed that all the normal mirrors are lossless. The PCM's are pumped by fields having frequency ν_0, and it is presumed that they provide gain, i.e., they conjugate and amplify an incident wave.[5,7] We shall neglect the transverse-mode distribution, that is, we shall suppose that the modes are plane waves so that the electric field between the mirrors in Fig. 1d satisfies the one-dimensional free-space wave equation:

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = 0 \tag{1}$$

subject to the appropriate boundary conditions at the PCM and at the complex mirror. In Eq. (1), z is a coordinate along the optical path and c is the speed of light. The nature of phase conjugation by means of four-wave mixing is such that an incident wave having frequency $\nu_0 + \delta\nu$ is reflected with frequency $\nu_0 - \delta\nu$.[7] With this in mind one can guess that an eigenmode may consist of fields having two frequencies that are symmetrically located about the pump frequency. A general eigenmode satisfying Eq. (1) would then have the form

$$\begin{aligned} E(z,t) = {} & [A_U \exp(ik_U z) + B_U \exp(-ik_U z)] \\ & \times \exp[-i(\nu_0 + \delta\nu)t] \\ & + [A_L \exp(ik_L z) + B_L \exp(-ik_L z)] \\ & \times \exp[-i(\nu_0 - \delta\nu)t], \end{aligned} \tag{2}$$

with $k_{U,L} = (\nu_0 \pm \delta\nu)/c$. Associated the PCM is a gain $g(\delta\nu)$ that may be dependent on the incident-wave frequency but that, we will suppose, is symmetric about the pump frequency so that $g(\delta\nu) = g(-\delta\nu)$. Then the PCM located at $z = 0$ imposes the following boundary conditions:

$$A_L = g(\delta\nu)B_U^*, \tag{3a}$$

$$A_U = g(\delta\nu)B_L^*. \tag{3b}$$

Here, we have considered the PCM to be ideal and not to induce any additional phase shifts of its own.[8,9] The generic complex mirror has electric-field amplitude reflection and transmission coefficients

$$r(\nu) = \rho(\nu)\exp[i\varphi_r(\nu)], \tag{4a}$$

$$t(\nu) = \tau(\nu)\exp[i\varphi_t(\nu)], \tag{4b}$$

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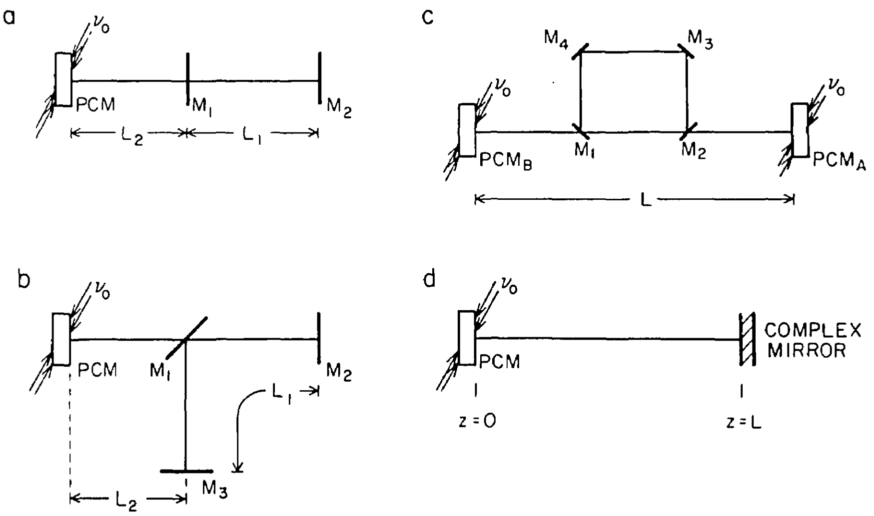

Fig. 1. Coupled resonator configurations. a, PCR coupled to a Fabry–Perot resonator; b, PCR coupled to a folded Fabry-Perot resonator; c, PCR coupled to a ring resonator; d, PCR coupled to a normal resonator represented as a complex mirror.

subject to the energy-conserving requirement that $\rho^2 + \tau^2 = 1$. Thus the mirror at $z = L$ imposes the boundary conditions

$$B_U = \rho(\nu_0 + \delta\nu)A_U \exp\{i[2k_U L + \varphi_r(\nu_0 + \delta\nu)]\}, \quad (5a)$$

$$B_L = \rho(\nu_0 - \delta\nu)A_L \exp\{i[2k_L L + \varphi_r(\nu_0 - \delta\nu)]\}. \quad (5b)$$

In order for the field amplitudes to meet the boundary conditions [Eqs. (3) and (5)], two conditions must be satisfied:

$$\text{(I)}\ g^2(\delta\nu)\rho(\nu_0 + \delta\nu)\rho(\nu_0 - \delta\nu) = 1$$

and

$$\text{(II)}\ \varphi(\nu_0 + \delta\nu) - \varphi(\nu_0 - \delta\nu) + 4\delta k L = 2n\pi \quad (n = 0, \pm1, \pm2, \ldots),$$

where $\delta k = \delta\nu/c$. Condition (I) is a familiar requirement for sustained oscillation; the saturated gain supplied by the gain medium must equal the resonator losses. Condition (II) says that the round-trip accumulated phase must be an integer multiple of 2π. This condition determines the eigenfrequencies of the resonator. Condition (II) is always satisfied for $\delta\nu = 0$. As an example, consider a simple mirror, that is, one with ρ and φ_r independent of frequency. A mode always occurs at the pump frequency ν_0. In addition, according to condition (II), there will be side modes flanking this fundamental mode with a frequency spacing of $n\pi c/2L$. This is in agreement with previous findings.[3–6]

Now consider the coupled resonator of Fig. 1a. Let the intensity reflection coefficient of M_2 be $\rho_2^2 = 1$. One can find the electric-field reflection coefficient for the complex mirror formed by the pair by assuming an incident field on M_1 and summing contributions from successive round trips within the resonator[8,9]:

$$r_0(\nu) = \frac{\tau_1^2 e^{i\epsilon}}{1 - \rho_1 e^{i\epsilon}}\,\rho_1, \quad (6)$$

where

$$\epsilon = (\nu - \omega_0)(2L_1/c) \quad (7)$$

and ω_0 is a resonance frequency of the two-mirrored cavity. The magnitude is $|r_0| = \rho_0 = 1$, and the phase $\varphi_r(\nu)$ varies rapidly near resonance. Now suppose that the resonator length L_1 is tuned to the pump frequency (or vice versa), that is, $\nu_0 = \omega_0$. Condition (II) will be satisfied not only for $\delta\nu = 0$ but also for a value near 0. The case in which $L_1 = L_2 \equiv L$ is identical with that treated by Spencer and Lamb for a normal coupled resonator.[10] In this case, condition (II) is satisfied by a frequency

$$\delta\nu \simeq \tau c/2L.$$

Unlike the normal coupled resonator, however, the frequency spectrum of this hybrid resonator is characterized by a frequency triplet in the region of the pump frequency and by doublets spaced $n\pi c/2L$ from the pump frequency. (The spectrum is more complicated if $L_1 \neq L_2$.) Notice that the splitting frequency $\tau c/2L$ is dependent on both L_2 and L_1. Thus a change in the PCM distance from M_1 will cause a change in the splitting frequency.

The folded normal resonator of Fig. 1b can also be replaced by a single complex mirror. We set the reflectivities of M_2 and M_3 to unity. The reflectivity (back toward the PCM) and transmission (away from the PCM and up in the figure) of the complex mirror are

$$r(\nu) = \frac{\tau_1^2}{1 - \rho_1^2 e^{i\epsilon}}\,, \quad (8a)$$

$$t(\nu) = \rho_1\left(\frac{\tau_1^2 e^{i\epsilon}}{1 - \rho_1^2 e^{i\epsilon}} - 1\right), \quad (8b)$$

where ϵ is now $(\nu - \omega_0)(2L_1/c)$ and L_1 is the unfolded cavity length. Notice that the reflectivity magnitude reaches one only at resonance. Therefore the PCM can sustain oscillations only when the pump frequency is at or near the cavity resonance frequency. In this sense, the present configuration behaves like a normal resonator with an internal atomic gain medium in which the cavity length must be tuned to the atomic resonance in order for lasing action to occur.

As a final example we treat the case of two PCM's coupled to a ring interferometer in order to reveal the rotation-sensing properties of the interferometer.[11] M_1 and M_2 are considered identical, and M_3 and M_4 have unit reflection coefficients. Again the normal mirrors forming the ring are represented as a single complex mirror located at $z = 0$ flanked by PCM's at $z = \pm L/2$. The left-hand side of the complex mirror represents mirror M_1 and the right-hand side replaces M_2. By anticipating the properties of the ring resonator, we allow for different transmission coefficients for waves traveling to the right (φ_+, toward PCM_A) and to the left (φ_-, toward PCM_B). Provided that there is no backscattering from the normal mirrors of the ring, the complex mirror has zero reflectivity (that is, no light is reflected back toward the PCM's). The assumed field is divided into the two regions on either side of the complex mirror:

$$E(z,t) = [A_\nu \exp(ik_\nu z) + B_\nu \exp(-ik_\nu z)] \times \exp[-i(\nu_0 + \delta\nu)t] + [A_L \exp(ik_L z) + B_L \times \exp(-ik_L z)] \exp[-i(\nu_0 - \delta\nu)t], \quad z > 0, \quad (9)$$

with a similar expression for $z < 0$. The field is subject to the appropriate boundary conditions at the four mirror surfaces. In order to find the eigenfrequencies for this arrangement of mirrors, the boundary requirements are combined, and they provide two separate conditions:

$$2\delta kL + \varphi_+(\nu_0 + \delta\nu) - \varphi_-(\nu_0 - \delta\nu) = 2n\pi, \quad (10a)$$

$$2\delta kL + \varphi_-(\nu_0 + \delta\nu) - \varphi_+(\nu_0 - \delta\nu) = 2n\pi. \quad (10b)$$

In order for field amplitudes A_U and B_L to nonzero, Eq. (10a) must be satisfied. In order for amplitudes A_L and B_U to be nonzero, Eq. (10b) must hold. (There are constraints on the gains as well, but we are interested here in the eigenfrequencies.) In general, the mode-frequency spectrum will be rather complicated, as it is for the hybrid resonator of Fig. 1a. We focus on the rotation sensitivity of this device by supposing that the pump frequency of the PCM's is tuned to a stationary ring-cavity resonance and center our attention around frequencies near this resonance ($n = 0$). If the ring is stationary, then both Eqs. (10a) and (10b) are satisfied for $\delta\nu = 0$. When the ring is rotating, the resonance frequencies for oppositely directed traveling waves shift:

$$\omega_\pm = \omega_0 \mp \delta\omega, \quad (11)$$

where $\delta\omega$ is given by the Sagnac formula[11]:

$$\delta\omega = \frac{4\pi A}{\lambda P}\Omega, \quad (12)$$

where A is the area enclosed by the ring, P is the perimeter of the ring, λ is the wavelength, and Ω is the rotation rate in radians per second. Thus the transmission phases become

$$\varphi_+(\nu) = \varphi_0(\nu - \delta\omega), \quad (13a)$$

$$\varphi_-(\nu) = \varphi_0(\nu + \delta\omega), \quad (13b)$$

where

$$\varphi_0(\nu) = \arg\{t\} = \arg\left\{\frac{\tau_1^2}{1 - \rho_1^2 e^{i\epsilon}}\right\}$$

and ϵ is given by Eq. (7) with $2L_1$ replaced by P, the perimeter of the ring. φ_0 is a sharply varying function of frequency near resonance for a ring cavity of high finesse. In this case, we can drop the first term appearing on the right-hand sides of Eqs. (10a) and (10b). If Ω is defined to be positive for counterclockwise rotation of the ring in Fig. 1c, then for $\Omega > 0$ only condition (10b) can be satisfied, and for $\Omega < 0$ only condition (10a) can be satisfied. In both cases, the required conditions hold for $\delta\nu = \delta\omega$. Thus the eigenmodes of this coupled system are oppositely directed traveling waves having a difference frequency equal to that for a normal ring resonator.

We have discussed three hybrid coupled-resonator structures. The mode spectra of the coupled systems were shown to depend on the normal resonator optical length. Although the first of the three systems considered has some interesting aspects, the other two should probably be considered more practical for applications in resonant interferometry. In each of the hybrid configurations discussed, we specialized to the case in which the pump frequency was tuned to normal cavity resonance or vice versa. To do this in an active way so that the pump and the resonator remain tuned together for an indefinite length of time requires some additional optics (and electronics), which were omitted from both the figures and the discussion.[12] In practical applications, then, these hybrid configurations used as interferometers become essentially equivalent to their normal counterparts wherein the PCM and its pump are replaced by a source laser. The distortion correcting, aligning, and self-oscillation properties of the PCM have yet to be demonstrated experimentally. Therefore whether the hybrid interferometer has any real advantages to offer remains to be seen.

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