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MOUNT WILSON AND PALOMAR OBSERVATORIES

CARNEGIE INSTITUTION OF WASHINGTON,
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Special Technical Report No. 3

A THEORY OF THE ROLE OF MAGNETIC ACTIVITY DURING STAR FORMATION

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A THEORY OF THE ROLE OF MAGNETIC ACTIVITY

DURING STAR FORMATION

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SUMMARY

Under the assumption that magnetic activity is due to the action of a magnetic field in a rotating star with a convective zone, it is possible to draw the following picture of a sequence of events in the Hertzsprung-Russell diagram.

The H-R diagram is divided into two regions by an almost vertical line. Stars of later type have a hydrogen convective zone of great extension (region C); stars of earlier type have no hydrogen convective zone or a convective zone of small extension (region D).

Stars in region C show stellar activity of electrodynamic origin like solar activity; stars in region D show little or no stellar activity.

When matter is ejected by a star, the magnetic field compels the matter to rotate with the star even at very large distances where it carries away large angular momentum per unit mass. Stars which reach the main sequence in region C may lose a large amount of angular momentum, while stars reaching the main sequence in region D lose only a little.

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This process explains the sharp difference between early and late type stars. Stars of type earlier than spectral type F2 rotate rapidly because they have reached the main sequence through region D, while stars of type later than F2 rotate slowly because they have reached the main sequence through region C.

Region C can be divided into two parts. In region CI, the star loses matter through its equatorial plane, but the rate of loss of angular momentum through magnetic interaction increases steadily towards the main sequences, until the star begins to slow down. The star then enters region CII where it does not lose any more matter through the equatorial plane, but via electromagnetic loss of angular momentum reaches the main sequence with only slow rotation. The slowing down of a rapidly rotating star can be achieved with a mass loss as small as 10^{-5} M⊙.

Magnetic activity in region CI may build, at the surface of the star, abnormally high abundances of certain elements (rare earths, lithium, etc.) showing what we shall call P characteristics (P for peculiar). When stars enter region CII, the abundance of these elements first increases, but then due to dilution by mixing in the convective zone, decreases and can be vanishingly small when the star reaches the main sequence. When stars cross region D, the elements which have been formed at the surface are dragged with the surface flow toward the equatorial plane and partially lost. However, stars which successively cross region CI, CII and reach the main sequence through region D, can attain a maximum of P characteristics. This explains why peculiar abundances are seen only in a narrow spectral range near A0 (peculiar A stars).

Magnetic fields built in region CI become essentially toroidal in the CII region, as a consequence of the action of the differential rotation; the magnetic fields within region CI remain essentially poloidal in region D, as a consequence of the shear flow towards the equator. The stretching of the lines of force leads also to a decrease in magnetic field; therefor magnetic stars should be more frequent in the neighborhood of type A0.

INTRODUCTION

The present discussion is an attempt to connect a certain number of facts, with the help of only a few new hypotheses. Since the starting point of this work is a discussion of the difference in the speed of rotation of stars on the upper and lower parts of the main sequence, it is worth while to understand clearly the logical implications of that well known fact.

The fact that early type stars have a larger equatorial velocity implies that they have been formed by contraction. Indeed, a contracting star always has a tendency to increase its angular velocity, unless some powerful mechanism acts as a brake. We can therefor rule out the suggestion that stars and interstellar matter have been formed by expansion of the same prestellar matter, since we cannot imagine any way of storing a large amount of angular momentum in a non-rotating fluid.

If we start with a sphere of low-density interstellar gas, we can estimate the density it must have to reach the main sequence with the maximum possible angular velocity, if the initial angular velocity

ω_o of the gas is the angular velocity of the sun around the galactic center. If R_o is the radius of the sphere of interstellar gas, ω_o its angular velocity, R the radius of the star, ω its angular velocity, $(2/5)MR_o^2$ the moment of inertia of the cloud, KMR^2 the moment of inertia of the star, we have:

$$KMR^2 \omega = \frac{2}{5} MR_o^2 \omega_o \quad . \qquad (1)$$

As we suppose that the mass has not changed, we have

$$\frac{R}{R_o} = \left(\frac{2}{5K} \frac{\omega_o}{\omega} \right)^{1/2} . \qquad (2)$$

With $\omega_o \approx 10^{-15}$, and $\omega^2 = \frac{4\pi}{3} G \bar{\rho}$, this leads to the value

$$\frac{\rho_o}{\bar{\rho}} = 10^{-16}, \qquad (3)$$

if we take $1/K = 13.5$, which corresponds roughly to the standard model.

This initial density is relatively high. A similar conclusion has been reached by Spitzer (1949). If we start with densities comparable to the densities of interstellar matter, we are sure the proto-star will become flat and lose matter through the equator long before it reaches the main sequence.

Therefor, we should expect all stars along the main sequence to have very fast rotation. Such a conclusion has been known for a long time; its first mention can probably be found in the criticism by Fouché and by Babinet of Laplace's cosmogony.

A homology transformation will help us find the track of a star during its contracting phase. If we suppose that the star has always the same internal structure, we have two relations. One expresses the fact that, at the equator, gravitational and centrifugal force are equal: -

$$\lambda_1 \frac{GM}{R^2} = \omega^2 R \,, \tag{4}$$

where λ_1 is a constant, of order of magnitude unity, which takes into account the flattening of the system. The other equations, which express the conservation of angular momentum, suppose that angular momentum is lost only at the equator, with an angular momentum $R^2\omega$ per gram:

$$\frac{d}{dt}\left(KMR^2\omega\right) = R^2\omega \frac{dM}{dt} \,. \tag{5}$$

Dividing both sides by $KMR^2\omega$ we obtain

$$\left(1 - \frac{1}{K}\right)\frac{1}{M}\frac{dM}{dt} + \frac{2}{R}\frac{dR}{dt} + \frac{1}{\omega}\frac{d\omega}{dt} = 0 \,. \tag{6}$$

Integrating equation (6) we obtain the homology relation

$$M^{1-\frac{1}{K}} R^2\omega = \text{constant}. \tag{7}$$

If we insert in equation (7) the relation (4), we obtain the homology relations

$$R \sim M^{\frac{2}{K}-3} \,, \tag{8}$$

$$\omega \sim M^{\frac{-3}{K}+5} \,.$$

As we have seen, $1/K$ is of the order of 13.5. Consequently, during that phase, there is little mass loss, since

$$M \sim R^{\frac{1}{24}} . \tag{9}$$

The angular velocity increases slowly, as

$$\omega \sim R^{-\frac{71}{48}} , \tag{10}$$

and there is an even slower increase of equatorial velocity

$$V = \omega R \sim R^{-\frac{23}{48}} . \tag{11}$$

Let us now consider the various suggestions which have been made to explain the loss of angular momentum during star formation.

It has been suggested that stars lose mass from their surface by corpuscular radiation; Fessenkov (1949) and Schatzman (1954a, b) have considered the possibility that such a phenomenon could explain the difference between the early and late type stars. However, three major criticisms can be made of that theory: (1) To slow down the motion of a star and make it evolve from type A to type F, it is necessary for it to lose about half of its mass. Such a large mass loss would hardly escape observation. (2) The theory does not explain the sharp difference between stars earlier and later than type F2. (3) The distribution among giants and subgiants, well explained by conservation of angular momentum (Oke and Greenstein, 1954), is left completely out of consideration by the theory of corpuscular radiation.

The theory that stars could lose angular momentum through magnetic interaction with the interstellar medium has had some attention (Spitzer, 1956), the sharp difference at type F2 possibly being due to the difference in size and density of the Strömgren spheres. However, if we consider the distance at which matter can be dragged with the rotating magnetic field, we find that it is proportional to $\rho^{-1/2}$; therefor, the rate of transfer of the angular momentum from the rotating interstellar matter to the matter at rest, through any kind of friction, is larger in an H II region than in an H I region.

Lüst and Schlüter (1955) have considered the loss of angular momentum from magnetic coupling between a star, its magnetic field and an envelope of conductive matter. The rate of loss of angular momentum is essentially related to the decay of the magnetic field and is a very slow and inefficient process. (See also L. Mestel, 1959 .)

ELECTROMAGNETIC COUPLING

Another mechanism, which combines coupling with the stellar magnetic field and mass loss has been already proposed by Schatzman (1959).

The idea is the following: we suppose that when ejection of matter occurs at the surface of a star which has a magnetic field, the ejected matter is forced to turn with the star up to a critical distance beyond which it is no longer dragged by the magnetic field. Since that distance is much larger than the radius of the star, the loss of angular momentum per unit mass is large; a small mass loss can produce a very large effect.

We could describe more precisely the mechanism in the following way: during flares, jets of matter are produced, which, after having locally disturbed the magnetic field, follow the magnetic lines of force up to a point where the magnetic stresses are no longer sufficiently large to force matter to follow the lines of force. At that point, it can be said that matter leaves the star and carries away some angular momentum.

In order to estimate the rate of loss of angular momentum, we proceed in the following way: if H is the magnetic field at distance r from the star, we suppose that the field drags the matter as long as the tension $\frac{H^2}{4\pi R_c}$ of the lines of force, of radius of curvature R_c, can resist the pressure gradient due to the Coriolis force acting on the ionized gas of density ρ and velocity V:

$$2\rho V\omega = \frac{H^2}{4\pi R_c} . \tag{12}$$

This is achieved where R_c is of the order of r;

$$2\rho V\omega = \frac{H^2}{4\pi r} . \tag{13}$$

If we suppose that the continuity equation can be written in two dimensions:

$$\frac{\rho}{\rho_o} = \frac{r_o}{r} , \tag{14}$$

where r_o is a constant of the order of the radius of the star, we can write

$$\frac{H^2}{8\pi} \approx \rho_o V r_o \omega . \tag{15}$$

If we suppose that the field is a two-dimensional dipole field, produced by a pair of linear magnetic spots, we have

$$\frac{H}{H_o} \approx \left(\frac{a}{r}\right)^2 \tag{16}$$

and therefor

$$r \approx a\left(\frac{H_o^2/8\pi}{\rho_o V r_o \omega}\right)^{1/4} . \tag{17}$$

Let us consider now the equation of conservation of angular momentum:

$$\frac{d}{dt}\,(KMR^2\omega) = \frac{r^2}{R^2}\,R^2\omega\,\frac{dM}{dt}\,, \tag{18}$$

which expresses the fact that the angular momentum is lost at a distance r from the star. If we insert the expression (17) into equation (18), we obtain the differential equation

$$\frac{1}{M}\frac{dM}{dt} + \frac{2}{R}\frac{dR}{dt} + \frac{1}{\omega}\,\frac{d\omega}{dt} = \frac{a^2}{KR^2}\left(\frac{H_o^2/8\pi}{\rho_o\,Vr_o}\right)^{1/2}\frac{1}{\omega^{1/2}}\,\frac{1}{M}\frac{dM}{dt} . \tag{19}$$

If a, R, H_o, ρ_o, V and r_o are constants, equation (19) can be integrated to give

$$\left(\frac{\omega}{\omega_o}\right)^{1/2} = 1 + \frac{1}{2K}\,\frac{a^2}{R^2}\left(\frac{H_o^2/8\pi}{\rho_o\,Vr_o}\right)^{1/2}\log\frac{M}{M_o}\,, \tag{20}$$

if we suppose that M changes very little. Numerically, if we take $\rho_o = 2 \times 10^{-21}$ gm/cm^3, $V = 10^8$ cm/sec, $\omega_o = 10^{-3.45}$ (the maximum possible angular velocity of a star of solar mass and radius), $H_o = 10^4$ gauss, $a/R = 0.1$, $1/K = 13.5$ we obtain

$$\left(\frac{\omega}{\omega_o}\right)^{1/2} = 1 - 10^{5.11} \left(\frac{\rho_o}{2 \times 10^{-21}} \ \frac{V}{10^8} \ \frac{10^8}{H_o^2} \right)^{-1/2} \frac{\Delta M}{M_o} , \qquad (21)$$

where ΔM is the mass loss.

We thus see that a very small mass loss can lead to an enormous loss of angular momentum, and that it would be even possible that the star be brought to rest. However, we shall see later that it would take an infinite length of time.

THE CONVECTIVE ZONE

The physical process we have been considering does not lead in itself to an explanation of the difference between early and late type stars. However, the process implies both an ejection of matter and a magnetic field. We shall suppose that the ejection of matter is due to surface activity, similar to the activity at the surface of the sun.

Surface activity is related to a cycle of magnetic activity in the sun. Ejection of matter is due to flares, which are produced by instabilities occurring in the vicinity of neutral lines, as has been proved by Severny's observations (1958), and as far as I can judge myself, on magnetograms taken with Babcock's apparatus (Howard, 1959; Leighton, 1959). If we accept Parker's picture (1955) of solar activity, we are bound to establish a correlation between the presence of the surface convective zone (hydrogen convective zone) and the surface activity.

We shall therefor assume that whenever there exists a convection zone, surface activity is a normal feature of the star, but that when the convective zone is absent or small, the surface activity is also small or absent even though a strong magnetic field might be present.

As is well known, G, K, and M stars have a convective zone of considerable thickness (hydrogen convective zone) and A and B stars have a thin convective zone or none, except that probably the early B stars have a helium convective zone.

The transition from a thick convective zone to a thin convective zone is quite sharp and probably takes place in the F type stars. For stars of a given chemical composition, the line which divides the Hertzsprung-Russell diagram into two regions, region C where stars have a convective zone, region D where stars have no convective zone, is almost parallel to the vertical axis. Starting in the F types on the main sequence it reaches the G types in the supergiant range (Figure 1).

It is supposed that when a contracting star goes from region C to region D (Figure 1), it stops losing angular momentum, and therefor can reach the main sequence with a high equatorial velocity. The helium convective zone (region C', Figure 1) can slow down the stars which happen to reach the main sequence in its range. However, due to its small extent, region C', has not the power to stop completely the stars which cross it. Therefor, we can only expect a small effect.

We therefor have found what seems to be the basic explanation of the distribution of equatorial velocities along the main sequence--the connection between the convective zone, the magnetic activity (surface activity) and the loss of angular momentum.

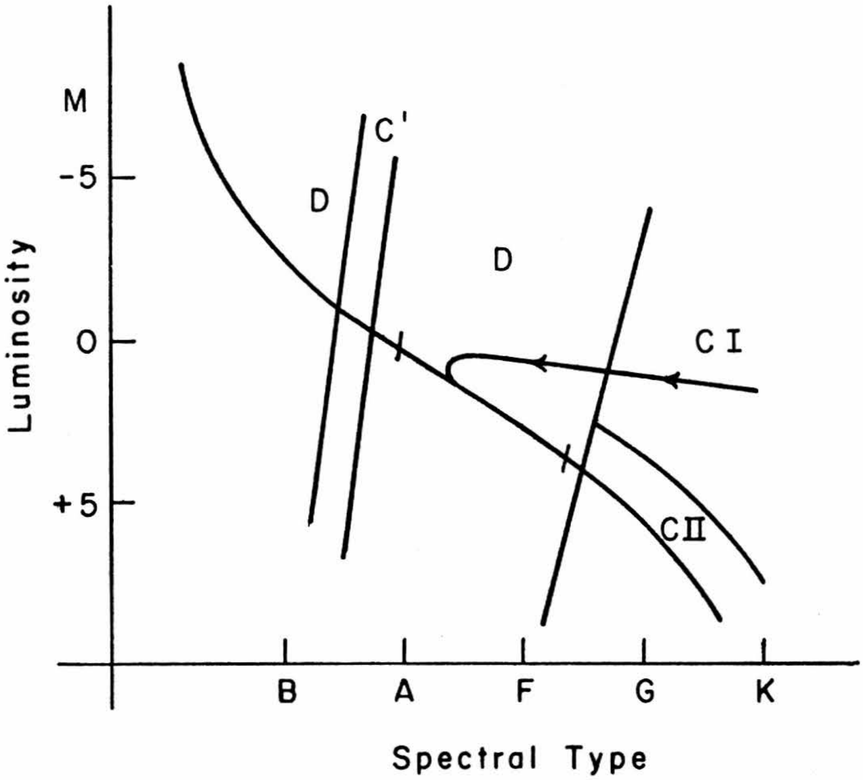

Fig. 1. Regions in the H-R diagram

ENERGY BALANCE

The energy needed for the exchange of angular momentum between the star and the ejected material is borrowed from the magnetic field. The total energy of a system of magnetic spots is of the order of $a^3 \frac{H_o^2}{8\pi}$. This has to be compared to the energy loss during flares:

$$\frac{1}{2} V^2 \delta M < \frac{a^3 H_o^2}{8\pi} .$$

With the value we have used earlier, we find that a group of magnetic spots has stored enough energy to allow a maximum mass loss $\delta M < 10^{20.9}$ gm. Let us consider a total mass loss of $10^{28.2}$ gm. This represents a series of 2×10^7 groups of flares, each group being produced in the same magnetic region. If that succession is to occur in

10^6 years, it requires 20 groups of flares a year. Such a frequency is not improbable, from what we know, for example in T Tauri stars. If a very small part of the magnetic energy is used to produce the ejection of matter, we can consider the possibility that the number of magnetic spots is larger, or that flares will start from many different places at the surface of the star. Even an efficiency of 10^{-2} would be sufficient to provide the necessary mass loss.

If we consider now the total magnetic energy of a magnetic star, of the order of 10^{40} ergs, we see that the energy stored in the magnetic field is quite sufficient to provide the energy necessary for the required mass loss.

MAGNETIC ACTIVITY

If we use Parker's picture (1955) we notice that the poloidal field is built by turbulence and the toroidal field is built by differential rotation. Parker writes for the rate of production of the vector potential (0, A, 0) of the poloidal field

$$\frac{dA}{dt} = \Gamma B + \frac{1}{\mu\sigma} \nabla^2 A, \tag{22}$$

where B is the toroidal component and σ the conductivity. The coefficient Γ has the dimensions of a velocity. We can introduce the coefficient Γ most simply by supposing that the kinetic energy of matter in any vortex is of the order of magnitude of the work done by the Coriolis force acting on the same matter. We therefor have

$$\frac{1}{2}\Gamma^2 = \frac{1}{2}C_o^2\,\omega\,v_t\,\ell_t, \tag{23}$$

where v_t is the velocity of turbulent motion, ℓ_t a characteristic length of the turbulence, and ω the angular velocity of the rotating star. C_o is an adjustable constant which is not given by the phenomenological theory. We thus write

$$\Gamma = C_o(\omega\,v_t\,\ell_t)^{1/2} \quad . \tag{24}$$

Parker suggests that activity is related to the formation of a migratory dynamo wave. Its wave number k is

$$k = \frac{2\omega'^2}{\Gamma\alpha} \ , \tag{25}$$

where ω' is the pulsation period of the migratory wave, and $\alpha = |\underset{\sim}{\nabla}\,\underset{\sim}{v}|$ is a measure of the gradient of the velocity in the differential rotation. If v is the gradient velocity ωR, the gradient of v is proportional to ω:

$$\alpha = C_1\,\omega\,, \tag{26}$$

where C_1 is a constant which is not given by the phenomenological theory.

As the migratory dynamo wave increases its amplitude with time, it is quite obvious that its wave length should not be much smaller than the radius of the star, otherwise its amplitude would increase by too large a factor during propagation. We make the following choice:

$$k \sim \frac{2\pi}{\frac{\pi}{2} R} = \frac{4}{R} \tag{27}$$

and derive the relation

$$\omega' = C_1^{1/2} \left(\frac{2\omega\Gamma}{R}\right)^{1/2} . \tag{28}$$

In the case of the sun, with $v_t = 10^4$, $\ell = 10^8$, $\omega = 10^{-5.58}$, we obtain

$$\omega'_\odot = 10^{-1.17} C_1^{1/2} C_o^{1/2} \omega = 10^{-2.15} \omega . \tag{29}$$

$C_1 C_o$ is consequently of the order of 10^{-2}, which seems a reasonable order of magnitude. We shall therefor write

$$\omega' = (2C_1C_o)^{1/2} (v_t \ell_t)^{1/4} \omega^{3/4} R^{-1/2}. \tag{30}$$

If the phenomenological relation is correct, we should expect a slow variation of the period of magnetic activity with time. However, the ratio T'/T varies about as $T^{-1/4}$ and is larger when T is smaller. A period of rotation of 1 day would thus correspond to a period of activity of 340 days. It has to be emphasized that in magnetic A stars, where there is no convective zone (or only a small one), we should not expect such a recurrent activity to be present. Such a period of magnetic activity should be expected only in region C (and possibly in region C').

The question of the magnitude of the magnetic field is not settled by this theory, since it is a linear theory of the formation of the poloidal and of the toroidal field. We can suppose that the magnetic field is constant. However, we might eventually suppose that it varies like R^{-2} as the star contracts. We shall see below that this is likely to give a reasonable picture.

RATE OF MASS LOSS

At first, we consider the rate of mass loss without contraction. We suppose that we have a period of activity both with a production of N active centers per cycle, each of these centers ejecting matter at the velocity V. An active center may eject ΔM in such a way that $\frac{1}{2}\Delta M \cdot V^2$ is the fraction $\frac{1}{n}$ of the magnetic energy of the active center:

$$\frac{1}{2}\frac{\Delta M}{\Delta t} = -\frac{N}{n}\frac{a^3}{6}\frac{H_o^2}{V^2}\frac{1}{P_{act}} , \tag{31}$$

where P_{act} is the period of activity. Therefor, we can write

$$\frac{dM}{dt} = -\frac{a^3}{3}\frac{H_o^2}{V^2}\frac{N}{n}\frac{1}{2\pi}\left(\frac{2C_oC_1\,\omega^{3/2}(v_t\,\ell_t)^{1/2}}{R}\right)^{1/2} . \tag{32}$$

If we use the relation (19) between mass and angular velocity which we now write as:

$$\frac{dM}{dt} = M\frac{KR^2}{a^2}\left(\frac{\rho_o\,Vr_o}{H_o^2/8\pi}\right)^{1/2}\frac{(d\omega/dt)}{\omega^{1/2}} , \tag{33}$$

we obtain the relation:

$$\frac{d\omega}{\omega^{5/4}} = -\frac{a^3}{3}\frac{H_o^2}{V^2}\frac{N}{nM}\frac{a^2}{KR^2}\left(\frac{H_o^2/8\pi}{\rho_o Vr_o}\right)^{1/2}\frac{1}{2\pi}\left(\frac{2C_oC_1(v_t\,\ell_t)^{1/2}}{R}\right)^{1/2} dt . \tag{34}$$

Let us consider the case where a star has reached the main sequence with the angular velocity ω_o. If we suppose that the lengths R_o and r_o are both of the order of magnitude of R, the radius of the star, we have

$$4\left(\frac{1}{\omega^{1/4}} - \frac{1}{\omega_o^{1/4}}\right)$$
$$= \frac{4a^5}{3R^3}\frac{1}{MV^{5/2}}\ \frac{N}{n}\left(\frac{H_o^2}{8\pi}\right)^{3/2}(2C_oC_1)^{1/2}\ \frac{(v_t\,\ell_t)^{1/4}}{K\rho_o^{1/2}}(t-t_o). \qquad (35)$$

If we put

$$t_o = \frac{K\rho_o^{1/2}}{(v_t\,\ell_t)^{1/4}}(2C_oC_1)^{-1/2}\left(\frac{H_o^2}{8\pi}\right)^{-3/2}\frac{n}{N}MV^{5/2}\,\frac{3R^3}{a^5}\ \frac{1}{\omega_o^{1/4}}, \qquad (36)$$

we can write

$$\frac{\omega}{\omega_o} = \left(1 + \frac{t - t_1}{t_o}\right)^{-4}. \qquad (37)$$

The time scale of the loss of angular momentum is essentially t_o. If we use the same values as before, we find

$$t_o = 16 \times 10^6 \text{ years } \frac{n}{N} .$$

However, if we want to apply this theory to the sun, we have to take more realistic values of H_o and a/R. If instead 10^4 gauss we take 2×10^3, we obtain a time scale of 2×10^9 years (for $n/N \approx 1$). As we can see, the parameters enter in equation (36) with large exponents (for example, a^5), so that many adjustments are possible to make the time scale fit with the age of the solar system. Therefor, we cannot be too cautious about the application of formula (36) to other stars.

LOSS OF ANGULAR MOMENTUM DURING CONTRACTION

Let us now follow a star during the contraction phase.

We start from equation (18), which we write again:

$$\frac{d}{dt}\left(KMR^2\omega\right) = a^2\left(\frac{H_o^2/8\pi}{\rho_o\,VR\,\omega}\right)^{1/2}\omega\,\frac{dM}{dt}\,, \qquad (38)$$

and we insert in (38) the expression (32) for the rate of mass loss:

$$\frac{d}{dt}\left(KMR^2\omega\right)_{mag} = -a^2\left(\frac{H_o^2/8\pi}{\rho_o\,VR\,\omega}\right)^{1/2}\omega\,\frac{4a^3}{3}\,\frac{H_o^2}{8\pi}\,\frac{1}{V}\,\frac{N}{n}\left(\frac{2C_oC_1\,\omega^{3/2}\,(v_t\ell_t)^{1/4}}{R}\right)^{1/2}. \qquad (39)$$

Let us consider more specifically for t_o the value t_m corresponding to the star of mass M, radius R_o on the main sequence, and magnetic field H_o, where H_o varies as R^{-2}. (The latter corresponds to conservation of magnetic flux in the star during contraction.) We then have

$$\frac{1}{KMR^2\omega}\,\frac{d}{dt}\left(KMR^2\omega\right)_{mag} = -\frac{4}{t_m}\left(\frac{\omega}{\omega_o}\right)^{1/4}\left(\frac{R_o}{R}\right)^4. \qquad (40)$$

The total loss of angular momentum can be written

$$\left(1-\frac{1}{K}\right)\frac{1}{M}\frac{dM}{dt}+\frac{2}{R}\frac{dR}{dt}+\frac{1}{\omega}\frac{d\omega}{dt} = -\frac{4}{t_m}\left(\frac{\omega}{\omega_o}\right)^{1/4}\left(\frac{R_o}{R}\right)^4. \qquad (41)$$

If we take into account the equations governing gravitational contraction, we have from the equality of gravitational and centrifugal force at the equator,

$$\frac{GM}{R^2} = C_o \omega^2 R. \qquad (42)$$

Given the relation between luminosity and gravitational contraction,

$$L = - C_2 \frac{GM}{R^2} \frac{dR}{dt}, \qquad (43)$$

and the mass luminosity relation,

$$L \sim M^i R^{-j}, \qquad (44)$$

we can eliminate $\frac{dR}{dt}$ and $\frac{d\omega}{dt}$ between these equations. We finally obtain

$$\left(\frac{3}{2} - \frac{1}{K}\right) \frac{1}{M} \frac{dM}{dt} = - \frac{4}{t_m} \left(\frac{\omega}{\omega_o}\right)^{1/4} \left(\frac{R_o}{R}\right)^{1/4} + \frac{1}{2t_c} \left(\frac{M}{M_o}\right)^{i-2} \left(\frac{R}{R_o}\right)^{1-j}, \qquad (45)$$

where

$$t_c = \frac{C_2 \, GM_o^2}{L_o R_o}, \qquad (46)$$

is the time scale of contraction for a star of mass $M_o R_o$ on the main sequence. Since the mass is changing very little during the final contracting phase, we can write

$$\left(\frac{3}{2} - \frac{1}{K}\right) \frac{1}{M} \frac{dM}{dt} = - \frac{4}{t_m} \left(\frac{R_o}{R}\right)^{i/4} + \frac{1}{2t_c} \left(\frac{R}{R_o}\right)^{1-j}. \qquad (47)$$

$\frac{dM}{dt}$ has to be negative. We see that when the star contracts, losing mass from the equator and by electromagnetic coupling, the rate of loss by electromagnetic coupling increases steadily, up to the point where it surpasses the rate of loss through the equator. Then, the loss of mass through the equator stops. On the left hand side of equation (47) the term $(1/K)$ which came only from the equatorial loss disappears expressing the fact that the loss of mass continues only through surface activity. The radius for which the change from one region to the other occurs is given by

$$\frac{R}{R_o} = \left(\frac{8t_c}{t_m} \right)^{1/\frac{33}{4} - j} . \tag{48}$$

The value of j is not very important, and we may as well write

$$\frac{R}{R_o} = \left(\frac{8t_c}{t_m} \right)^{1/8} . \tag{49}$$

With a ratio $(8t_c/t_m) \approx 100$, as can be inferred from the values we have used, we find

$$\frac{R}{R_o} = 1.77. \tag{50}$$

Naturally, the numerical value in (50) has no precision, and if the magnetic time scale t_m was larger than t_c (which could be the case, if the magnetic field were very weak), the loss of matter through the equator would last until the star reaches the main sequence. However, it seems likely that region C has to be divided into two by a line running roughly parallel to the main sequence. Above that line, stars do lose mass through the equator (region CI); below the line, stars undergo only loss of mass through surface activity (region CII). We could, in the same way expect a division of region C' into two regions, C'I and C'II. We have sketched in Figure 2, the shape of the stars when in these regions.

C I, D C II

Fig. 2. Shapes of contracting stars in different parts of the H-R diagram.

NUCLEAR REACTION AT THE SURFACE OF STARS

We shall suppose that nuclear reactions, due to magnetic activity, occur at the surface of the stars. Therefor, it is essentially only during the crossing of region C that the star can accumulate abnormal amounts of the elements on its surface. The high values of the abundances of certain normally rare elements determined spectroscopically in certain stars has led to their classification as peculiar. We shall speak of these stars as having P characteristics.

When a star contracts and loses matter through the equatorial plane, lines of flow carry away part of the surface material. If some were left, it would be in the vicinity of the pole*. If the final contracting phase

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* The detailed pattern of the lines of flow has been calculated.

without activity lasts sufficiently long, we may expect that most of the P characteristics have been diluted beyond recognition. We can thus explain that in the upper part of the main sequence, the P characteristics do not appear and we can predict some P characteristics for stars which have just crossed region C' before reaching the main sequence.

In region C, where the convective zone is present, the higher abundance of the P elements results from an equilibrium between the formation of the elements at the surface and their dilution inside the convective zone. During the final stage of the contraction, when the star rotates more and more slowly, the magnetic activity decreases and the rate of formation of the P elements vanishes; the P characteristics are diluted by mixing with the material of the convective zone, and finally the essentially normal abundance of the elements is again found at the surface.

In between the two regions, the type A stars meet the most favorable conditions for the P characteristics. On both sides of A stars, the conditions are less favorable; B stars have lost matter from the surface, F and G stars have diluted their P elements in the convective zone.

A rough calculation can be made, leading to a semi-quantitative prediction. We shall suppose that the rate of surface nuclear reactions is proportional to $(\frac{dM}{dt})_{mag}$; we can suppose that the loss of P elements through the equator is proportional to their number N and to the equatorial mass loss $(\frac{dM}{dt})_{equ}$; we shall also suppose that the rate of loss by dilution in the convective zone is proportional to the number of atoms N. We can write

$$\frac{dN}{dt} = -\alpha\left(\frac{dM}{dt}\right)_{mag} + \beta N\left(\frac{dM}{dt}\right)_{equ} - \gamma N. \tag{51}$$

In region CI we can use the stationary solution

$$N_o = \frac{-\alpha\left(\frac{dM}{dt}\right)_{mag}}{\gamma - \beta\left(\frac{dM}{dt}\right)_{equ}} \quad . \tag{52}$$

In region D we have essentially the time-dependent solution

$$N = N_o\, e^{-\gamma t} - \alpha e^{-\gamma t} \int_o^t e^{\gamma t} \left(\frac{dM}{dt}\right)_{mag} dt. \tag{53}$$

In the case where region CII has merged into the main sequence (small magnetic field), we can use relation (37). We can thus write

$$\left(\frac{dM}{dt}\right)_{mag} = -\frac{M}{t_2}\,\frac{1}{(1 + t/t_o)^3} \quad . \tag{54}$$

If we combine such an expression with (53) we note that the second term

$$J_2 \equiv \alpha e^{-\gamma t} \int_o^t e^{\gamma t}\, \frac{M}{t_2}\left(\frac{1}{1 + t/t_o}\right)^3 dt \tag{55}$$

has a maximum. If we put $\gamma(t + t_o) = y$, we have

$$J_2 = \frac{\alpha M}{t_2}\, t_o^3\, \gamma^2\, e^{-y} \int_{\gamma t_o}^{y} \frac{e^y}{y^3}\, dy, \tag{56a}$$

$$J_2 = \frac{\alpha M}{t_2}\, t_o^3\, \gamma^2\, e^{-y} \left[-\frac{2e^y}{y^2} - \frac{2e^y}{y} + Ei(y)\right]_{\gamma t_o}^{y} \quad . \tag{56b}$$

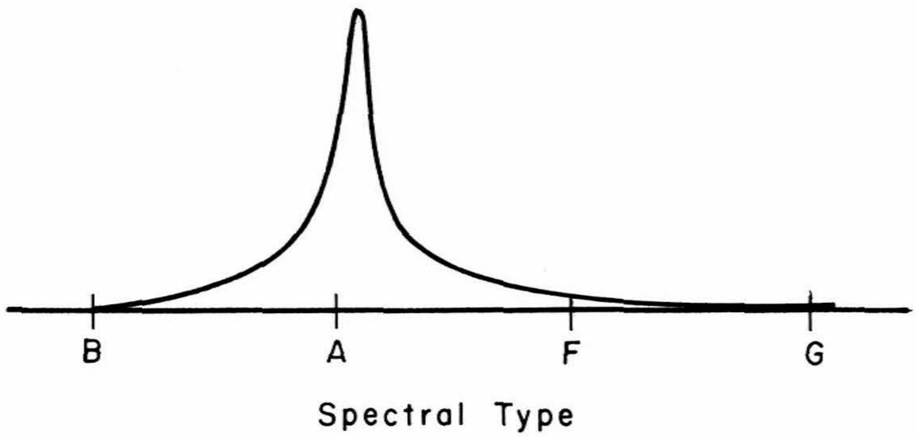

Fig. 3. Expected probability of finding stars with P characteristics

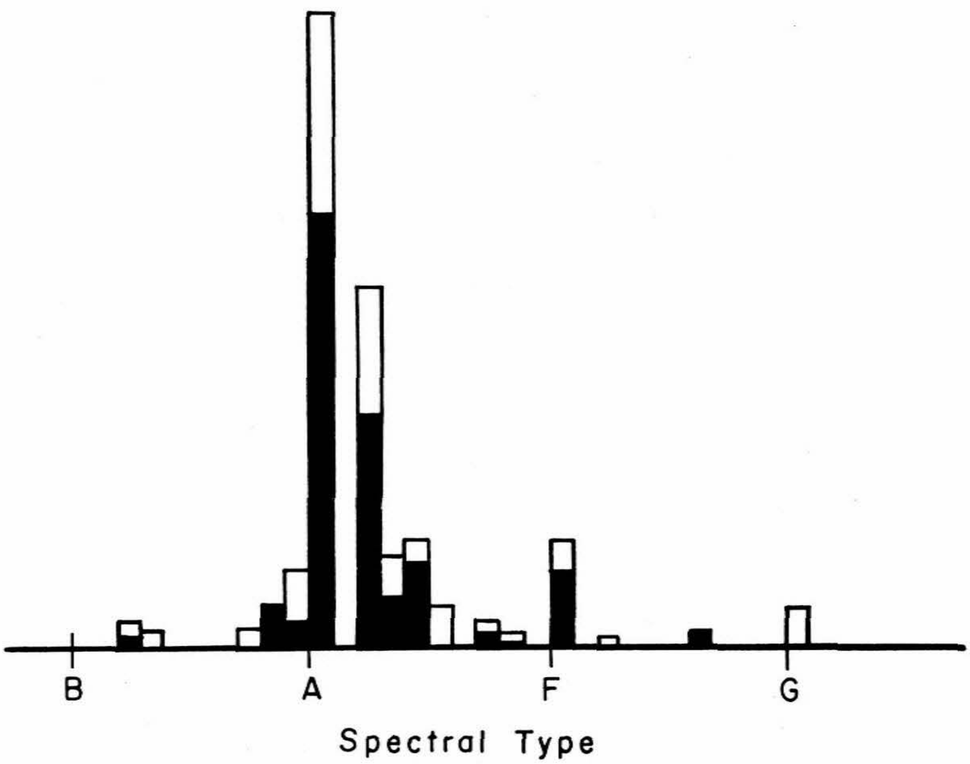

Fig. 4. Babcock's observations of the frequency of stars with P characteristics. Solid bars are magnetic; open bars are probably magnetic.

We can therefor expect that stars which just cross the top of region CII will further increase their amount of P elements.

As a function of spectral type, we should expect the P characteristics to vary as sketched in Figure 3.

We conclude that the probability of finding a peculiar star is larger for the AO spectral type. Note the great analogy of the histogram of Babcock's magnetic and probably magnetic stars (1958) (Figure 4) with the predicted curve.

If we assume that there is a statistical distribution of initial magnetic fields, we must conclude that there should also be a statistical distribution of P characteristics. The greater the P characteristics, the higher the magnetic field. We can conclude that these stars had a larger magnetic activity than the others, and this activity might have continued in region D for a while. Therefor, the A-peculiar stars should rotate more slowly than the others (electromagnetic loss of angular momentum), in agreement with the observed effect.

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