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Direct measurement of the D_s branching fraction to $\phi\pi$

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The Beijing Spectrometer (BES) Collaboration has observed exclusive pair production of D_s mesons at the Beijing Electron-Positron Collider (BEPC) at a center-of-mass energy of 4.03 GeV. The D_s mesons are detected in the $\phi\pi^+$, $\bar{K}^{*0}K^+$, and $\bar{K}^0K^+$ decay modes; two fully reconstructed events yield the value $(3.9\ ^{+5.1}_{-1.9}\ ^{+1.8}_{-1.1})\%$ for the D_s branching fraction to $\phi\pi$. This is the first direct, model-independent measurement of this quantity.

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Many fixed target and e^+e^- experiments have observed inclusive production of D_s mesons, and the branching fractions of many decays are known relative to the branching fraction to $\phi\pi$, $B_{\phi\pi}$ [1]. However, subsequent estimates of absolute branching fractions have required theoretical input. Either the normalization has been based on an estimate of total D_s production, or a theoretical estimate of the ratio $\Gamma(D_s^+ \to \phi e^+\nu)/\Gamma(D^+ \to \bar{K}^{*0}e^+\nu)$ [2] and the measured D_s^+ and D^+ lifetimes have been used to relate $B_{\phi\pi}$ to $B(D^+ \to K^-\pi^+\pi^+)$ [3]. Published results for $B_{\phi\pi}$ [1] range from 3.1 to 5.1 %, despite agreement among the measurements of $B(D_s^+ \to \phi e^+\nu)/B_{\phi\pi}$ (0.49–0.61) from which they are derived [3,4].

In the present experiment, a total integrated luminosity of 22.3 pb^{-1} was accumulated at c.m. energy 4.03 GeV where D_s pairs were exclusively produced in e^+e^- annihilation. This energy was chosen since it is below D_s^* threshold, and since the D_s pair production cross section

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is predicted to reach a local maximum there [5]. It follows that if a D_s meson decay is fully reconstructed in a given event, the recoil system must also correspond to the decay of a D_s meson. Events for which only one D_s decay is fully reconstructed are termed singly tagged, while those for which both decays are fully reconstructed are termed doubly tagged. For a particular decay mode, it follows that the relative rate of detection of doubly and singly tagged events provides a direct measurement of the absolute branching fraction which is independent of the value of the pair production cross section and the integrated luminosity for the experiment. This technique was first used by the Mark III experiment to measure absolute D^0 and D^+ branching fractions [6], and to set the first model-independent upper limit on $B_{\phi\pi}$ [7].

At c.m. energy 4.03 GeV, the Beijing Electron-Positron Collider (BEPC) attains a peak luminosity of 6×10^{30} $\text{cm}^{-2}\,\text{s}^{-1}$. Interaction products are detected in the Beijing Spectrometer (BES), a conventional cylindrical detector described in detail in [8]. A four-layer central drift chamber (CDC) just outside the beam pipe surrounds the interaction region and provides trigger information. Outside the CDC, the 40-layer main drift chamber (MDC) provides tracking and energy loss (dE/dx) information on charged tracks over ~85% of the total solid angle. The momentum resolution is $\sigma_p/p = 0.017\sqrt{1+p^2}$ (p in GeV/c), and the dE/dx resolution is ~11% for hadron tracks. An array of 48 scintillation counters surrounds the MDC and measures the time of flight (TOF) of charged tracks with a resolution of ~450 ps for hadrons. Outside the TOF system, a 12 radiation length, lead-gas barrel shower counter (BSC), operating in self-quenching streamer mode, measures the energies of electrons and photons over ~80% of the total solid angle. Surrounding the BSC is a solenoidal magnet giving a 0.4 T magnetic field in the central tracking region of the detector. Three double layers of muon counters instrument the magnet flux return, and serve to identify muons of momentum greater than 0.5 GeV/c. End cap time-of-flight and shower counters extend coverage to the forward and backward regions.

In defining candidate track combinations for the decay $D_s \to \phi\pi$, only the decay mode $\phi \to K^+K^-$ was considered. Each of the three charged tracks was required to originate from within 1.2 cm of the beam position in the transverse plane, and from within 15 cm of the center of the interaction region along the beam direction. In addition, the polar angle, θ, was required to satisfy $|\cos\theta| < 0.8$ in order to ensure reliable tracking and barrel TOF information. The resulting dE/dx and TOF measurements were required to be consistent with the mass interpretation assigned to the track. Furthermore, a kaon (pion) assignment required $\chi^2_{K(\pi)} < \chi^2_{\pi(K)}$, where the χ^2 function combined the available dE/dx and TOF information for the track in question. Surviving $K^+K^-\pi^\pm$ candidates were subjected to a one-constraint (1C) kinematic fit requiring overall event four-momentum conservation and that the unmeasured recoil system have the same invariant mass as the candidate three-track system. Candidates with a fit confidence level >1%, a K^+K^- mass within 18 MeV of the nominal ϕ mass, and $|\cos\theta_K| > 0.25$ [9] were retained. The resulting distribution in $\phi\pi$ mass, calculated using fitted momentum vectors, is shown in Fig. 1, and exhibits a clear signal at the D_s mass position. An unbinned maximum likelihood fit [10] to this distribution yields a D_s mass value of 1968.7±0.6 MeV and a D_s signal, $N_{\phi\pi}$, of 40.8±7.2 events. The curve shown in Fig. 1 corresponds to the fit result.

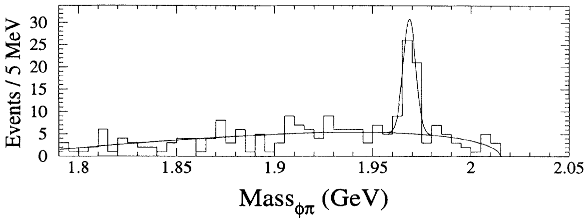

FIG. 1. The distribution in $\phi\pi$ invariant mass, calculated using fitted momentum vectors, for candidates satisfying the 1C fit described in the text at a confidence level greater than 1%; the selection criteria on $M_{K^+K^-}$ and the helicity of the K^+ in the ϕ rest frame have also been applied.

The product of the cross section for D_s pair production, $\sigma_{D_s^+D_s^-}$, and $B_{\phi\pi}$ is related to $N_{\phi\pi}$ by

$$\sigma_{D_s^+D_s^-} \cdot B_{\phi\pi} = \frac{N_{\phi\pi}}{2\epsilon_{\phi\pi}\int \mathcal{L}dt}, \tag{1}$$

where $\epsilon_{\phi\pi}$ is the efficiency for reconstructing $D_s^+ \to \phi\pi^+$ with $\phi \to K^+K^-$, and $\int \mathcal{L}dt$ is the integrated luminosity for the analyzed data set. The reconstruction efficiency was estimated by Monte Carlo simulation to be 8.14% [including $B(\phi \to K^+K^-)$], and an integrated luminosity of 22.3±0.9 pb^{-1} was obtained from the study of wide-angle Bhabha scattering events. Equation (1) then yields $\sigma_{D_s^+D_s^-} \cdot B_{\phi\pi} = 11.2 \pm 2.0(\text{stat}) \pm 2.5(\text{syst})$ pb. The systematic uncertainty contains contributions from the uncertainty in the integrated luminosity measurement, from the sensitivity of the D_s signal to quantitative changes in the selection criteria, and from variation of the background parametrization in the fit to the data of Fig. 1. The present model-dependent world average for $B_{\phi\pi}$ is 3.5±0.4% [1]; this, together with the above value of $\sigma \cdot B_{\phi\pi}$, gives $\sigma_{D_s^+D_s^-} = 320 \pm 56(\text{stat}) \pm 81(\text{syst})$ pb. This is significantly smaller than the predicted value at this energy (~750 pb) [5].

The direct measurement of $B_{\phi\pi}$ in the present experiment requires, in addition to $N_{\phi\pi}$, a measurement of the number of pair production events in which both D_s decays are fully reconstructed. If the search for such events were restricted to only the $\phi\pi$ decay mode, the values of $N_{\phi\pi}$, the reconstruction efficiency, and the world average for $B_{\phi\pi}$ [1] imply that fewer than 0.1 doubly tagged events would be expected in the present experiment [see Eq. (2) below]. It follows that additional D_s decay modes must be incorporated into the analysis in order to significantly increase the probability of obtaining a nonzero result. For this reason, the decay modes $D_s^+ \to \bar{K}^{*0}K^+$, $\bar{K}^{*0} \to K^-\pi^+$, and $D_s^+ \to \bar{K}^0K^+$, $K^0 \to K_s \to \pi^+\pi^-$

are included in the search for doubly tagged events [11]. The expected number of doubly tagged events, $\langle N\rangle$, is then expressed in terms of $N_{\phi\pi}$, and $B_{\phi\pi}$ as

$$\begin{aligned}\langle N\rangle &= [\sigma_{D_s^+D_s^-} B_{\phi\pi}]\,(\textstyle\int\mathcal{L}\,dt)\,B_{\phi\pi}\sum b_i b_j \epsilon_{ij} + N_{\text{bg}}\\ &= \frac{N_{\phi\pi}}{2\epsilon_{\phi\pi}}\cdot B_{\phi\pi}\cdot\sum b_i b_j\epsilon_{ij} + N_{\text{bg}}\\ &= \kappa\cdot N_{\phi\pi}\cdot B_{\phi\pi} + N_{\text{bg}},\end{aligned} \tag{2}$$

where Eq. (1) has been used to eliminate the explicit dependence on $\sigma_{D_s^+D_s^-}$ and the integrated luminosity, and the summation is over the six possible final states corresponding to the decay of either D_s to $\phi\pi$, $\bar K^{*0}K$, or $\bar K^0 K$. The factor ϵ_{ij} in the summation is the efficiency for reconstructing the $D_s \to$ mode i vs $\bar D_s \to$ mode j final state, determined from Monte Carlo simulation, and the b_i are the branching ratios $B(D_s \to \text{mode } i)/B_{\phi\pi}$ taken from [1]. The term N_{bg} represents the background level in the doubly tagged signal region, and the constant, κ, which combines the detector efficiencies and branching ratios, has the value 1.14±0.04 for this experiment.

In the search for doubly tagged D_s events, charge-balanced events with exactly six tracks were kinematically fit, subject to four-momentum conservation and the condition that the invariant masses of both candidate D_s systems in an event be the same. Events with a fit confidence level less than 0.1% were rejected. Tracks in the remaining events were required to have TOF and dE/dx information consistent with their assigned mass hypotheses, and the K^+K^-, $K^-\pi^+$, and $\pi^+\pi^-$ submasses were required to be within 18, 50, and 20 MeV of the nominal values of the ϕ, $\bar K^{*0}$, and K^0 masses, respectively. Additionally, the helicity angle of the K^- from the decay of the $\bar K^{*0}$ was required to satisfy $|\cos\theta_{K^*\to K\pi}| > 0.4$. The mass distribution, calculated using the 5C fitted momentum vectors, for events passing the selection criteria is shown in Fig. 2; two events lie in the signal region, which is defined to be within 6 MeV of the nominal value of the D_s mass [12]. Table I compares the invariant mass and resonant submasses of the doubly tagged candidates calculated using both 1C and 5C fitted momentum vectors; the 1C and 5C fits yield consistent results. As confirmation, the same two events were also selected by an analysis where the 5C fit was replaced by selection criteria applied to the overall momentum and to the energy of each D_s candidate. The background level in the signal region was estimated to be 0.1–0.3 events using the nonsignal regions in the mass distributions resulting from the kinematic fit (Fig. 2) and the confirming analysis. Additionally, Monte Carlo generated $e^+e^- \to D^{*+}D^-, D^{*0}\bar D^0, D^{*+}D^{*-}$, and $D^{*0}\bar D^{*0}$ events were analyzed using the full kinematic fit procedure and gave an estimate of 0.1 background events in the signal region. Combining these results, the background level in the D_s signal region, N_{bg}, was conservatively estimated as 0.2±0.2 events.

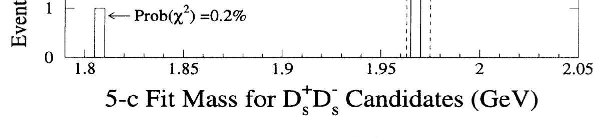

FIG. 2. The distribution in candidate D_s invariant mass, calculated using fitted momentum vectors, for the events which satisfy the 5C fit described in the text at a confidence level greater than 0.1%; the 1C fit masses corresponding to the events in the D_s signal region are given in Table I.

TABLE I. Doubly-tagged $D_s^+D_s^-$ events.

Event	Using 1C fit momentum vectors D_s mass (MeV)	 Resonant submass (MeV)	Using 5C fit momentum vectors D_s mass (MeV)	 Resonant submass (MeV)
$\phi\pi$ vs	1969	1014	1968	1012
$\bar K^{o*}K$	1963	847		853
$K^{o*}\bar K$ vs	1968	876	1967	875
$\bar K^{o*}K$	1962	915		918

The value of $B_{\phi\pi}$ corresponding to two events in the signal region was obtained by means of a maximum likelihood fit. The likelihood function, $L(B_{\phi\pi}, N_{\phi\pi})$, was constructed from Eq. (2) with $N_{\text{bg}} = 2$ using a Poisson distribution to describe the number of doubly tagged events and a Gaussian distribution to describe the singly tagged sample:

$$L(B_{\phi\pi}, N_{\phi\pi}) = \frac{\langle N\rangle^{N_{\text{obs}}} e^{-\langle N\rangle}}{N_{\text{obs}}!}\,\frac{\exp[-\frac{1}{2}(\frac{N_{\phi\pi}-N_{\phi\pi,\text{obs}}}{\sigma_{\phi\pi}})^2]}{\sqrt{2\pi}\cdot\sigma_{\phi\pi}}, \tag{3}$$

where N_{obs} is the measured number of doubly tagged events ($N_{\text{obs}} = 2$), and $N_{\phi\pi,\text{obs}} \pm \sigma_{\phi\pi}$ is the result of the fit to the number of singly tagged events ($N_{\phi\pi,\text{obs}} \pm \sigma_{\phi\pi} = 40.8\pm7.2$). The value of the likelihood function is marginalized by integrating over $N_{\phi\pi}$,

$$L(B_{\phi\pi}) = \int L(B_{\phi\pi}, N_{\phi\pi})\, dN_{\phi\pi}, \tag{4}$$

and it is shown as a function of $B_{\phi\pi}$ in Fig. 3. The maximum likelihood solution is $B_{\phi\pi} = (3.9\,^{+5.1}_{-1.9})\%$, where the

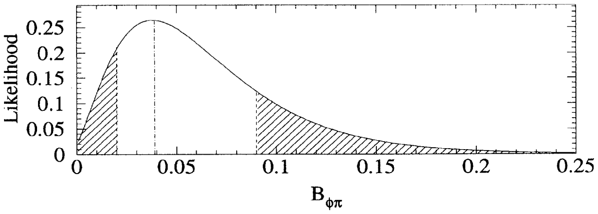

FIG. 3. The variation of the marginalized likelihood function described in the text with $B_{\phi\pi}$; the dashed lines correspond to the ±1σ statistical error estimates.

TABLE II. Contributions to the systematic error on $B_{\phi\pi}$.

Quantity	Error on $B_{\phi\pi}$ (%)	
Selection criteria	+1.5	-0.4
κ	+0.1	-0.1
N_{bg}	+0.5	-0.4
$N_{\phi\pi}$ (systematic uncertainty only)	+0.9	-0.9
Total	+1.8	-1.1

statistical errors are obtained by integrating the function; the area under the curve between the peak value and the -1σ ($+1\sigma$) value corresponds to 68.3% of the total area below (above) the peak position.

The systematic uncertainty in $B_{\phi\pi}$ has contributions from the uncertainties in the background level, N_{bg}, in the detector efficiencies and D_s branching ratios combined together in κ, in the number of singly tagged D_s mesons, $N_{\phi\pi}$, and in the sensitivity of the value of $B_{\phi\pi}$ to the selection criteria. These contributions are summarized in Table II. After combining the systematic errors, the final result for $B_{\phi\pi}$ obtained in this experiment is

$$B_{\phi\pi} = [3.9\ ^{+5.1}_{-1.9}(\text{stat})\ ^{+1.8}_{-1.1}(\text{syst})]\%.$$

In conclusion, the product of the D_s production cross section at c.m. energy 4.03 GeV and the $\phi\pi$ branching fraction has been measured by the Beijing Spectrometer (BES) Collaboration to be 11.2±2.0±2.5 pb. This result, when combined with the 1994 world average for $B_{\phi\pi}$, implies a D_s pair production cross section a factor of 2 smaller than expected [5]. Doubly tagged D_s events have been reconstructed, and the first direct measurement of the D_s branching fraction to $\phi\pi$ has been obtained: $B_{\phi\pi} = (3.9\ ^{+5.1}_{-1.9}\ ^{+1.8}_{-1.1})\%$. This result is consistent with the present world average of values resulting from indirect or model-dependent procedures, $B_{\phi\pi} = (3.5\pm0.4)\%$ [1]. It may be possible to reduce the uncertainties in the present measurement by including additional D_s decay modes; however, a significant increase in integrated luminosity would be necessary to improve the precision to that of the present world average.

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[1] A summary of experimental measurements of D_s branching ratios can be found in the meson full listings of the 1994 Review of Particle Properties by the Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).

[2] Throughout the paper, reference to a specific charge configuration also implies reference to the conjugate charge configuration.

[3] For a description of the technique, see F. Butler *et al.*, Phys. Lett. B **324**, 255 (1994).

[4] F. Muheim and S. Stone, Phys. Rev. D **49**, 3767 (1994); P. L. Frabetti, Phys. Lett. B **313**, 253 (1993); H. Albrecht, *ibid.* **255**, 634 (1991); J. Alexander *et al.*, Phys. Rev. Lett. **65**, 1531 (1990); W. Braunschweig and R. Gerhards, Z. Phys. C **35**, 317 (1987).

[5] E. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980); W. S. Lockman, Mark III memorandum, "D and D_s Production in the Range $3.8 < \sqrt{s} < 4.5$ GeV," 1987 (unpublished).

[6] J. Adler *et al.*, Phys. Rev. Lett. **60**, 89 (1988).

[7] J. Adler *et al.*, Phys. Rev. Lett. **64**, 169 (1990).

[8] J. Z. Bai *et al.*, Nucl. Instrum. Methods A **344**, 319 (1994).

[9] Since the D_s meson has spin zero, the ϕ meson is longitudinally polarized. Therefore, the angle between the K^+ momentum in the ϕ rest frame and the ϕ momentum in the D_s rest frame follows a $\cos^2\theta_K$ distribution.

[10] A Gaussian function was assumed for the signal. The background shape used was the product of a phase space function, accounting for the production of the $\phi\pi$ system and its equal mass recoil, and a linear function of the $\phi\pi$ mass.

[11] J. Z. Bai *et al.*, Phys. Rev. Lett. **74**, 4599 (1995).

[12] The rms deviation of the invariant mass distribution calculated with 5C fit momentum vectors was found from Monte Carlo simulation to be 2 MeV.