

${}^7\text{Be}(p, \gamma){}^8\text{B}$ cross section and the properties of ${}^7\text{Be}$

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We study the nonresonant part of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction using a three-cluster resonating group model that is variationally converged and virtually complete in the ${}^4\text{He} + {}^3\text{He} + p$ model space. The importance of using adequate nucleon-nucleon interaction is demonstrated. We find that the low-energy astrophysical S factor is linearly correlated with the quadrupole moment of ${}^7\text{Be}$. A range of parameters is found where the most important ${}^8\text{B}$, ${}^7\text{Be}$, and ${}^7\text{Li}$ properties are reproduced simultaneously; the corresponding S factor at $E_{c.m.} = 20$ keV is 24.6–26.1 eV b.

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The flux of high-energy neutrinos generated in the solar core is directly proportional to the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction rate. Thus, knowledge of S_{17} , the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ S factor at solar energies (center-of-mass energy $E \approx 20$ keV), is crucial to conclusions drawn from present (Homesake, Kamiokande) and future (SNO, Superkamiokande) solar neutrino experiments [1,2]. Despite extensive experimental efforts, the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ cross section is still the most uncertain nuclear input to the standard solar model [1,3] due to a significant spread among the values of S_{17} deduced from the various experiments (direct capture [4]: $S_{17} = 18\text{--}28$ eV b and Coulomb breakup [5]: $S_{17} = 16.7 \pm 3.5$ eV b). Theoretical estimates also vary ($S_{17} = 16\text{--}30$ eV b) [6], making these predictions rather unreliable. Some of the theory underlying our understanding of this reaction can be found in Refs. [7,8].

The aim of this paper is to constrain more tightly the theoretical value of S_{17} . To this end, we study the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction in a microscopic three-cluster (${}^4\text{He} + {}^3\text{He} + p$) approach. This model is currently the closest approximation to a full solution of the microscopic eight-nucleon problem with a consistent treatment of bound and scattering states. As we will demonstrate below, our approach is superior (at least theoretically) to all previous studies of the low-energy ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction, and allows us to investigate correlations between S_{17} and the properties of the participating nuclear systems, similar to the approach of Ref. [9] for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction.

Adopting a microscopic three-cluster (${}^4\text{He} + {}^3\text{He} + p$) ansatz for the eight-nucleon system, our trial function reads

$$\Psi = \sum_{(ij)k, S, l_1, l_2, L} \mathcal{A} \left\{ \left[\left[\Phi^i(\Phi^j \Phi^k) \right]_S \chi_{[l_1 l_2]L}^{i(jk)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \right]_{JM} \right\}, \quad (1)$$

where the indices i , j , and k denote any of the labels ${}^4\text{He}$, ${}^3\text{He}$, and p . The intercluster antisymmetrizer is \mathcal{A} , the cluster internal states Φ are translationally invariant harmonic oscillator shell model states, the $\boldsymbol{\rho}$ vectors are the intercluster Jacobi coordinates, [...] denotes angular momentum coupling, and the sums over S , l_1 , l_2 , and L include all angular momentum configurations of any significance. This same model was used in Ref. [10] in

the study of the ground state of ${}^8\text{B}$; further details on the model space and other aspects can be found there. The intercluster dynamics is determined by inserting (1) into the eight-nucleon Schrödinger equation using the two-nucleon strong and Coulomb interactions. In addition to the full model space calculation, which contains all three possible arrangements of the three clusters, we also present a restricted calculation involving only (${}^4\text{He}{}^3\text{He}$) p configurations (${}^7\text{Be} + p$ type model space), analogous to simple ${}^7\text{Be} + p$ potential model studies such as those of Refs. [8,11].

It is well known that the low-energy ${}^7\text{Be}(p, \gamma){}^8\text{B}$ cross section is strongly dominated by $E1$ capture. Previous microscopic calculations have shown that $M1$ capture only plays a role in the vicinity of the 1^+ resonance at $E = 640$ keV and is negligible at astrophysical energies [12], while $E2$ capture is tiny at $E < 500$ keV and can safely be ignored. Our calculations confirm that these multipolarities are unimportant at low energies. We have therefore calculated the $E1$ capture cross section into the ${}^8\text{B}$ ground state in perturbation theory [12], describing the initial scattering states and the ${}^8\text{B}$ ground state by the many-body wave functions determined in our microscopic three-cluster approach.

The capture cross section depends upon the bound (${}^8\text{B}$) and the scattering (${}^7\text{Be} + p$) wave functions. At energies far below the Coulomb barrier, the capture takes place at large ${}^7\text{Be} - p$ distances, so that these wave functions must be accurate to distances of a few hundred fm, which requires a reliable method to determine the unknown relative motion functions χ in (1). We expand these functions in terms of products of basis functions of the Jacobi coordinates, which allow us to reduce the three-cluster wave functions (1) to equivalent two-cluster forms [13].

We use the variational Siegert method to determine the ${}^8\text{B}$ bound state [14]. The trial state contains tempered Gaussian functions [15] plus a term with the correct outgoing Whittaker asymptotics in the ${}^7\text{Be} + p$ partitions. Using such a trial function in a linear variational method leads to a transcendental equation for the binding energy, which can be solved iteratively. To be able to calculate every many-body matrix element analytically, we match the external Whittaker functions with internal Gaussians, using a modified version of the technique

described in Ref. [16]. The numerical accuracy of this procedure is better than 1–2% in S_{17} .

The scattering wave functions were calculated using the variational Kohn-Hulthén method [16], which ensures the correct scattering asymptotics. To achieve high accuracy we avoid the use of complex wave functions and so neglect channel coupling between different angular momentum channels; this approximation is certainly justified at astrophysical energies, where the capture occurs far outside the range of the strong forces. The present scattering solution is numerically well conditioned for $E > 3$ keV, and its numerical accuracy is better than 0.1%.

The bulk of our calculations use the Minnesota (MN) effective nucleon-nucleon interaction [17], which contains central and spin-orbit terms. This force reproduces the most important properties of the low-energy $N + N$ and ${}^4\text{He} + N$ scattering phase shifts and the low-energy ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction cross section well enough to appear suitable for the problem at hand. However, we also present calculations with other effective NN interactions. Note that the tensor component of the effective NN interaction in microscopic cluster models is not well constrained [10] and is usually ignored. Nevertheless, we have also performed a calculation including a tensor force, which, at the least, gives the correct low-energy order of the triplet-odd $N + N$ phase shifts [10].

The free parameters in our model are the size parameter (β) in the ${}^4\text{He}$ and ${}^3\text{He}$ cluster model functions (technical reasons force us to use the same value for both ${}^4\text{He}$ and ${}^3\text{He}$), the exchange mixture parameter of the central part of the effective NN interaction, and the strength of the spin-orbit force. It is generally preferable to adjust these parameters to independent data. However, a meaningful study of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction at low energies requires the *exact* reproduction of the experimental ${}^8\text{B}$ binding energy (137 keV) as this determines the asymptotic behavior of the bound state in the ${}^7\text{Be} + p$ channel. We have guaranteed this by the appropriate choice of the exchange mixture parameter. The strength of the spin-orbit force was adjusted to the experimental splitting between the $3/2^-$ and $1/2^-$ ${}^7\text{Be}$ states. We have varied β , thus changing our description of the ${}^7\text{Be}$ properties.

As is demonstrated by the open circles in Fig. 1, S_{17} scales linearly with the quadrupole moment of ${}^7\text{Be}$, $Q_{7\text{Be}}$. This linear dependence can be understood as follows. As the capture process takes place at very large ${}^7\text{Be} - p$ distances, where the bound state wave function must be proportional to a fixed Whittaker function, the low-energy cross section depends almost exclusively on the square of the asymptotic normalization factor, \bar{c} [7,18]. Let us compare calculations with different ${}^7\text{Be}$ wave functions, which give different ${}^7\text{Be}$ radius, quadrupole moment, etc., but with fixed binding energy of ${}^8\text{B}$. The effective local potentials between ${}^7\text{Be}$ and p have different radii, which means that the height of the Coulomb barrier is larger if the potential radius (and the ${}^7\text{Be}$ radius) is smaller. Consequently, the probability of finding the proton in the outside region decreases as the size of the ${}^7\text{Be}$ nucleus becomes smaller. But as the shape of the external wave function is fixed, this smaller probability must stem from

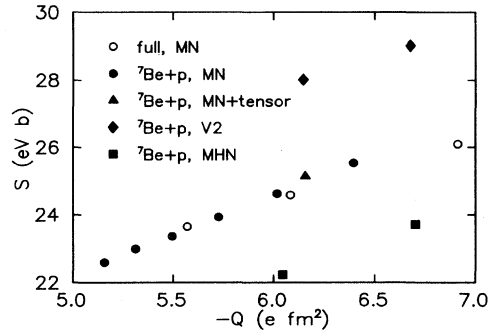


FIG. 1. The astrophysical S factor of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction as a function of the negative of the ${}^7\text{Be}$ quadrupole moment. The symbols are explained in the text.

a smaller normalization constant \bar{c} . It is easy to see that this leads \bar{c}^2 , and consequently S_{17} , to be linearly proportional to either $r_{7\text{Be}}^2$ or $Q_{7\text{Be}}$. Note that this relation is not changed if a tensor component is added to the MN interaction (see triangle in Fig. 1). We find the same linear $S_{17} - Q_{7\text{Be}}$ relation in our truncated calculation considering only the ${}^7\text{Be} + p$ model space. Results of these restricted calculations are shown in Fig. 1 as full circles. We note that the ${}^3\text{He}({}^4\text{He}, \gamma){}^7\text{Be}$ reaction shows a similar correspondence between $S_{7\text{Be}}$ and $Q_{7\text{Be}}$ [9]. However, the present case is more complicated since various subsystems like ${}^7\text{Be}$, ${}^5\text{Li}$, and ${}^4\text{Li}$ have nonzero quadrupole moments. In particular, the ${}^7\text{Be}$ core has large nontrivial contributions to the ${}^8\text{B}$ quadrupole moment [19], which makes a study of the $S_{17} - Q_{8\text{B}}$ correlation rather inconclusive.

Unfortunately the linear relation is not sufficient to determine S_{17} indirectly by measuring the ${}^7\text{Be}$ quadrupole moment, as this relation depends upon the effective NN interaction used. To demonstrate this, we have performed calculations within the ${}^7\text{Be} + p$ model space using the Volkov force V2 and the modified Hasegawa-Nagata (MHN) force, both of which have been used in previous microscopic cluster calculations of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction at low energies [12,20,21]. While both forces also show the linear dependence between S_{17} and the ${}^7\text{Be}$ quadrupole moment, the V2 force yields larger values for S_{17} for a given $Q_{7\text{Be}}$ (diamonds in Fig. 1), while the MHN force yields smaller values (squares). These differences can be traced to the different quality of the description of the $N + N$ systems (phase shifts, energy and radius of the deuteron) by these forces. For example, while the MN force well reproduces the experimental deuteron energy and radius, the V2 force underbinds the deuteron by 1.6 MeV (however, it unphysically binds the singlet dinucleon states) and the MHN force overbinds it by 4.4 MeV. We note that the M3Y interaction, which was used in Ref. [22] in an external capture approach to predict a very small ${}^7\text{Be}(p, \gamma){}^8\text{B}$ cross section ($S_{17} = 16.5$ eV b), also overbinds the deuteron. Motivated by its successful description of the NN system and the various two-cluster subsystems, we adopt the Minnesota (MN) force for a detailed study of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction. We note, however, that the MHN force also gives a good description of the various subsystems and S_{17} values less than 10%

smaller. Cluster calculations using the V2 force should be regarded with care (see also [23]).

Accepting the MN force as adequate for the eight-nucleon problem, our result for S_{17} could be read off Fig. 1 if the ${}^7\text{Be}$ quadrupole moment were known. Absent this information, we will estimate a best S_{17} value by constraining the ${}^4\text{He}$ and ${}^3\text{He}$ cluster size parameter to reproduce (i) the binding energy of ${}^7\text{Be}$ with respect to ${}^4\text{He} + {}^3\text{He}$; (ii) the squared sum of the ${}^4\text{He}$ and ${}^3\text{He}$ radii; (iii) the quadrupole moment of ${}^7\text{Li}$ (as a surrogate for the unknown quadrupole moment of the analog nucleus ${}^7\text{Be}$). These requirements ensure that both the ${}^7\text{Be}$ bound states and the ${}^4\text{He} - {}^3\text{He}$ relative motion are well described. The second requirement is fulfilled by choosing $\beta = 0.4 \text{ fm}^{-2}$. With this choice, the ${}^7\text{Be}$ ground state is slightly underbound by 200 keV, while the excitation energy of the $1/2^-$ state is reproduced ($E^* = 0.43 \text{ MeV}$). The calculated energies and widths of the first $7/2^-$ ($E^* = 4.77 \text{ MeV}$, $\Gamma = 0.28 \text{ MeV}$) and $5/2^-$ states (5.85 MeV, 0.9 MeV) are in good agreement with experiment ($E^* = 4.57 \text{ MeV}$, $\Gamma = 0.18 \text{ MeV}$ and $E^* = 6.7 \text{ MeV}$, $\Gamma = 1.2 \text{ MeV}$, respectively.) The quadrupole moment of ${}^7\text{Li}$ is calculated as -4.10 e fm^2 , to be compared with the experimental value $-4.05 \pm 0.08 \text{ e fm}^2$ [24]. We calculate the ${}^5\text{Li} + {}^3\text{He}$ threshold at 3.39 MeV, close to the experimental value of 3.69 MeV. Our model predicts the width of the ${}^5\text{Li}$ ground state as 1.64 MeV, while the experimental value is 1.5 MeV. We use the exchange mixture parameter $u = 1.025$. This value, close to a Serber mixture ($u = 1$), indicates that the trial wave function describes the nuclear system properly [25]. For the squared sum of the ${}^3\text{He} + {}^4\text{He}$ point nucleon matter radii we obtain 5.31 fm^2 .

We conclude that our model gives a good description of the $p + {}^3\text{He} + {}^4\text{He}$ system. We then obtain an S_{17} value of 26.1 eV b, while the ${}^7\text{Be}$ quadrupole moment is -6.9 e fm^2 . Our approach calculates the quadrupole moment of ${}^8\text{B}$ as 7.45 e fm^2 , while the experimental value is $(6.83 \pm 0.21) \text{ e fm}^2$ [26]. Even if one concludes from these comparisons that our ${}^7\text{Be}$ quadrupole moment is also slightly too large, we note that a 10% reduction in this quantity would only decrease S_{17} to 24.8 eV b.

If we use the same cluster size parameter in the restricted ${}^7\text{Be} + p$ space as in the full calculation ($\beta = 0.4 \text{ fm}^{-2}$), we find that the ${}^7\text{Be}$ nucleus is overbound (by 600 keV) [27], while its quadrupole moment is reduced to -6.0 e fm^2 . To compensate for the reduced flexibility of the trial wave function, the exchange mixture parameter had to be increased to $u = 1.085$. The quadrupole moments of ${}^7\text{Li}$ (-3.46 e fm^2) and ${}^8\text{B}$ (6.55 e fm^2) are slightly smaller than the experimental values. In this restricted calculation we find S_{17} to be 24.6 eV b.

Since both the full and restricted ${}^7\text{Be} + p$ model spaces predict the same linear dependence of S_{17} on the ${}^7\text{Be}$ quadrupole moment and these calculations bracket the experimental ${}^7\text{Li}$ and ${}^8\text{B}$ quadrupole moments, we conclude that the microscopic three-cluster calculations predict S_{17} to be between 24.6 and 26.1 eV b. We note that previous microscopic cluster calculations, although employing different NN interactions, obtained similarly large values for S_{17} (Refs. [12,20,21,28]), in contrast to

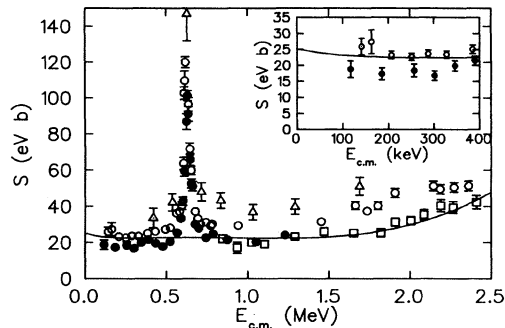


FIG. 2. Energy dependence of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ astrophysical S factor. The symbols denote the experimental data of Ref. [31] (open circles), Ref. [33] (filled circles), Ref. [32] (squares), and Ref. [34] (triangles). The inset shows the low-energy part on a magnified scale.

the smaller predictions (16.5 eV b [22,23], 16.9 eV b [29], and 17 eV b [30]). We also note that S_{17} deduced from our model is consistent with the value deduced from the direct capture data ($22 \pm 2 \text{ eV b}$ [21], and $24 \pm 2 \text{ eV b}$ [4], respectively).

Less elaborate microscopic cluster calculations have been presented in Refs. [12,20,21,28]. While the two earlier studies [12,28] were restricted to a simple ${}^7\text{Be} + p$ model space, Ref. [20] recently improved these studies by including a ${}^5\text{Li} + {}^3\text{He}$ rearrangement channel. However, in Ref. [20] the ${}^7\text{Be}$ nucleus is described by only one Gaussian basis function between ${}^4\text{He}$ and ${}^3\text{He}$, which means that the three-cluster wave function is not free for the variational method. A more flexible trial function would result in the collapse of the artificially fixed wave function. Moreover, in Ref. [20] the description of the ${}^7\text{Be}$ nucleus is rather unphysical, as it is unbound relative to the ${}^4\text{He} + {}^3\text{He}$ threshold. In Ref. [21] there are two basis functions for ${}^7\text{Be}$, with carefully chosen parameters, and the most important angular momentum configurations of the ${}^7\text{Be} + p$ type partition are present. In the present model we use six states for ${}^7\text{Be}$ (and ten in the ${}^7\text{Be} - p$ relative motion, and six in all other relative motions) and include all relevant angular momentum channels. Our test calculations showed that the present three-cluster model space is virtually complete, which means that our results are free from the artifacts of an unconverged or incomplete model. Although the incompleteness of the previous works makes the comparison difficult, our results are qualitatively in good agreement with Refs. [20] and [21]. Referring to Fig. 1, this is, however, not surprising as the ${}^7\text{Li}$ quadrupole moment, and thus presumably also $Q_7\text{Be}$, is well described in these studies.

In Fig. 2, we show the energy dependence of the S factor, calculated with the ${}^7\text{Be} + p$ model space, the MN force, and $\beta = 0.4 \text{ fm}^{-2}$. At low energies our calculated S factor is in rather close agreement with the direct capture data of Refs. [31] and [34], but it is higher than those of Ref. [33] and the preliminary results deduced from a Coulomb dissociation experiment [5]. Although our S factor appears to agree well with the data of Ref. [32] for $E > 1 \text{ MeV}$, this is likely to change, if the coupling between the different angular momentum channels in the

${}^7\text{Be} + p$ scattering and, more importantly, other multipoles ($M1$ and $E2$) are taken into account.

In summary, we have studied the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction in a microscopic model that is virtually complete in the three-cluster model space at low energies. We found that the low-energy astrophysical S factor is strongly correlated with the properties of ${}^7\text{Be}$ (e.g., its quadrupole moment). For a set of parameters that reproduce simultaneously the most important properties of ${}^7\text{Be}$, ${}^7\text{Li}$, and ${}^8\text{B}$, we predict $Q_{{}^7\text{Be}}$ to be between $-6.0 e\text{fm}^2$ and $-6.9 e\text{fm}^2$ and find $S_{17} = 24.6\text{--}26.1 e\text{Vb}$, in agreement with direct capture results and the currently adopted value in the standard solar model. If it turns out that the S factor is considerably lower than our present value [5], then the present three-cluster approach is inappropriate

and physics beyond our model (larger eight-body model space, improved effective interaction) has to be invoked. Although we found that the Minnesota force was suitable for the present work, the construction and use of other high quality interactions would be useful. We also note that a precise measurement of the ${}^7\text{Be}$ quadrupole moment or radius could test the self-consistency of our conclusions.

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