Supplementary Material for 'A Physical Model for Seismic Noise Generation from Sediment Transport in Rivers'

Victor C. Tsai, Brent Minchew, Michael P. Lamb, and Jean-Paul Ampuero

1 Evaluation of Phase Velocity

With shear wave speed as given in Eq. (4) of the main text as $v_s = v_0(z/z_0)^\alpha$, and assuming Rayleigh-wave sensitivity that decays with depth proportional to e^{-kz}, then the Rayleigh-wave phase velocity can be approximately expressed as

$$v_c \approx \frac{\int_0^\infty v_s(z)e^{-kz}dz}{\int_0^\infty e^{-kz}dz} = \frac{kv_0}{z_0^\alpha}\int_0^\infty z^\alpha e^{-kz}dz = \frac{v_0\Gamma(1+\alpha)}{z_0^\alpha k^\alpha}. \tag{1}$$

Substituting $k = 2\pi f/v_c$, one can then solve for v_c as

$$v_c = \left[\frac{v_0\Gamma(1+\alpha)}{z_0^\alpha(2\pi f)^\alpha}\right]^{1/(1-\alpha)} = \left[\frac{v_0\Gamma(1+\alpha)}{(2\pi z_0 f_0)^\alpha}\right]^{1/(1-\alpha)}\left[\frac{f}{f_0}\right]^{-\alpha/(1-\alpha)} \tag{2}$$

2 Approximation of $\chi(\beta)$

$\chi(\beta)$ is defined in Eq. (8) of the main text as

$$\chi(\beta) \equiv \int_{-\infty}^{\infty}\frac{1}{\sqrt{1+s^2}}e^{-\beta\sqrt{1+s^2}}ds. \tag{3}$$

To approximately evaluate this integral, we consider the limits $\beta \ll 1$ and $\beta \gg 1$. For $\beta \ll 1$, $\chi(\beta)$ can be approximated as

$$\chi(\beta) \approx 2\int_0^1\frac{1}{\sqrt{1+s^2}}ds + 2\int_1^\infty\frac{1}{s}e^{-\beta s}ds = 2\sinh^{-1}1 + 2\Gamma(0,\beta), \tag{4}$$

where $\Gamma(0,\beta)$ is the incomplete gamma function. $\Gamma(0,\beta)$ can be approximated as $\Gamma(0,\beta) \approx e^{-\beta}\log(1+1/\beta)$. For $\beta \ll 1$, $\Gamma(0,\beta) \gg \sinh^{-1} 1$, so we finally have

$$\chi(\beta) \approx 2e^{-\beta}\log(1+1/\beta), \beta \ll 1. \tag{5}$$

On the other hand, when $\beta \gg 1$, $\chi(\beta)$ can be approximated as

$$\chi(\beta) \approx 2\int_0^\infty e^{-\beta(1+s^2/2)} ds = e^{-\beta}\sqrt{\frac{2\pi}{\beta}}, \beta \gg 1. \tag{6}$$

Smoothly transitioning between Eq. (5) and (6) with an exponential weighting then results in

$$\chi(\beta) \approx 2\log\left(1+\frac{1}{\beta}\right)e^{-2\beta} + (1-e^{-\beta})e^{-\beta}\sqrt{\frac{2\pi}{\beta}}. \tag{7}$$

3 Estimation of Local Water Depth

For given slope S, channel width W, and channel depth H, the total water flux $Q = WHU$, where average velocity U is given in the main text and can be written as $U \propto H^{2/3}S^{1/2}$ so that $Q \propto WH^{5/3}S^{1/2}$. Thus,

$$H_2 = H_1\left(\frac{Q_2}{Q_1}\frac{W_1}{W_2}\frac{S_1^{1/2}}{S_2^{1/2}}\right)^{3/5}, \tag{8}$$

where subscripts refer to different locations. Since W and S can be estimated from imagery, and *Lave and Avouac* [2001] provide estimates of Q along the Trisuli, we can estimate H_2 near the seismic stations of interest relative to the H measured in the town of Betrawati. Our estimates are that $Q_2/Q_1 \approx 800\text{m}^3/\text{s}/1000\text{m}^3/\text{s} \approx 0.8$, $W_2/W_1 \approx 35\text{m}/70\text{m} \approx 0.5$, and $S_2/S_1 \approx 0.025/0.010 \approx 2.5$ so that $H_2 \approx 1.0H_1$. This means that the water level records at Betrawati can be used, without modification, as estimates of the water levels near the seismic stations of interest.

4 Estimation of Grain Size Distribution

To estimate the grain size distribution of the Trisuli River close to the seismic stations of interest, we assume that the distribution is similar to that of the nearby Marsyandi River at a location with similar drainage area and slope. Based on the descriptions of the two rivers in Plate 6 of *Lave and Avouac* [2001], we find that a location just north of the main central thrust (MCT) along the Marsyandi has similar slope (2.5°) and $Q_{10} \approx 10^3$ m^3/s as the region of the Trisuli of interest, and we choose this location as representative. *Attal and Lave* [2006] provide average grain size distributions on the Marsyandi both upstream and downstream of the MCT, so we take an average of these two zones as representative. For this average, the distribution of grain sizes larger than 5 cm is approximately 17%, 38%, 39%, 6% and 0% in bins of 5-8 cm, 8-16 cm, 16-32 cm, 32-64 cm, and > 64 cm, respectively. Performing a best-fit to these data using the log-'raised cosine' distribution discussed in the main text, we obtain a median grain size $D_{50} = 0.15$ m and an equivalent normal standard deviation of $\sigma_g = 0.525$ (i.e., $s = 1.45$). This best-fitting model results in a grain size distribution of 12%, 42%, 38%, 8% and 0% for the same grain size bins.

As stated in the main text, we choose to use the log-'raised cosine' distribution rather than the more commonly used log-normal distribution because the log-normal distribution has an unrealistically long tail at large (and small) grain sizes. Since our model is quite sensitive to the largest grain sizes (with an approximate D^3 dependence of P_v), having a realistic tail at the high end of the grain size distribution is therefore important for the model prediction. To provide a sense for how different the log-'raised cosine' and log-normal distributions are, we note that the log-normal distribution plotted in Fig. 3a of the main text has a grain size distribution with 10%, 43%, 38%, 7% and 0.3% in the same grain size bins described above. Thus, while the distributions are very similar (and fit the measured grain size distribution nearly equally as well), the 0.3% at very large grain sizes (> 64 cm) would result in significant seismic power predicted for those grains (from the sensitivity of the model to large grain sizes) despite the very small percentage, and would therefore bias our prediction somewhat. For any model (like the one presented here) that is sensitive to the tails of a grain size distribution that is known to be bounded, we recommend use of the log-'raised cosine' over the log-normal distribution.