New Techniques for Constructing EC/AUED Codes

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Abstract

We present two new techniques for constructing t-EC/AUED codes. The combination of the two techniques reduces the total redundancy of the best constructions by one bit or more in many cases.

SUMMARY

The problem of finding error correcting/all unidirectional error detecting codes (EC/AUED) has received wide attention in recent literature [1]-[3].
Given two binary vectors $\underline{u}$ and $\underline{v}$ of length n, denote by $N(\underline{u},\underline{v})$ the number of $1 \rightarrow 0$ transitions from $\underline{u}$ to $\underline{v}$. We say that $\underline{u}$ is contained in $\underline{v}$ ($\underline{u} \subseteq \underline{v}$) if $N(\underline{u},\underline{v}) = 0$.
Assume that $\underline{u}$ is transmitted but $\hat{\underline{u}}$ is received. We say that $\underline{u}$ has suffered unidirectional errors if either $\underline{u} \subseteq \hat{\underline{u}}$ or $\hat{\underline{u}} \subseteq \underline{u}$.
We are interested in codes that can correct up to t errors and detect all unidirectional errors when the number of unidirectional errors is greater than t. The next theorem [3] gives necessary and sufficient conditions for a code to be t-EC/AUED.

Theorem 1 Let C be a subset of $\{0,1\}^n$. Then C is a t-EC/AUED code if and only if, for any pair of vectors $\underline{u} \in C$ and $\underline{v} \in C$, $N(\underline{u},\underline{v}) \geq t+1$ and $N(\underline{v},\underline{u}) \geq t+1$.

Given k information bits, the way most authors construct a t-EC/AUED code C of length n is as follows: first the information bits are encoded into a t-EC (error-correcting) code C' of length n', with n' as small as possible; then, a tail of length r is added as further redundancy. The length of the code is then $n = n' + r$, and the total redundancy is $n' - k + r$. The tail is a function of the weight of the vector. The goal is to obtain a tail with r as small as possible.
Next, we give a general construction for t-EC/AUED codes. We need a definition first. We denote by $\lceil x \rceil$ ($\lfloor x \rfloor$) the smallest (largest) integer j such that $j \geq x$ ($j \leq x$).

Definition 1 A descending tail matrix of strength s is an $m \times r$ $\{0,1\}$-matrix with rows $\underline{t}_i$, $0 \leq i \leq m-1$, such that for all $0 \leq i \leq j \leq m-1$,

$$N(\underline{t}_i, \underline{t}_j) \geq \min\{s, \lceil (j-i)/2 \rceil\}.$$

Construction 1 Let C' be a t-EC of length n' and let T be a $T(n'+1, r; t+1)$ descending tail matrix with rows $\underline{t}_0, \underline{t}_1, \ldots, \underline{t}_{n'}$. Let C be the following code of length $n'+r$:

$$C = \{(\underline{v}, \underline{t}_{w(\underline{v})})\}$$

where $w(\underline{v})$ denotes the Hamming weight of $\underline{v}$. Then C is a t-EC/AUED code.

Proving that C is a t-EC/AUED is relatively easy using Theorem 1 [1].
Some of the best descending tail matrices are given in [1]. As said before, the goal is to make r as small as possible. The construction in [1] heavily depends on the best asymmetric error-correcting codes available.
In this talk, we propose two different techniques to reduce the redundancy of t-EC/AUED codes. The two methods can be used together. The first one involves using t-EC codes that contain the all-1 vector (for instance, BCH codes and the Golay code have this property). When choosing a codeword, we take either a codeword or its complement, according to which of the two has smaller weight. We have to pay a bit for this operation, but the weight distribution is reduced by half. We then append a tail in the way described by Construction 1. Overall, we will often gain in redundancy.
The second technique, involves improving upon the tail matrices given in [1].
Using the two methods either together or separetly, we often improve upon the best t-EC/AUED codes known. We also give efficient encoding and decoding algorithms for the new codes.

References

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