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Calabi–Yau mirror symmetry as a gauge theory duality

Mina Aganagic† and Andreas Karch‡

† California Institute of Technology, Pasadena, CA 91125, USA
‡ Center for Theoretical Physics, MIT, Cambridge, MA 02139, USA

E-mail: mina@theory.caltech.edu and karch@ctp.mit.edu

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Abstract. Using brane set-ups we construct dual gauge theories in two dimensions with $\mathcal{N} = (2, 2)$ supersymmetry. Two different dualities are realized. One is basically a consequence of three-dimensional mirror symmetry. The nonlinear sigma model with a Calabi–Yau target space on the Higgs branch of the gauge theory is mapped into an equivalent non-linear sigma model on the Coulomb branch on the dual, realizing a T-dual target space with torsion. The second dual is genuine to two dimensions. In addition to swapping Higgs and Coulomb branches it trades twisted for untwisted multiplets, implying a sign flip of the left-moving $U(1)_R$ charge. Successive application of both dualities leads to geometric mirror symmetry for the Calabi–Yau target space.

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1. Introduction

The mirror symmetry of Calabi–Yau (CY) manifolds is one of the remarkable predictions of type II string theory. The $(2, 2)$ superconformal field theory associated with a string propagating a Ricci-flat Kähler manifold has a $U(1)_L \times U(1)_R$ R-symmetry group, such that the Hodge numbers of the manifold correspond to the charges of (R, R) ground states under the R-symmetry. There is a symmetry in the conformal field theory which is a flip of the sign of the $U(1)_L$ current, $J_L \rightarrow -J_L$; this trivial symmetry, translates into a mirror flip $H^{p,q} \rightarrow H^{d-p,q}$ for the Hodge diamond. If physically realized, this implies the existence of pairs of manifolds, $(\mathcal{M}, \mathcal{W})$ which have 'mirror' hodge diamonds, $H^{p,q}(\mathcal{M}) = H^{d-p,q}(\mathcal{W})$, and give rise to exactly the same superconformal field theory.

However, while this observation predicts the existence of mirror pairs of Calabi–Yau d-folds, it is not constructive: one would like to know how to find $\mathcal{W}$ if one is given $\mathcal{M}$. The mirror construction has been proposed on purely mathematical grounds by Batyrev, for Calabi–Yau manifolds which can be realized as complete intersections in toric varieties.

In a beautiful paper [1], Strominger, Yau and Zaslow (SYZ) argued that, for mirror symmetry to extend to a symmetry of non-perturbative string theory, $(\mathcal{M}, \mathcal{W})$ must be T^d fibrations, with fibres which are special Lagrangian and, furthermore, that mirror symmetry is T^d duality of the fibres. The argument of SYZ is local and is very well understood only for smooth fibrations. To be able to fully exploit the idea, one must understand degenerations of special Lagrangian tori or, more generally, limits in which the Calabi–Yau manifold itself becomes singular. Some progress on how this is supposed to work has been made in [1–3], and local mirror symmetry appears to be the simplest to understand in this context.

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In an *a priori* unrelated development initiated by [4], 'mirror pairs' of gauge theories were found. In the field theory context, a duality in the sense of the infrared (IR) equivalence of two gauge theories is usually referred to as a 'mirror symmetry' if:

- the duality swaps Coulomb and Higgs branch of the theories, trading FI terms for mass parameters;
- the R-symmetry of the gauge theory has the product form $G_L \times G_R$ and duality swaps the two factors.

The example of [4] studies $\mathcal{N} = 4$ SUSY theories in three dimensions. Recently, it was shown [5] that this duality can even be generalized to an equivalence of two theories at all scales, a relation that will prove to be crucial for our applications.

The purpose of this paper is to obtain geometrical mirror pairs by a 'worldsheet' construction. Since, at the present stage, there seems to be little hope that one can do so directly in the nonlinear sigma model (NLσM), we study linear sigma models which flow in the IR to nonlinear sigma models with Calabi–Yau's as their target spaces. String theory offers a direct physical interpretation of these models as a worldvolume theory of a D-string probe of the Calabi–Yau manifolds. Gauge theory mirror duality for the linear sigma models reduces to geometric mirror symmetry for the Calabi–Yau target spaces.

Using brane constructions T-dual to the D1-brane probe on $\mathcal{M}$ one can easily construct gauge theories which flow to mirror manifolds. We will show that two different dualities of $d = 2$, $(2, 2)$ supersymmetric gauge theories arise: one is realized as an 'S-duality' of 'interval' brane set-ups. This duality is a consequence of the mirror symmetry of $d = 3$, $\mathcal{N} = 2$ field theories where both the Coulomb branch and the Higgs branch are described by non-compact Calabi–Yau manifolds. This duality in $d = 3$ maps a Calabi–Yau manifold to an identical one, so while it is a non-trivial field theory statement, this does not provide a linear σ-model construction of the mirror Calabi–Yau manifold. There exists another $d = 2$, $(2, 2)$ field theory duality obtained as an S-duality of the diamond brane constructions of [3]. This duality maps a theory whose Coulomb branch is dual to the Calabi–Yau manifold $\mathcal{M}$ in the 'boring' way described above, to a theory whose Higgs branch is the mirror manifold $\mathcal{W}$. The composition of these two dualities, therefore, flows to the Calabi–Yau mirror symmetry. While we consider in this paper a very particular family on non-compact Calabi–Yau manifolds, the generalization to arbitrary affine toric varieties is expected to be possible.

This paper is organized as follows. In the next section we discuss the different possible dualities in two dimensions as obtained via brane constructions for the case of the conifold. Section 3 generalizes this discussion to sigma models built from branes dual to more general non-compact CY manifolds and to the non-Abelian case, describing N D-brane probes on the singular CY. In the final section we present a detailed study of the moduli space after combined application of both dualities, indeed finding the mirror Calabi–Yau.

2. Mirror duality in two-dimensional gauge theories

2.1. From three to two dimensions

The original $D = 3$ $\mathcal{N} = 4$ of [4] upon compactification also automatically implies a duality relation in $\mathcal{N} = (4, 4)$ theories in two dimensions, as noted in [6, 7]. Using the recent results of [5] these results can be made more precise. The nature of this duality with eight supercharges will teach us how we should understand the $\mathcal{N} = (2, 2)$ examples. Since in two dimensions the concept of a moduli space is ill-defined, equivalence of the IR physics does not require the moduli spaces and metrics to match point by point, but only that the NLσMs on the moduli space† are equivalent, as we will see in several examples.

Start with the three-dimensional (3D) theory compactified on a circle. This is the set-up analysed in [10]. It is governed by two length scales, g_{YM}^{-2}, the 3D Yang–Mills coupling, and R_2, the compactification radius. To flow to the deep IR is equivalent to sending both length scales to zero. However, physics still might depend on the dimensionless ratio

$$\gamma = g_{YM}^2 R_2.$$

As shown in [10], while the Higgs branch metric is protected, the Coulomb branch indeed does depend on γ. For $\gamma \gg 1$ we first have to flow into the deep IR in 3D and then compactify, resulting in a two-dimensional (2D) NLσM on the 3D quantum-corrected Coulomb branch. The resulting target space is best described in terms of the dual photon in 3D, a scalar of radius γ. For 'the mirror of the quiver' ($U(1)$ with N_f electrons) it turns out to be an ALF space with radius γ. For small γ we should first compactify (express the theory in terms of the Wilson line) a scalar of radius $1/\gamma$, and obtain as a result a tube metric with torsion, corresponding to the metric of an NS5-brane on a transverse circle of radius $1/\gamma$ [10, 11]. Indeed, these two NLσMs are believed to be equivalent [12] and exchanging the dual photon for the Wilson line amounts to the T-duality of NS5-branes and the ALF space in terms of the IR NLσM‡.

In order to obtain a linear σ-model description of this scenario, one has to use the all-scale mirror symmetry of [5]. This shows that g_{YM}^2 maps to a Fermi-type coupling in the mirror theory, or more precisely, one couples the gauge field via a baptized Fermi (BF) coupling to a twisted gauge field, the gauge coupling of the twisted gauge being baptized Fermi coupling. For the case of the quiver theory with Fermi coupling, one obtains the same ALF space, this time on the Higgs branch.

In the same spirit we will present two kinds of dualities for $(2, 2)$ theories. In both cases we start with the LσM leading to a non-compact CY on the Higgs branch. The dual gauge theories have an equivalent NLσM with torsion on their Coulomb branch. Studying the dual Coulomb branch in terms of the 3D dual photon in order to have a CY interpretation, we find that in the boring case obtained from the interval this is the same Calabi–Yau we started with, while in the more exciting examples constructed via brane diamonds we obtain the mirror Calabi–Yau§.

2.2. Mirror symmetry from the interval

One way to 'derive' field theory mirror duality is to embed the field theory into string theory and then to use a string theory duality. A construction of this sort was the implementation of the $\mathcal{N} = 4$ theory in $d = 3$ as a brane construction in [13]. One uses an interval construction with the three basic ingredients: NS5 along 012345, D5 along 012789 and D3 along 0126. The two R-symmetries are $SU(2)_{345}$ and $SU(2)_{789}$. D3-brane segments between NS5-branes give rise to a vector multiplet, with the three scalars in the three of $SU(2)_{345}$. D3-brane segments between D5-branes are hypermultiplets with the four scalars transforming as two doublets of $SU(2)_{789}$.

† Or in the non-compact CY examples we are considering the two disjoint CFTs of the Coulomb and Higgs branch [8, 9].

‡ This picture is indeed obvious from the string theory perspective. Studying a D2-D6 system on a circle, going to the IR first lifts us to an M2 on an ALF space which becomes a fundamental string on the ALF, while going to 2D first makes us T-dualize to D1-D5, leaving us with the σ model of a string probing a 5-brane background.

§ Roughly speaking the interval mirror Coulomb branch is a tube metric with torsion one T-duality away, which is undone by going to the dual photon, while the diamond mirror is two T-dualities away, which conspire with the third from going to the dual photon to become the mirror symmetry of [1].

Under S-duality the D5-branes turn into NS5-branes and vice versa. The D3-branes stay invariant. One obtains the same kind of set-up but with D5- and NS5-branes interchanged. To be precise, just performing S-duality one ends up with NS5-branes along 012789 and D5-branes along 012345. The theory will really be a gauge theory of twisted hypers coupled to twisted vectors†. If, in addition, one performs a 90° rotation, taking 345 into 789 space, one actually swapped the two R-symmetries as advertised and is back to a theory written in terms of vectors and hypers.

Now let us move on to the 2D theories. The brane realization of this duality is via an interval theory in IIA with NS- and NS′-branes as above and a D2-branes along 016 [14]. The IIA analogue of S-duality, the 2–10 flip‡, takes this into a D4-brane along 01789 and a D4′-brane along 01457. The following parameters define the interval brane set-up and the gauge theory:

- The separation of the NS- and NS′-brane along 7 is the FI term. It receives a complex partner, the 10 separation which maps to the 2D theta angle.
- The separation of the D5-branes along 3 gives twisted masses to the flavours.

Mirror symmetry maps the FI term to the twisted masses. A twisted mass sits in a background vector multiplet and has to be contrasted with the standard mass from the superpotential which sits in a background chiral multiplet. Like the real mass in $\mathcal{N} = 2$ theories, in $d = 3$ it arises from terms like

$$\int \mathrm{d}^4\theta \; Q^\dagger \mathrm{e}^{m\theta\bar{\theta}} Q.$$

Example. As an example let us discuss the interval realization of the small resolution of the conifold. As shown in [15, 16], by performing T_6 T-duality on a D-string probe of the conifold we obtain an interval realization of the conifold gauge theory in terms of an elliptic IIA set-up with D2-branes stretched on a circle with one NS- and one NS′-brane. In this IIA set-up the separation of the NS-branes in 67 is the small resolution, while turning on the diamond mode would be the deformation of the conifold.

The gauge group on the worldvolume of the D-string on the conifold is [17] a $U(1) \times U(1)$ gauge group with two bifundamental flavours A_1, A_2, B_1 and B_2. We can factor out the decoupled centre-of-mass motion, the diagonal $U(1)$ which does not have any charged matter and hence is free in the IR. We are left with an interacting $U(1)$ with two flavours. The scalar in the decoupled vector multiplet is the position of the D1-brane in the 23 space transverse to the conifold. While the Coulomb branch describes separation into fractional branes, the Higgs branch describes motion on the internal space and reproduces the conifold geometry. The gauge theory on the D1 string is nothing else but the standard linear sigma model [18]. The complexified blowup mode for resolving the conifold is the FI term and the θ angle.

After the 2–10 flip, the dual brane set-up is again an elliptic model, this time with one D4- and one D4′-brane. The gauge theory is a single $\mathcal{N} = (8, 8)$ $U(1)$ from the D2-brane with two additional $\mathcal{N} = (2, 2)$ matter flavours from the D4- and D4′-brane. That is to say we have:

- Three ‘adjoints’, that is singlet fields X, Y and Z.
- Matter fields Q, $\tilde{Q}$, T and $\tilde{T}$ with charges +1, −1, +1, −1.

† The only difference between the twisted and untwisted multiplets is, under which R-symmetry they transform. As long as we do not start introducing twisted and untwisted fields simultaneously, the difference is really just a matter of convention.

‡ Which once more should be accompanied with a rotation in 345–789.

- They couple via a superpotential

$$W = QX\tilde{Q} + TX\tilde{T}.$$

- The singlet Z is decoupled and corresponds to the centre-of-mass motion.

Turning on the FI term and the θ angle in the original theory is a motion of the NS-brane along the 7 and 10 directions, respectively. It maps into a 23 motion for the D4-brane, giving a twisted mass to Q and $\tilde{Q}$.

This analysis can also be performed by going to the T-dual picture of D1-branes probing D5-branes intersecting in codimension two, that is to say over four common directions. Aspects of this set-up and its T-dual cousins in various dimensions have already been studied by numerous authors, e.g. for the D3 D7 D7′ system in [19] or for the D0 D4 D4′ system in [20]. The resulting gauge theory agrees with what we have found by applying the standard interval rules.

2.3. Mirror symmetry from diamonds

A second T-dual configuration for D1-brane probes of singular CY manifolds is D3-branes ending on a curve of NS-branes, called diamonds in [3]. These set-ups are the T_{48}-duals of D1-brane probes of the $\mathcal{C}_{kl}$ spaces. Indeed, it was this relation that allowed us to derive the diamond matter content to begin with [3]. In order to use the diamond construction to see mirror symmetry, we use the S-duality of string theory, as in the original work of [13]. Let us first consider the parameters defining a diamond and how they map under S-duality.

- The complex parameter defining the NS-brane diamond contains the FI term and again we pair it up with the 2D θ angle.
- The complex parameter defining the S-dual D5-brane diamond is a *complex* mass, as we will show momentarily.

Due to the changed identifications, the background twisted chiral multiplet containing the FI term maps to a background chiral multiplet containing the complex mass as compared to once more a background twisted multiplet containing the twisted mass, as we saw above. This implies a totally different map of operators under the two version of duality.

Example. Let us start once more with the simplest example, the D1 string on the blowup of the conifold; that is, we consider a single diamond, one NS- and one NS′-brane, on a torus. After S-duality† this elliptic model with an NS5- and NS5′-brane turns into an elliptic model with a D5- and D5′-brane. Since we have only D-branes in this dual picture, the matter content can be analysed by basically perturbative string techniques. To shortcut, we perform T_{48} duality to the D1 D5 D5′ system as in the interval set-up. For the special example of the conifold the two possible mirrors do not differ in the gauge and matter content, only in the parameter map. This will not be the case in the more general examples considered below.

As analysed above, the corresponding dual gauge theory is a $U(1)$ gauge group with three neutral fields X, Y and Z and two flavours Q, $\tilde{Q}$, T and $\tilde{T}$ with charges +1, −1, +1, −1, respectively. The superpotential in the singular case is $W = QX\tilde{Q} + TY\tilde{T}$.

By S-duality, as in the NS NS′ set-up, turning on the D5-brane diamonds corresponds turning on vevs for the $d = 4$ hypermultiplets from the D5 D5′ strings. Under $\mathcal{N} = (2, 2)$ these hypermultiplets decompose into background chiral multiplets and hence appear as parameters in the *superpotential*. If we call those chiral multiplets h and $\tilde{h}$, the corresponding

† Accompanied with a rotation of 235 space into 679 space.

superpotential contributions are $Qh\tilde{T} + \tilde{Q}\tilde{h}T$ [20], so that, all in all, the full superpotential reads

$$W = QX\tilde{Q} + TY\tilde{T} + Qh\tilde{T} + \tilde{Q}\tilde{h}T.$$

3. More mirror pairs

3.1. Other singular CY spaces

According to the analysis of [3, 15] D1-brane probes on the blowup of spaces of the form

$$G_{kl}: \quad xy = u^k v^l$$

are T_6 dual to an interval set-up with k NS- and l NS′-branes. The gauge group is a $U(1)^{k+l-1}$ with bifundamental matter. It is straightforward to construct interval mirrors via the 2–10 flip in terms of a $U(1)$ with two singlets and $k + l$ flavours. The $k + l - 1$ complexified FI terms map into the $k + l - 1$ independent twisted mass terms (one twisted mass can be absorbed by redefining the origin of the Coulomb branch).

Similarly, we can construct diamond mirrors for D1-brane probes of all C_{kl} spaces. The gauge group for the D1-brane probe is $U(1)^{2kl-1}$. Note that like in the case of $\mathbb{C}^3/\Gamma$ orbifolds the gauge theory on the D1-brane is not just the naive linear sigma model on C_{kl}, which is just a $U(1)^{(k+1)(l+1)-3}$. However, the D1-brane probe theory yields the same moduli space as its Higgs branch and especially the $2kl - 1$ FI terms only lead to $(k + 1)(l + 1) - 3$ independent blowup parameters. The mirror is once more a single $U(1)$ with two singlets and $k + l$ flavours. This time the $(k + 1)(l + 1) - 3$ complexified FI terms map to superpotential masses.

3.2. Generalization to non-Abelian gauge groups

Our realization in terms of brane set-ups gives us for free the non-Abelian version of the story, that is the mirror dual of N D1-branes sitting on top of the conifold. Let us spell out the dual pairs once more in the simple example of the conifold. Generalization to arbitrary G_{kl} and C_{kl} spaces is straightforward. The gauge group on N D1-branes on the blowup of the conifold is [17]

$$SU(N) \times SU(N) \times U(1)$$

where we have already omitted the decoupled centre-of-mass VM. The matter content consists of two bifundamental flavours $A_{1,2}$, $B_{1,2}$. They couple via a superpotential

$$W = A_1B_1A_2B_2 - A_1B_2A_2B_1.$$

The diamond mirror of this theory is a single $U(N)$ gauge groups with three adjoints† X, Y and Z and two fundamental flavours Q, $\tilde{Q}$, T, $\tilde{T}$ coupling via a superpotential:

$$W = X[Y, Z] + QX\tilde{Q} + TY\tilde{T} + hQ\tilde{T} + \tilde{h}\tilde{Q}T$$

where h and $\tilde{h}$ are the same background parameters determining the diamond as in the Abelian case.

† And here by adjoint we really mean a $U(N)$ adjoint, that is an $SU(N)$ adjoint and a singlet. The singlet in Z once again corresponds to the overall centre-of-mass motion and decouples.

4. Geometric mirror symmetry from linear sigma models

Our basic conjecture is that applying both dualities successively maps the LSM for a given Calabi–Yau to the LSM on the mirror. In both cases the Calabi–Yau is realized on the Higgs branch. The parameter map we have presented above implies that the dual theory is formulated in terms of twisted multiplets, realizing the required flip in the R-charge. Since we only know in very simple examples how to do the full map explicitly, in order to verify our conjecture we calculate, using the holomorphicity arguments of [21], the exact quantum-corrected Coulomb branch for the 'exciting' mirror from the diamonds in the 3D gauge theory. Since the interval mirror directly derives from 3D mirror symmetry, we know that it will realize the same target space as its Higgs branch.

Let us perform this calculation for the single D1-brane probe on a $\mathcal{C}_{kl}$ space. By construction, the Higgs branch of the gauge theory we start with is the blown-up $\mathcal{C}_{kl}$ space. The mirror theory is a $U(1)$ gauge theory coupled to $k + l$ flavours Q, $\tilde{Q}$ and T, $\tilde{T}$, and two singlet fields X and Y. The superpotential takes the form

$$W = \sum_{i=1}^{k} Q_i(X - a_i)\tilde{Q}^i + \sum_{a=1}^{l} T_a(Y - b_a)\tilde{T}^a + \sum_{ia} Q_i h^i_a \tilde{T}^a + \tilde{Q}^i \tilde{h}^a_i T_a,$$

where h and $\tilde{h}$ are background hypermultiplets parametrizing the diamonds and the a_i and b_a are the relative positions of the D5- and D5′-branes in the D1 D5 D5′ picture along 45 and 89, respectively; $\sum a_i = \sum b_a = 0$.

First let us study the classical moduli space. The D-term equations require

$$\sum_{i=1}^{k} |Q_i|^2 - |\tilde{Q}^i|^2 + \sum_{a=1}^{l} |T_a|^2 - |\tilde{T}^a|^2 = 0,$$

the F-term requirements for the Q, T, $\tilde{Q}$ and $\tilde{T}$ fields are

$$N \begin{pmatrix} Q \\ T \end{pmatrix} = 0$$

$$(\tilde{Q}, \tilde{T}) N^T = 0$$

where N is the $k + l$ by $k + l$ matrix

$$N = \left(\begin{array}{c|c} \mathrm{diag}\{X - a_1, X - a_2, \ldots, X - a_k\} & h \\ \tilde{h} & \mathrm{diag}\{Y - b_1, Y - b_2, \ldots, Y - b_l\} \end{array} \right).$$

In addition, the scalar potential contains the standard

$$2|\sigma|^2 \left(\sum_{i=1}^{k} |Q_i|^2 + |\tilde{Q}^i|^2 + \sum_{a=1}^{l} |T_a|^2 + |\tilde{T}^a|^2 \right)$$

piece from the coupling of the scalar σ in the vector multiplet to the matter fields and the D-terms for X and Y. These equations have the obvious solution

$$Q = \tilde{Q} = T = \tilde{T} = 0$$

with X, Y and σ arbitrary. This is the Coulomb branch. The Higgs branch corresponds to vevs for Q, $\tilde{Q}$, T and $\tilde{T}$. In order to have non-trivial solutions with non-vanishing vevs for those matter fields, the determinant of N has to vanish. Since N is the mass matrix for the flavours this requirement that at least one of the eigenvalues is zero is what we expect for the possibility of Higgs branches, which will be further constrained by X and Y D-flatness. The classical Coulomb branch is hence complex three dimensional and has singularities along the curve

$$\det(N) = 0$$

where fundamental flavours come down to zero mass. Higgs branches can touch the Coulomb branch only along this curve. (Note that this is nothing but the defining equation of the curve of the NS5-branes wrap, the diamond [3]. It is also the defining equation of the complex structure of the local mirror manifold for the blown-up $\mathcal{C}_{kl}$, the deformed $\mathcal{G}_{kl}$, whose defining equation is obtained by adding the 'quadratic pieces' $UV - \det(N) = 0$ which do not change the complex structure.)

We expect the quantum Coulomb branch to be G_{kl} in the 3D limit. So we have to consider a 3D $U(1)$ gauge theory with $k+l$ flavours, two singlets and a superpotential as above, written in a compact notation

$$W = \mathrm{Tr}(NM)$$

where N is the matrix we introduced above and M is the $k+l$ by $k+l$ matrix formed out of the gauge-invariant mesons

$$M = \begin{pmatrix} Q_i\tilde{Q}^j & Q_i\tilde{T}^b \\ T_a\tilde{Q}_j & T_a\tilde{T}^b \end{pmatrix}.$$

As shown in [21] the Coulomb branch of this $U(1)$ theory with $N_f = k + l$ flavours is parametrized by two chiral fields V_+ and V_- and an effective superpotential

$$W_{eff} = -N_f (V_+ V_- \det(M))^{1/N_f}$$

which captures the relevant physics, at least away from the origin.

Adding the tree level $\mathrm{Tr}(NM)$ to this effective superpotential, the M F-term equations describing our quantum Coulomb branch read

$$N_{\beta\gamma} + (V_+V_-)^{1/N_f}\frac{H_{\beta\gamma}}{\det(M)^{1-1/N_f}} \tag{1}$$

where

$$H_{\beta\gamma} = \frac{\partial \det(M)}{\partial M^{\beta\gamma}}.$$

Taking the determinant in equation (1) we arrive at

$$\det(N) = V_+V_-$$

which is precisely the space we are after. Since the origin $V_+ = V_- = X = Y = 0$ is no longer part of this branch of moduli space, we arrive at a smooth solution even though we started from the effective superpotential of [21] which is singular at the origin.

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