

# Dark Matter in Classically Scale-Invariant Two Singlets Standard Model

Koji Ishiwata

*California Institute of Technology, Pasadena, CA 91125, USA*

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We consider a model where two new scalars are introduced in the standard model, assuming classical scale invariance. In this model the scale invariance is broken by quantum corrections and one of the new scalars acquires non-zero vacuum expectation value (VEV), which induces the electroweak symmetry breaking in the standard model, and the other scalar becomes dark matter. It is shown that TeV scale dark matter is realized, independent of the value of the other scalar's VEV. The impact of the new scalars on the Higgs potential is also discussed. The Higgs potential is stabilized when the Higgs mass is over  $\sim 120$  GeV.

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The hierarchy problem is one of the unsolved problems in standard model of particle physics. In the standard model, Higgs boson is the only scalar and it has a mass scale. This mass is highly unstable if radiative corrections are taken into account, which means that it must be fine-tuned for the electroweak (EW) symmetry breakdown to occur at  $O(100)$  GeV. Even if one tolerates the fine-tuning, the Higgs sector has another problem. Higgs quartic coupling might become large and have Landau pole or get negative at high scale below Planck scale [1–7]. In order to avoid a Landau pole up to Planck scale and make the EW vacuum cosmologically stable, the Higgs mass has to be about 115 GeV–180 GeV [7, 8]. Otherwise, there should be new physics below Planck scale. High energy collider experiments are aimed at searching for the Higgs boson. The LEP experiments give a lower bound on the Higgs mass as  $m_h \geq 114.4$  GeV ( $m_h$  is standard-model Higgs mass) [9]. The region  $156 \text{ GeV} \leq m_h \leq 177 \text{ GeV}$  is excluded by the Tevatron experiment [10]. At the LHC, most of the region in  $145 \text{ GeV} \leq m_h \leq 466 \text{ GeV}$  is excluded [11].

The existence of dark matter in the universe is another mystery to be answered. Dark matter (DM) accounts for about five times of the baryon energy density in the universe according to the recent observations [12]. However, the standard model does not have a good candidate for DM.

Probably most popular and elegant solution to the both problems is supersymmetry. In a supersymmetric model, the fine-tuning for the Higgs mass parameter is avoided when the mass of superparticles are below about a TeV. Moreover, since the Higgs quartic coupling is written in terms of gauge coupling constants squared, it never has a Landau pole or gets negative up to the Planck scale. In addition, the lightest superparticle (LSP) is stable under  $R$ -parity. Such a stable LSP, with a mass of the order a TeV, can be the dark matter, which is the so-called weakly interacting massive particle (WIMP) scenario. However, the recent null result of the search of the superparticles at the LHC [13, 14] indicates that typical scale of superparticle mass should be above around TeV. This result causes a tension in supersymmetric model in terms of hierarchy problem.

Another approach for the solution of the hierarchy problem is to consider scale invariance in theory. Even if a theory has scale invariance classically, the scale invariance would be broken by quantum corrections in Coleman-Weinberg mechanism [15]. Then a mass scale for scalar field is induced (known as dimensional transmutation). This mass is protected from quantum corrections due to the scale invariance at classical level although the scale invariance is anomalous [16, 17]. However, possibility of Coleman-Weinberg mechanism in the standard-model Higgs sector is already excluded due to large top Yukawa coupling. In the framework of scale-invariant theory, several simple extensions of the standard model are proposed recently. In Refs. [18, 19], a new scalar charged with non-zero  $B - L$  ( $B$  and  $L$  are baryon and lepton numbers) breaks  $U(1)_{B-L}$  gauge symmetry radiatively and then the braking induces the EW symmetry breaking. Another extension is adding new scalar singlets to the standard model [20, 21]. It is described in their works, however, that dark matter can not be explained without considering additional fields.<sup>1</sup>

In this Letter, we consider two new scalar singlets in classically scale-invariant standard model, and study a possibility that one of the scalars becomes dark matter. The mass of the scalar dark matter is provided by the non-zero vacuum expectation value (VEV) of the other scalar. The non-zero VEV is induced by quantum corrections to break classical scale invariance, and then the EW symmetry is broken as shown in Ref. [20]. It is shown that the singlet can explain the present energy density of dark matter with a mass of TeV for various values of the VEV. It turns out that the scalar which has the non-zero VEV is sequestered from the other sectors, then the properties of the singlet DM becomes similar to the one in the standard model with a singlet scalar [22–25]. The

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<sup>1</sup> It is mentioned in Refs. [18, 19] an additional scalar is needed for dark matter candidate, while a scenario where dark matter in mirror standard model is considered in Ref. [20]. In Ref. [21], heavy right-handed neutrino is discussed as a candidate for dark matter and further cosmological consequence of the scale-invariant theory is given.

impact of the new scalars on Higgs potential is also discussed. The Higgs lower mass bound from the stability of the Higgs potential gets smaller and it turns out to be 120–125 GeV due to the existence of the new scalars.

Let us start off with the framework of our model. We consider the theory which has scale invariance at classical level, *i.e.* there is no mass term in Lagrangian. In addition to the Higgs boson, we introduce two real new standard-model singlet scalars,  $\phi_1$  and  $\phi_2$ , in scalar sector. Then, the renormalizable potential of the scalar sector is written as

$$V = \frac{1}{8}\lambda_1\phi_1^4 + \frac{1}{8}\lambda_2\phi_2^4 + \frac{1}{4}\kappa_{12}\phi_1^2\phi_2^2 + \frac{1}{2}\lambda_H(H^\dagger H)^2 + \frac{1}{2}\kappa_{H1}H^\dagger H\phi_1^2 + \frac{1}{2}\kappa_{H2}H^\dagger H\phi_2^2, \quad (1)$$

where  $H$  is Higgs doublet. Here we assume  $Z_2$  symmetry, *i.e.* the potential is invariant under  $\phi_2 \rightarrow -\phi_2$ . Then a term, such as  $\phi_1\phi_2H^\dagger H$ , is forbidden. As to couplings, we consider all the couplings are positive, except for  $\kappa_{H1}$ . However, a sizable negative value for  $\kappa_{H1}$  might cause instability of the potential. Thus, in this Letter, we assume very small  $|\kappa_{H1}|$ . In addition, we limit our study in the region where all the couplings are under control in perturbation up to Planck scale.

First let us focus on the new scalars. Even  $\kappa_{12} > 0$ , the spontaneous breakdown of scale invariance would be induced by quantum corrections. Either  $\phi_1$  or  $\phi_2$  could have a VEV. For the EW symmetry breakdown in the standard model sector (by the negative  $\kappa_{H1}$ ), we consider the case where  $\phi_1$  gets the non-zero VEV. The evaluation of the VEV is performed by the use of renormalization group (RG) improved potential at one-loop level [26]:

$$V = \frac{1}{8}\lambda_1(t)\phi_1^4 + \frac{1}{8}\lambda_2(t)\phi_2^4 + \frac{1}{4}\kappa_{12}(t)\phi_1^2\phi_2^2 + \dots, \quad (2)$$

where  $t = \log(\mu/M)$ . Here  $\mu$  is renormalization scale and  $M$  is arbitrary scale. The RG equations at one-loop level for  $\lambda_1$ ,  $\lambda_2$  and  $\kappa_{12}$  are given by

$$16\pi^2 \frac{d\lambda_1}{dt} = 9\lambda_1^2 + \kappa_{12}^2 + 4\kappa_{H1}^2, \quad (3)$$

$$16\pi^2 \frac{d\lambda_2}{dt} = 9\lambda_2^2 + \kappa_{12}^2 + 4\kappa_{H2}^2, \quad (4)$$

$$16\pi^2 \frac{d\kappa_{12}}{dt} = 4\kappa_{12}^2 + 3\kappa_{12}(\lambda_1 + \lambda_2) + 4\kappa_{H1}\kappa_{H2}. \quad (5)$$

The RG equations for the other couplings are given in Appendix. Hereafter we take  $M = \langle\phi_1\rangle$ . ( $\langle\phi_1\rangle$  denotes the VEV of  $\phi_1$ .) For the evaluation to find stationary point where  $\phi_1$  has the VEV (and  $\langle\phi_2\rangle=0$ ), taking  $\mu = \phi_1$  is a good approximation. Then  $\partial V/\partial\phi_1|_{\phi_1=\langle\phi_1\rangle} = 0$  gives a condition,

$$\left[ \frac{d\lambda_1}{dt} + 4\lambda_1 \right] \Big|_{t=0} = 0, \quad (6)$$

and it leads to

$$\lambda_1(0) \simeq -\frac{1}{64\pi^2}(\kappa_{12}^2(0) + 4\kappa_{H1}^2(0)). \quad (7)$$

Consequently, the mass parameter of  $\phi_1$  is calculated by the second derivative of the potential with respect to  $\phi_1$  at  $\phi_1 = \langle\phi_1\rangle$ :

$$\begin{aligned} m_{\phi_1}^2 &= \left. \frac{\partial^2 V}{\partial^2 \phi_1} \right|_{t=0} \\ &\simeq -2\lambda_1(0)M^2 \\ &\simeq \frac{1}{32\pi^2}(\kappa_{12}^2(0) + 4\kappa_{H1}^2(0))M^2. \end{aligned} \quad (8)$$

Therefore, the negative value of  $\lambda_1(0)$  gives the right sign for the mass term of  $\phi_1$ . One may wonder the negative  $\lambda_1(0)$  would induce a deeper minimum at some higher scale. However, since  $|\lambda_1(0)|$  is required to be small enough (see Eq. (7)), the potential is stabilized.

The important aspect of the spontaneous symmetry breaking in  $\phi_1$  sector is that the symmetry breaking also induces the EW symmetry breakdown in the Higgs sector. By replacing  $\phi_1$  as  $M + \phi_1$  in the potential, relevant terms for the EW symmetry breaking are given by

$$V = \frac{1}{2}\lambda_H(H^\dagger H)^2 + \frac{1}{2}\kappa_{H1}M^2H^\dagger H + \dots \quad (9)$$

Then it is seen that the EW symmetry is broken (by the negative  $\kappa_{H1}$ ). In the evaluation of the Higgs VEV, taking renormalization scale to be the Higgs field is a good approximation. Then  $\lambda_H$  and  $\kappa_{H1}$  relates at the scale of  $\mu = v$  ( $v \simeq 246$  GeV) as

$$\mu_h^2(t_v) = -\kappa_{H1}(t_v)M^2. \quad (10)$$

Here  $\mu_H^2(t) = \lambda_H(t)v^2$  and  $t_v = \log(v/M)$ .

Due to the VEVs of  $\phi_1$  and Higgs, both fields mix with each other. Plugging  $H = (v+h)/\sqrt{2}$  ( $h$  is Higgs field) in the potential, the mass terms for the mixed states are

$$V_{h\phi_1}^{\text{mass}} = \frac{1}{2} \begin{pmatrix} \phi_1 & h \end{pmatrix} \begin{pmatrix} m_{\phi_1}^2 & \Delta m^2 \\ \Delta m^2 & \mu_h^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ h \end{pmatrix}, \quad (11)$$

where  $\Delta m^2 = \kappa_{H1}vM$ . Then the mass eigenstates, denoted as  $s_a$  and  $s_b$ , are parametrized as

$$\begin{pmatrix} s_a \\ s_b \end{pmatrix} = \begin{pmatrix} \cos\theta_s & \sin\theta_s \\ -\sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} \phi_1 \\ h \end{pmatrix}. \quad (12)$$

Here and hereafter we take  $m_{s_a} < m_{s_b}$  ( $m_{s_a}$  and  $m_{s_b}$  are masses of  $s_a$  and  $s_b$ , respectively) without loss of generality. When  $m_{s_a} < 114.4$  GeV, the mass of  $s_a$  would be constrained by the LEP experiment. The constraints are given by in terms of  $\xi^2 = (g_{sZZ}/g_{HZZ}^{\text{SM}})$  ( $g_{sZZ}$  is new scalar- $Z$ - $Z$  coupling and  $g_{HZZ}^{\text{SM}}$  is  $h$ - $Z$ - $Z$  coupling in the standard model) [9]. In our case it is equal to  $\sin^2\theta_s$  and the scalar which has  $\xi \gtrsim 0.01$  is constrained when it is lighter than 114.4 GeV. Thus we calculate  $\xi^2$  to evaluate the constraints by the LEP. On the other hand,  $\phi_2$  does not mix with the other scalars. Its mass is induced by the  $\langle\phi_1\rangle$ , and it is easily obtained as

$$m_{\phi_2}^2 = \frac{1}{2}\kappa_{12}(0)M^2. \quad (13)$$

Therefore,  $m_{\phi_2} \gg m_{s_a}, m_{s_b}$  is expected in our scenario.

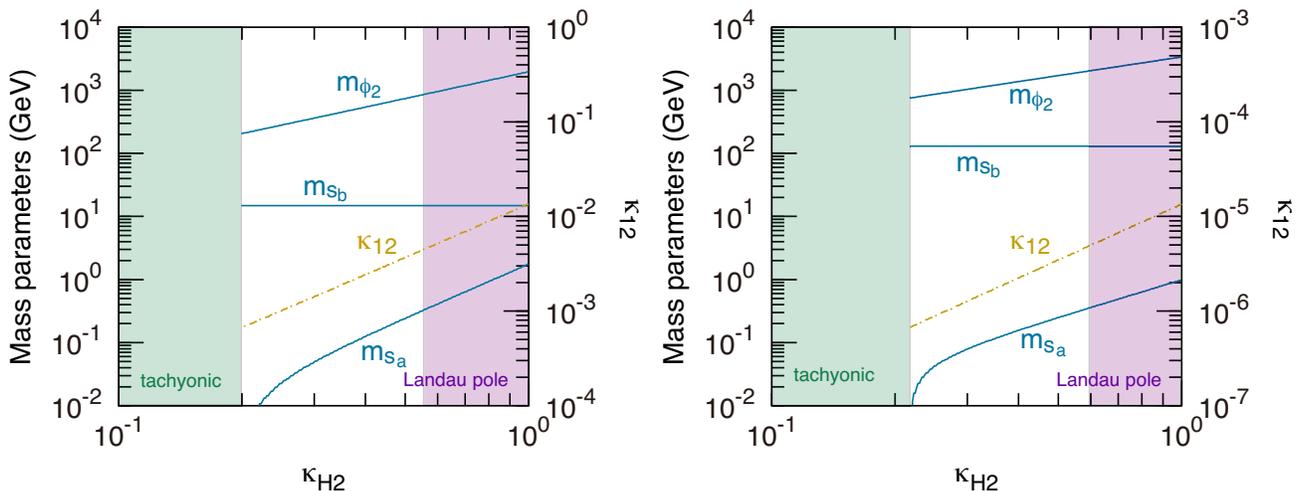


FIG. 1: Masses of  $s_a$ ,  $s_b$  and  $\phi_2$  (left vertical axis) and  $\kappa_{12}$  (right vertical axis) as the function of  $\kappa_{H2}(\mu = m_{\phi_2})$ . Here we take  $\mu_h(t_v) = 130$  GeV and  $|\kappa_{H1}(t_v)| = 10^{-5}$  (left panel),  $10^{-8}$  (right panel). Light green shaded region shows that the lightest scalar becomes tachyon, while in dark purple shaded region  $\kappa_{H2}$  has Landau pole.

Now we are ready to discuss phenomenological aspects of our model. Since the  $Z_2$  symmetry for  $\phi_2$  is unbroken,  $\phi_2$  is stable and becomes a good candidate for dark matter. In the early universe,  $\phi_2$  is produced thermally and the relic abundance is determined by its annihilation cross section. The annihilation cross section depends on  $\kappa_{12}$ ,  $\kappa_{H2}$  and  $\langle\phi_1\rangle$ . Relevant processes are  $\phi_2\phi_2 \rightarrow s_a s_a, s_b s_b, s_a s_b, WW, ZZ$  and  $f\bar{f}$ . (Here  $f$  stands for standard-model fermion.) When  $\kappa_{H2}$  is negligible, main process which contributes the annihilation is  $\phi_2\phi_2$  to the scalar pairs. In such a case, however, the annihilation cross section is too small, which leads to overclosure of the universe. Therefore we consider finite value of  $\kappa_{H2}$ , which allows the annihilation channels to the standard-model gauge bosons and fermion pairs. In our model, when  $\kappa_{H1}$ ,  $\kappa_{H2}$  and  $\kappa_{12}$  are fixed for a given value of  $\mu_h$ , all the mass parameters are determined. Then the annihilation cross section and the relic abundance can be calculated. Therefore we impose  $\Omega_{\phi_2} = \Omega_{\text{DM}}$  ( $\Omega_{\phi_2}$  and  $\Omega_{\text{DM}}$  are the energy densities of  $\phi_2$  and dark matter, respectively) in our numerical calculation.

The mass spectra to realize  $\phi_2$ -DM scenario are given in Fig. 1. Here we take  $\mu_h(t_v) = 130$  GeV, and  $|\kappa_{H1}(t_v)| = 10^{-5}$  and  $10^{-8}$  are taken in left and right panels, respectively. Light shaded region is forbidden because  $s_a$  becomes tachyonic, while  $\kappa_{H2}$  has a Landau pole in dark shaded region. In the evaluation, we have solved the RG equations given in Eqs. (3)-(5) and (14)-(16) for  $\mu \geq m_{\phi_2}$ . When  $\mu \leq m_{\phi_2}$ , we consider the effective theory in which  $\phi_2$  is integrated out. Thus, for  $\lambda_H$  and  $\kappa_{H1}$ , we have solved the RG equations given in Eqs. (17) and (18). (The RG evolution of  $\lambda_H$  at low energy is important to determine the Higgs pole mass. See later discussion.) Here we note that horizontal axis in

the Figure should be interpreted as a value at the scale of  $\mu = m_{\phi_2}$  because energy scale of annihilation process is characterized by  $\phi_2$  mass. Meanwhile, we have checked running effect of  $\kappa_{12}$  is very small. Thus we have used values of  $m_{\phi_1}$  and  $m_{\phi_2}$  at  $\mu = M$  for the calculation of the pole masses.  $\Delta m$  is evaluated by using  $\kappa_{H1}(t_v)$ . In the allowed region, the mass of  $\phi_2$ -DM turns out to be a few hundred GeV to a few TeV. This result indicates that  $\phi_2$ -DM annihilates into the standard-model particles via  $\kappa_{H2} \sim O(0.1-1)$ . Then, as in the standard WIMP scenario, particle with a mass of TeV scale give the right amount relic to explain the present DM abundance. For the other scalars, we found that  $s_a \simeq \phi_1$  and  $s_b \simeq h$ . The behavior of  $m_{s_a}$  can be understood as follows. From Eqs. (10) and (13),  $\kappa_{12} \sim 10^2 \kappa_{H1}$  for  $\mu_h \sim 100$  GeV and  $m_{\phi_2} \sim 1$  TeV. Then the mass of  $s_a$  is estimated as  $m_{s_a} \simeq m_{\phi_1} \sim 10^3 \sqrt{|\kappa_{H1}|}$  GeV, which roughly agrees with the numerical result given in the Figure. It is seen that the mass of the lightest scalar is far below of the LEP bound. However, we have checked that  $\xi^2$  for  $s_a$  is much smaller than 0.01. Therefore, the LEP experiment does not exclude the lightest scalar. Finally as to a magnitude of  $\kappa_{H1}$ , its upper bound is expect as  $|\kappa_{H1}| \lesssim O(10^{-3})$  from perturbativity, using  $\kappa_{12} \sim 10^2 |\kappa_{H1}| \lesssim O(0.1)$ . On the other hand, lower bound is implicitly given by  $|\kappa_{H1}| \gtrsim \mu_h^2 / M_{\text{pl}}^2$  ( $M_{\text{pl}} = 2.4 \times 10^{18}$  GeV) from Eq. (10).

Since  $|\kappa_{H1}|$  is very small but  $\kappa_{H2}$  is sizable, the properties of  $\phi_2$ -DM is similar to those of a singlet DM in new minimal standard model (NMSM) which is studied in Refs. [22–25]. Phenomenology of the NMSM has been studied by many works up to the present. It is pointed out possibility of direct detection of singlet DM. (See Ref. [29] for the updated analysis, in which DM with a mass of less than 200 GeV is considered.) We calculated

spin independent cross section of  $\phi_2$ -DM with nucleon by following [30] to include gluon contribution with QCD correction. Then the cross section turns out to be about  $10^{-45}$  cm<sup>2</sup> for  $m_h = 125$  GeV. The result has less dependence on  $\kappa_{H2}$  in the region where  $|\kappa_{H1}| \lesssim 10^{-5}$ . Thus it may be difficult to detect  $\phi_2$ -DM in the near future direct detection experiments.

The new singlet scalars also have impact on the RG evolution of the Higgs quartic coupling. In the standard model,  $\lambda_H$  gets negative below Planck scale for  $\mu_h(t_v) \lesssim 135$  GeV at the one-loop level RG equations. In careful evaluation of the Higgs pole mass at two-loop level, it is shown the Higgs pole mass should be larger than about 130 GeV in order for the Higgs potential to be stable up to Planck scale [5, 6].<sup>2</sup> In the recent work the Higgs lower mass bound in the NMSM is calculated carefully at one-loop level. It is shown in Ref. [28] that the Higgs lower mass bound turns out to be around 130 GeV in the NMSM. In order to see the impact of the new scalars to the Higgs potential in our model, we solved the RG equations given in Eqs. (3)-(5) and (14)-(16) for  $\mu \geq m_{\phi_2}$  and Eqs. (17) and (18) for  $\mu \leq m_{\phi_2}$ . In the present scenario sizable  $\kappa_{H2}$  tends to increase  $\lambda_H$ , then it turns out that  $\lambda_H$  is positive up to Planck scale when  $\mu_H(t_v) \gtrsim 120$  GeV, taking possible largest value for  $\kappa_{H2}$ . Following the procedure given in Ref. [5], we calculated the Higgs pole mass at one-loop level,<sup>3</sup> and obtained  $m_h \simeq 123$  GeV for  $\mu_h = 120$  GeV. This means that the Higgs potential is stabilized up to Planck scale when the Higgs pole mass is larger than about 120 GeV. However, since this result is based on the one-loop RG calculation, the lower Higgs mass bound might be changed by a few GeV in more accurate calculation of the pole mass. Such an analysis is beyond the scope of this Letter. It will be given elsewhere.

Finally we note on cosmological problem in our model. Since there is also  $Z_2$  symmetry under transformation,  $\phi_1 \rightarrow -\phi_1$ , domain wall is formed as a topological defect when the discrete symmetry breaks spontaneously. If the domain wall is stable or long-lived, it would affect the primordial density fluctuation in the early universe to give unobserved anisotropy in cosmic microwave background (CMB). However, such a problem might be avoided if one consider higher dimensional operator which is suppressed by Planck mass which explicitly breaks the  $Z_2$  symmetry, *e.g.*,  $\phi_1^5/M_{\text{pl}}$ , assuming that scale invariance is broken at Planck scale. Then, two degenerate vacua splits to true and false vacua. The energy density of the domain wall is roughly estimated as  $\rho_{\text{DW}} \sim \sigma/R$  where  $\sigma$  is the tension of the domain wall and  $R = M_{\text{pl}}/TT_{\text{sb}}$  is the scale of the domain wall. Here  $T$  is temperature and  $T_{\text{sb}}$  is the

temperature at the discrete symmetry breaking [31]. If the energy difference is larger than that of the domain wall, the false vacuum is destroyed by the true vacuum before the domain wall dominates the energy density of the universe. Consequently, the anisotropy in the CMB is avoided. In our case  $\sigma \sim M^3$  and  $T_{\text{sb}} \sim M$ . Thus at the temperature  $\rho_{\text{DW}} \sim \rho_R(\sim T^4)$ ,  $\rho_{\text{DW}}$  would be  $\sim (M^4/M_{\text{pl}})^{4/3}$ . On the other hand, the energy difference between true and false vacuum is  $M^5/M_{\text{pl}}$ . Therefore, the requirement for avoiding the domain wall problem is sufficiently satisfied in our model. Another way to avoid the domain wall problem is to consider reheating temperature less than the VEV of  $\phi_1$ . Then the large entropy production at the reheating dilutes the domain wall.

In conclusion we have studied classically scale-invariant standard model with the new scalar singlets, as a solution for the hierarchy problem. In our model one scalar has the VEV and breaks the scale invariance, then the EW symmetry breaking occurs. Although this scalar mixes with the standard-model Higgs boson, the mixing is so small that the Higgs phenomenology at the collider is unchanged. The other scalar acquires its mass from the VEV and becomes a good candidate for dark matter. It is found that dark matter with a mass of a TeV scale is realized for various value of the VEV. This scalar has sizable coupling to the Higgs and it may be possible to stabilize the Higgs potential when the Higgs mass is larger than  $\sim 120$  GeV.

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## Appendix

In the appendix, we give the RG equations for  $\kappa_{H1}$ ,  $\kappa_{H2}$  and  $\lambda_H$  and the annihilation cross section of  $\phi_2$ .

The RG equations for  $\kappa_{H1}$ ,  $\kappa_{H2}$  and  $\lambda_H$  are obtained as

$$16\pi^2 \frac{d\kappa_{H1}}{dt} = 4\kappa_{H1}^2 + \kappa_{H2}\kappa_{12} + \kappa_{H1}(2\gamma_h + 3\lambda_1 + 6\lambda_H), \quad (14)$$

$$16\pi^2 \frac{d\kappa_{H2}}{dt} = 4\kappa_{H2}^2 + \kappa_{H1}\kappa_{12} + \kappa_{H2}(2\gamma_h + 3\lambda_2 + 6\lambda_H), \quad (15)$$

$$16\pi^2 \frac{d\lambda_H}{dt} = 12\lambda_H^2 + 4\lambda_H\gamma_h - 12y_t^4 + \frac{3}{4}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4) + \kappa_{H1}^2 + \kappa_{H2}^2. \quad (16)$$

Here  $\gamma_h = -(9/4)g_2^2 - (3/4)g_1^2 + 3y_t^2$  ( $g_1$ ,  $g_2$  and  $y_t$  are

<sup>2</sup> When  $m_h \lesssim 130$  GeV, the Higgs potential has another deeper minimum below Planck scale. However, lifetime of the EW vacuum is longer than the age of universe when  $m_h \gtrsim 115$  GeV [8].

<sup>3</sup> In Ref. [5], they solve RG equations at two-loop level.

$U(1)_Y$ ,  $SU(2)_L$  gauge couplings and top Yukawa coupling, respectively). In the calculation, we also solve the RG equations for gauge and top Yukawa couplings. In the evaluation of the running of top Yukawa, we use initial condition,  $y_t(\mu = m_t) = \sqrt{2}m_t(1 + 4\alpha_s(m_t)/3\pi)^{-1}/v$  where  $m_t = 171$  GeV and  $\alpha_s(m_t)$  is strong coupling at the scale of  $\mu = m_t$  [6]. For  $\mu \leq m_{\phi_2}$ ,  $\phi_2$  is integrated out. Since we need to know the running of  $\lambda_H$  in this energy region for the determination of the Higgs pole mass, we use RG equations:

$$16\pi^2 \frac{d\lambda_H}{dt} = 12\lambda_H^2 + 4\lambda_H\gamma_h - 12y_t^4 + \frac{3}{4}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4) + \kappa_{H1}^2, \quad (17)$$

$$16\pi^2 \frac{d\kappa_{H1}}{dt} = 4\kappa_{H1}^2 + \kappa_{H1}(2\gamma_h + 3\lambda_1 + 6\lambda_H). \quad (18)$$

Although we have solved the RG equation for  $\kappa_{H1}$ , it turns out that the influence of  $\kappa_{H1}$  on  $\lambda_H$  is very small. Thus the running of  $\lambda_H$  is almost the same as in the standard model for  $\mu \leq m_{\phi_2}$ .

The annihilation cross sections of  $\phi_2$  for each channel,  $\phi_2\phi_2 \rightarrow s_a s_a$ ,  $s_b s_b$ ,  $s_a s_b$ ,  $WW$ ,  $ZZ$  and  $f\bar{f}$ , are given as follows:

$$\sigma v(\phi_2\phi_2 \rightarrow s_a s_a) = \frac{\beta_i(m_{s_a}, m_{s_a})}{64\pi m_{\phi_2}^2} \mathcal{M}_{aa}^2, \quad (19)$$

$$\sigma v(\phi_2\phi_2 \rightarrow s_b s_b) = \frac{\beta_i(m_{s_b}, m_{s_b})}{64\pi m_{\phi_2}^2} \mathcal{M}_{bb}^2, \quad (20)$$

$$\sigma v(\phi_2\phi_2 \rightarrow s_a s_b) = \frac{\beta_i(m_{s_a}, m_{s_b})}{32\pi m_{\phi_2}^2} \mathcal{M}_{ab}^2, \quad (21)$$

$$\begin{aligned} \sigma v(\phi_2\phi_2 \rightarrow WW) &= \frac{g_2^2 \beta_i(m_W, m_W)}{16\pi m_{\phi_2}^2} \mathcal{M}^2 \\ &\times \frac{m_W^2}{m_{\phi_2}^2} \left[ 1 + \frac{1}{2} \left( 1 - \frac{2m_{\phi_2}^2}{m_W^2} \right)^2 \right], \end{aligned} \quad (22)$$

$$\begin{aligned} \sigma v(\phi_2\phi_2 \rightarrow ZZ) &= \frac{g_2^2 \beta_i(m_Z, m_Z)}{32\pi m_{\phi_2}^2} \mathcal{M}^2 \\ &\times \frac{m_Z^2}{m_{\phi_2}^2} \left[ 1 + \frac{1}{2} \left( 1 - \frac{2m_{\phi_2}^2}{m_Z^2} \right)^2 \right], \end{aligned} \quad (23)$$

$$\sigma v(\phi_2\phi_2 \rightarrow f\bar{f}) = \frac{y_f^2 \beta_i(m_f, m_f)}{8\pi m_{\phi_2}^2} \left( 1 - \frac{m_f^2}{m_{\phi_2}^2} \right) \mathcal{M}^2. \quad (24)$$

Here  $g_Z = \sqrt{g_1^2 + g_2^2}$ , and  $m_W$  and  $m_Z$  are  $W$  and  $Z$  boson masses, respectively.  $m_f$  is fermion mass and its Yukawa coupling is given by  $y_f = \sqrt{2}m_f/v$ .  $\beta_i$  is defined as  $\beta_i^2(m_1, m_2) = (s^2 - 2s(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2)/s^2$  with  $s = 4m_{\phi_2}^2$ . The matrix elements of the scattering,  $\mathcal{M}_{aa}$ ,  $\mathcal{M}_{bb}$  and  $\mathcal{M}$ , are obtained as

$$\mathcal{M}_{aa} = -(\kappa_{12} \cos^2 \theta_s + \kappa_{H2} \sin^2 \theta_s) + \frac{2(\kappa_{12} \cos \theta_s M + \kappa_{H2} \sin \theta_s v)^2}{2m_{\phi_2}^2 - m_{s_a}^2}, \quad (25)$$

$$\mathcal{M}_{bb} = -(\kappa_{12} \sin^2 \theta_s + \kappa_{H2} \cos^2 \theta_s) + \frac{2(-\kappa_{12} \sin \theta_s M + \kappa_{H2} \cos \theta_s v)^2}{2m_{\phi_2}^2 - m_{s_b}^2}, \quad (26)$$

$$\mathcal{M}_{ab} = (\kappa_{12} - \kappa_{H2}) \cos^2 \theta_s \sin^2 \theta_s - (\kappa_{12} \cos \theta_s M + \kappa_{H2} \sin \theta_s v)(-\kappa_{12} \sin \theta_s M + \kappa_{H2} \cos \theta_s v) \times \left[ \frac{1}{t - m_{\phi_2}^2} + \frac{1}{u - m_{\phi_2}^2} \right],$$

$$\mathcal{M} = m_{\phi_2} \left[ \frac{(\kappa_{12} \cos \theta_s M + \kappa_{H2} \sin \theta_s v) \sin \theta_s}{4m_{\phi_2}^2 - m_{s_a}^2} + \frac{(-\kappa_{12} \sin \theta_s M + \kappa_{H2} \cos \theta_s v) \cos \theta_s}{4m_{\phi_2}^2 - m_{s_b}^2} \right]. \quad (27)$$

Here  $t = m_{\phi_2}^2 + m_{s_a}^2 - 2m_{\phi_2}(m_{s_a}^2 + m_{\phi_2}^2 \beta_i^2(m_{s_a}, m_{s_b}))^{1/2}$  and  $u = m_{\phi_2}^2 + m_{s_b}^2 - 2m_{\phi_2}(m_{s_b}^2 + m_{\phi_2}^2 \beta_i^2(m_{s_a}, m_{s_b}))^{1/2}$ .

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- [1] N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B **158**, 295 (1979).  
[2] M. A. B. Beg, C. Panagiotakopoulos and A. Sirlin, Phys. Rev. Lett. **52**, 883 (1984).  
[3] M. Lindner, Z. Phys. **C31**, 295 (1986).  
[4] G. Altarelli, G. Isidori, Phys. Lett. **B337**, 141-144 (1994).  
[5] J. A. Casas, J. R. Espinosa, M. Quiros, A. Riotto, Nucl.

- Phys. **B436**, 3-29 (1995). [arXiv:hep-ph/9407389 [hep-ph]].  
[6] J. A. Casas, J. R. Espinosa, M. Quiros, Phys. Lett. **B342**, 171-179 (1995). [hep-ph/9409458].  
[7] T. Hambye, K. Riesselmann, Phys. Rev. **D55**, 7255-7262 (1997). [hep-ph/9610272].  
[8] J. Ellis, J. R. Espinosa, G. F. Giudice, A. Hoecker, A. Riotto, Phys. Lett. **B679**, 369-375 (2009). [arXiv:0906.0954

- [hep-ph]].
- [9] R. Barate *et al.* [ LEP Working Group for Higgs boson searches and ALEPH and DELPHI and L3 and OPAL Collaborations ], Phys. Lett. **B565**, 61-75 (2003). [hep-ex/0306033].
- [10] D. Benjamin [for the CDF and D0 and The TEVNP Working Group (The Tevatron New Phenomena and Higgs Working Group) Collaborations], arXiv:1108.3331 [hep-ex].
- [11] Based on the talk “Higgs Searches at the LHC” given by Monica L. Vazquez Acosta at SUSY11, 2011
- [12] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].
- [13] G. Aad *et al.* [ATLAS Collaboration], arXiv:1109.6572 [hep-ex].
- [14] S. Chatrchyan *et al.* [CMS Collaboration], [arXiv:1109.2352 [hep-ex]].
- [15] S. R. Coleman, E. J. Weinberg, Phys. Rev. **D7**, 1888-1910 (1973).
- [16] W. A. Bardeen, FERMILAB-CONF-95-391-T.
- [17] R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D **77**, 035006 (2008) [arXiv:0709.2750 [hep-ph]].
- [18] S. Iso, N. Okada, Y. Orikasa, Phys. Lett. **B676**, 81-87 (2009). [arXiv:0902.4050 [hep-ph]].
- [19] S. Iso, N. Okada, Y. Orikasa, Phys. Rev. **D80**, 115007 (2009). [arXiv:0909.0128 [hep-ph]].
- [20] R. Foot, A. Kobakhidze and R. R. Volkas, Phys. Rev. D **82**, 035005 (2010) [arXiv:1006.0131 [hep-ph]].
- [21] L. Boyle, S. Farnsworth, J. Fitzgerald and M. Schade, arXiv:1111.0273 [hep-ph].
- [22] V. Silveira and A. Zee, Phys. Lett. B **161**, 136 (1985).
- [23] J. McDonald, Phys. Rev. D **50**, 3637 (1994) [hep-ph/0702143 [HEP-PH]].
- [24] C. P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B **619**, 709 (2001) [hep-ph/0011335].
- [25] H. Davoudiasl, R. Kitano, T. Li, H. Murayama, Phys. Lett. **B609**, 117-123 (2005). [hep-ph/0405097].
- [26] M. Sher, Phys. Rept. **179**, 273-418 (1989).
- [27] E. W. Kolb and M. S. Turner, “ The Early Universe”, Westview Press, 1990.
- [28] M. Gonderinger, Y. Li, H. Patel and M. J. Ramsey-Musolf, JHEP **1001**, 053 (2010) [arXiv:0910.3167 [hep-ph]].
- [29] M. Asano and R. Kitano, Phys. Rev. D **81**, 054506 (2010) [arXiv:1001.0486 [hep-ph]].
- [30] J. Hisano, K. Ishiwata and N. Nagata, Phys. Rev. D **82**, 115007 (2010) [arXiv:1007.2601 [hep-ph]].
- [31] J. Preskill, S. P. Trivedi, F. Wilczek and M. B. Wise, Nucl. Phys. B **363**, 207 (1991).