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Cryogenic shock tube

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Two shock tubes have been developed which allow for partial or full immersion of the test sections within a cryogenic bath. One tube is used for the study of shock-wave interactions with strong density gradients, and the other to obtain very large shock Mach numbers with ideal gas conditions in all flow regions.

The shock Mach number M_s in an ideal shock tube depends upon the ratio of the specific heats, speeds of sound and pressures of the driver (subscript 4) and driven gases (subscript 1)

$$M_s = M_s(\gamma_1, \gamma_4, a_4/a_1, p_4/p_1).$$

If the same monatomic gas is used in the driver and driven sections,

$$M_s \leq 4(T_4/T_1)^{1/2} \{1 + \frac{1}{16}(T_1/T_4) + O[(\frac{1}{4}T_1/T_4)^2]\}.$$

To increase M_s one commonly increases T_4 keeping T_1 at room temperature. During the last three years we have developed tubes in which T_4 is kept at room temperature and T_1 decreased to cryogenic temperatures. Using helium as the working gas T_1 can be decreased without much difficulty to 2 °K or so and limiting shock Mach numbers of the order of 50 are possible.

Figure 1 shows an earlier version of such a shock tube which has been used by one of us (V.C.R.) to study the strengthening of shock waves propagating into a density gradient.¹ Figure 2 shows the latest version of a tube capable of achieving a temperature "jump" across the

FIG. 1. 3.8-cm diam cryogenic shock tube.

FIG. 2. 2.5-cm diam cryogenic shock tube.

FIG. 3. Diaphragm pressure ratio versus shock Mach number with T_1 as a parameter. Solid lines represent ideal shock tube theory while the data is from the 2.5-cm tube.

diaphragm. Using a flexible diaphragm roll in a manner suggested by D. Coles of this laboratory, it is possible to fire 60 shocks without having to open the tube to atmospheric contamination. Finally, Fig. 3 demonstrates the results obtained with this tube. Gas imperfections are absent even for the helium behind a shock reflected from the end wall since it can easily be shown that the temperature T_5 satisfies

$$T_5 \leq 12T_4,$$

i.e., in the present case

$$T_5 \leq 3600^\circ\text{K}.$$

The present tube is intended to study the effect of well-controlled pressure and temperature pulses on cryogenic materials. As the first such application the tube is being used to generate finite amplitude first and second sound in liquid helium II.

This research effort has been sponsored by the Air Force Office of Scientific Research, Grant No. AF-AFOSR-71-2092.

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Quasineutral scaling laws

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(Received 25 April 1972; final manuscript received 23 June 1972)

Utilizing the Boltzmann equation we develop scaling laws for a quasineutral plasma. Any binary collision process is allowed but radiative processes, three-body collisions, and self-magnetic fields are not allowed. The independent variables appropriate to a cylindrical discharge are given.

We present a new type of scaling law which makes use of quasineutrality. We consider plasmas in which the distribution function f_j for each of the j types of particles of a plasma is governed by

$$\frac{\partial f_j}{\partial t} + \frac{q_j}{m_j} (\mathbf{E} + \mathbf{v} \times \mathbf{B}_a) \cdot \frac{\partial f_j}{\partial \mathbf{v}} + \mathbf{v} \cdot \frac{\partial f_j}{\partial \mathbf{x}} = \sum_k J_{jk}(f_j, f_k), \quad (1)$$

$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}_a}{\partial t}, \quad (2)$$

and

$$\sum_j q_j \int d^3\mathbf{v} f_j = 0, \quad (3)$$

where $\mathbf{v}$ is the velocity of the particle, $\mathbf{x}$ is the position of the particle, t is the time, $\mathbf{E}$ is the sum of an applied electric field, and an internal field, $\mathbf{B}_a$ is an applied magnetic field ($\nabla \cdot \mathbf{B}_a = 0$), and we assume that we may neglect the self-magnetic field. The mass of the j th type of particle is m_j ; q_j is the charge of the j th

type of particle and $J_{jk}(f_j, f_k)$ is the bilinear collision integral representing the interaction of the j th and k th types of particles. The first equation is the Boltzmann equation for the j th type of particle; Eq. (2) is one of Maxwell's equations without displacement currents, and Eq. (3) is the condition of quasineutrality which replaces Poisson's equation under quasineutral conditions. Implicit in these equations is the neglect of the self-magnetic field, radiative processes, and three-body collisions.

Our claim is that the above set of equations exhibit an invariance property. For each $\alpha > 0$, the equations remain invariant under the transformation

$$\mathbf{x}' = \mathbf{x}/\alpha, \quad \mathbf{v}' = \mathbf{v}, \quad t' = t/\alpha,$$

$$f_j'(\mathbf{x}', \mathbf{v}', t') = \alpha f_j(\mathbf{x}, \mathbf{v}, t), \quad (4)$$

$$\mathbf{E}'(\mathbf{x}', t') = \alpha \mathbf{E}(\mathbf{x}, t), \quad \mathbf{B}_a'(\mathbf{x}', t') = \alpha \mathbf{B}_a(\mathbf{x}, t).$$