y Scaling in Electron-Nucleus Scattering

D. B. Day, J. S. McCarthy, Z. E. Meziani,[a] R. Minehart, R. Sealock, and S. T. Thornton
Institute of Nuclear and Particle Physics and Department of Physics, University of Virginia, Charlottesville, Virginia 22901

J. Jourdan[b] and I. Sick
Institut für Physik, Universität Basel, CH-4056 Basel, Switzerland

B. W. Filippone, R. D. McKeown, R. G. Milner, and D. H. Potterveld
Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

and

Z. Szalata
American University, Washington, D.C. 20016
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Data on inclusive electron scattering from $A=4$, 12, 27, 56, 197 nuclei at large momentum transfer are presented and analyzed in terms of y scaling. We find that the data do scale for $y<0$ $(x>1)$, and we study the convergence of the scaling function with the momentum transfer Q^2 and A.

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The nuclear response function measured by inclusive quasielastic electron-nucleus scattering is an important observable in the study of the wave function of the bound nuclear system and the properties of its constituents. In the impulse approximation (IA) it relates the inclusive cross sections to the free electron-nucleon cross sections σ_{ep} and σ_{en}, and to the spectral function[1] $\rho(k,E)$ which gives the probability to find a nucleon in the nucleus with momentum k and separation energy E. On general grounds, the inclusive cross section $\sigma(q,\omega)$ depends on electron energy loss ω and momentum transfer q independently.

Much of the information in $\sigma(q,\omega)$ can only be extracted if the reaction mechanism of the scattering process is understood. Both nuclear properties and the reaction mechanism can be studied in detail if the inclusive cross section can be shown to scale,[2] i.e., depend on a single variable $y(q,\omega)$ rather than on ω and q separately. The scaling property yields information on the reaction mechanism, and in the limit $q\rightarrow\infty$ the scaling function provides a direct measurement of the longitudinal-momentum distribution of the nucleon constituents.

Since the prediction of the scaling behavior by West[2] and the demonstration of this behavior in light nuclei,[3,4] there has been considerable discussion of the properties of y scaling. This interest has been motivated by the importance of extracting nuclear momentum distributions from the experimental data and by the potential of this scaling law to tell us how the system we study at finite q differs (by the nature of the scale breaking) from the ideal one defined by the IA. Discussions have appeared in the literature on this convergence,[5] on different scaling variables, and on the importance of final-state interactions (FSI) in the determination of the momentum distributions.[6] For ^{3}He y scaling has been used[7,8] to set limits on the modification of the free-nucleon properties by the nuclear medium.

Experimental data suitable for such an interpretation and the study of scaling previously have been limited to $A\leq 3$. Detailed analysis has been performed for ^{3}He only, the case where the occurrence of y scaling was first demonstrated.[3] In this Letter we present data for a range of A from 4 to 197. These data, taken at very large momentum transfer and comparatively small energy transfer, complement the more extensive data on deep-inelastic scattering at large ω, for which the value of the Bjorken variable $x=Q^2/2m\omega$ is less than 1.

Scaling is a concept valid only in the limit of very large momentum transfer; the scale-breaking quantities are the nuclear binding energies and nucleon momenta. With the present experiment we cover a large enough range of q to study the convergence with increasing q, and to investigate the role of other scale-breaking effects like nucleon FSI and deviations from the free-nucleon response.

The experiment was done at SLAC with the nuclear physics injector. Data were taken at 2.02 GeV at 15° and 20°, and at 3.595 GeV at 16°, 20°, 25°, 30°, and 39° (Fe only). The scattered electrons were detected in the 8-GeV/c magnetic spectrometer. To span the region from the threshold over the quasielastic peak to the region of the first nucleon resonance, overlapping spectra were taken at many magnetic field settings of the spectrometer. Electron trajectories were determined by ten planes of proportional wire chambers and electron identification was provided by a lead glass shower counter and a gas Cherenkov counter. Electron contributions from background processes such as π^0 decay and pair production are small at the final electron energies of interest in this experiment.

The targets included a 15-cm-long cell through which liquid hydrogen was recirculated, and a 25-cm-long cell of recirculating ^{4}He at 25 atm and 21 K. The assembly included empty target cells and solid targets of natural isotopic abundances. Data were taken with 0.02-radiation-length (r.l.) C, Al, and Fe and 0.06-r.l Au targets. In order to maintain reasonable counting rates for the highest q value a 0.06-r.l. Fe target was used as well.

Variations in the ^{4}He target density with beam intensity were studied by our taking high-statistics calibration data at several beam intensities and beam repetition rates. The density changes were reproducible and varied linearly with the average beam current. The largest correction applied to the ^{4}He density was 18% at 14 μA.

The measured cross sections were corrected for radiative effects with use of the formulas of Stein and of Mo and Tsai.[9] To test the accuracy of radiative corrections, the spectra at 3.595 GeV and 16° were taken with both 0.02- and 0.06-r.l. Fe targets; agreement was found within <1% in σ. The radiative tail from the elastic peak was calculated for the worst case, ^{4}He at 2.02 GeV and 15°, and was found to contribute negligibly to the measured cross sections.

As a test of the overall efficiency of our detection system at each q, measurements of elastic scattering from the proton were compared with parametrizations of the available data. We found our results to agree within the uncertainties (3%) of the parametrizations.[10] Systematic uncertainties in the cross sections (only statistical errors are shown in the figures) were due to spectrometer acceptance ($\pm 1.5\%$), beam-intensity monitoring ($\pm 0.5\%$), and radiative corrections ($\pm 3.0\%$). The target thicknesses were known to better than $\pm 2.0\%$ for ^{4}He and $\pm 1.0\%$ for the solid targets.

The data set for C is shown in Fig. 1. The cross section $d^2\sigma/d\Omega\, d\omega$, plotted versus ω, varies by almost 6 orders of magnitude over the range of Q^2 and ω indicated. The quasielastic peak is seen clearly at low Q^2. At higher Q^2 the quasielastic peak is obscured by inelastic processes that include Δ production, nonresonant π production, excitation of higher nucleon resonances, and deep-inelastic electron-nucleon scattering. For nuclei heavier than C, the quasielastic peak is less well defined because the larger Fermi motion smears the inelastic processes into the quasielastic region. For ^{4}He, the quasielastic peak is distinct to $Q^2 > 1$ (GeV/c)^2. The kinematic extents of the data sets of ^{4}He, Al, and Au are similar to the one shown in Fig. 1. The data for Fe cover a larger range of ω and Q^2.

The scaling variable y can be determined if we assume that the impulse approximation is valid, and that the electron interacts with a single off-shell nucleon. These

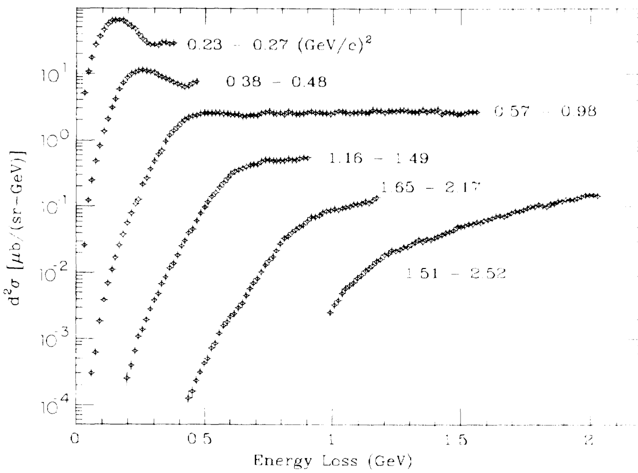

FIG. 1. Experimental values of the cross section from C vs energy loss. The Q^2 range (Q^2 decreases with increasing energy loss) is shown for each line.

assumptions allow us to write the energy and momentum conservation as

$$\omega + M_A = [(\mathbf{k}+\mathbf{q})^2 + M]^{1/2} + [\mathbf{k}^2 + (M_{A-1} + E_{A-1})^2]^{1/2}, \quad (1)$$

where E_{A-1} is the internal energy of the recoiling residual nucleus and $\mathbf{k}$ is the momentum of the knocked out nucleon before scattering. The initial nucleon is bound, with a total energy (kinetic and binding) equal to the negative of the separation energy.

In the limit of very large momentum transfer, $q \gg k_F$, Eq. (1) can be simplified. Terms not involving q and ω are small and can be replaced by their average values (ϵ for the binding energy and $\bar{k}_\perp^2$ for the perpendicular momentum). In the limit of very large momentum transfer, ω and q then are no longer independent variables. They are connected via the nucleon momentum component k. This momentum component, calculated from Eq. (1), $k_\parallel = y(q,\omega)$, is the scaling variable.

To obtain the scaling function $F(y)$, we use

$$F(y) = (d^2\sigma/d\omega\, d\Omega)(q,\omega) \times [Z\sigma_{ep}(q) + N\sigma_{en}(q)]^{-1} d\omega/dy. \quad (2)$$

In the IA, $F(y)$ represents the probability to find nucleons with momentum component $k_\parallel = y$ in the nucleus. The point $y=0$ corresponds to the maximum of the quasielastic peak and $y<0$ corresponds to the low-ω side. In terms of the Bjorken scaling variable x, $y=0$ occurs at $x \simeq 1$, while the $y<0$ region covers the domain $1 < x < A$.

We note that our scaling variable, as defined in Eq. (1) and Ref. 3, differs in two respects from the one introduced by West. When writing down energy and momentum conservation, West assumed the Fermi-gas model for the nucleus, with nucleons that in the initial state have a positive energy $k^2/2M$; Eq. (1) accounts for the off-shell nature of the bound nucleons. Moreover, West used the nonrelativistic energy-momentum relation for the recoil nucleon. For large momentum transfer (where scaling is valid) the resulting large recoil necessitates relativistic treatment. An alternative expression of y has been discussed by Gurvitz and Rinat.[6] By including effects of FSI one can hope to increase the rate of convergence of $F(y)$ with q at the expense of introducing some model dependence of $F(y)$.

In Fig. 2 the data of Fig. 1 are presented in terms of the scaling function $F(y)$. Nucleon FSI effects are greatest at small energy transfers; hence for the scaling analysis we use only the data with energy loss 50 MeV above breakup threshold. The electron-nucleon cross sections, σ_{ep} and σ_{en}, in Eq. (2) are the off-shell expressions by deForest,[11] while the form factors are the ones of Simon *et al.*[10] and Blatnik and Zovko[12] for the proton and neutron, respectively. The values for the parameters ϵ and $\bar{k}_\perp^2$ are taken from Fermi-gas fits to low-Q^2 quasielastic scattering data.[13]

The data shown in Fig. 2 exhibit a scaling behavior. At large q the cross sections which extend many decades in value collapse onto a narrow band defining $F(y)$.

Inspection of Fig. 2 reveals that the data do not scale for $y > 0$. This region corresponds to the high-ω side of the quasielastic peak where the reaction mechanism is not simple nucleon knockout. Inelastic processes such as π production, excitation of Δ's, and deep-inelastic electron-nucleon scattering dominate. Contributions from these processes are not expected to scale in y—they cannot be described in terms of the form factors implicit in Eq. (2) or the kinematics described by Eq. (2). Similar nonscaling is obtained at $y < 0$ if calculated cross sections for processes dominated by FSI or by meson-exchange currents are included. These observations emphasize the usefulness of the scaling property as a means to identify the reaction mechanism experimentally. Scaling has been demonstrated only for IA-dominated processes involving single-nucleon knockout.

In the region $y < 0$, the scaling behavior is not exact. $F(y)$ falls with increasing q, and converges to a single curve as required by the impulse approximation at very large q. Figure 3 shows the convergence for Fe for two values of y where the Q^2 dependence was determined by our fitting $F(y)$ at each energy and angle with phenomenological parametrization. The fit was restricted to values of y within ± 0.1 of the points shown. For light nuclei, convergence is faster in accordance with the smaller k_F. For ^4He the difference in $F(y)$ ($y = -0.4$) between the maximum and minimum q is only 22%. Figure 3 emphasizes that scaling is a concept valid only in the limit of $q \gg k_F$. While the absolute values quoted for $F(y)$ may depend upon the parameters ϵ and k^2, and the particular definition of y and $F(y)$, the trend in Q^2 is little affected.

The approach to convergence of $F(y)$ with Q^2 is compatible with the one obtained by Gurvitz and Rinat[6] as a result of the nucleon FSI. It is not clear, however, that FSI is the main cause for the change of $F(y)$ with Q^2. Even at $y < 0$ the cross section receives some contribution from the smeared inelastic processes which can also lead to a residual q dependence of $F(y)$.

It is of interest to study the behavior of the scaling

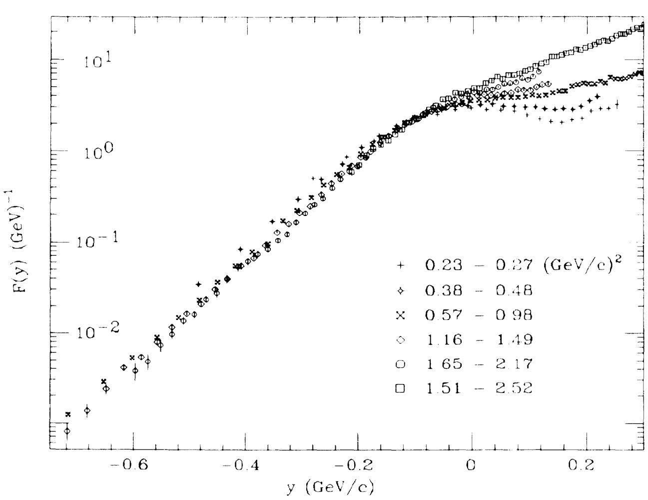

FIG. 2. $F(y)$ for data of Fig. 1 through Eq. (2). Only data that are more than 50 MeV above the threshold for breakup and have fractional errors less than 0.3 are shown.

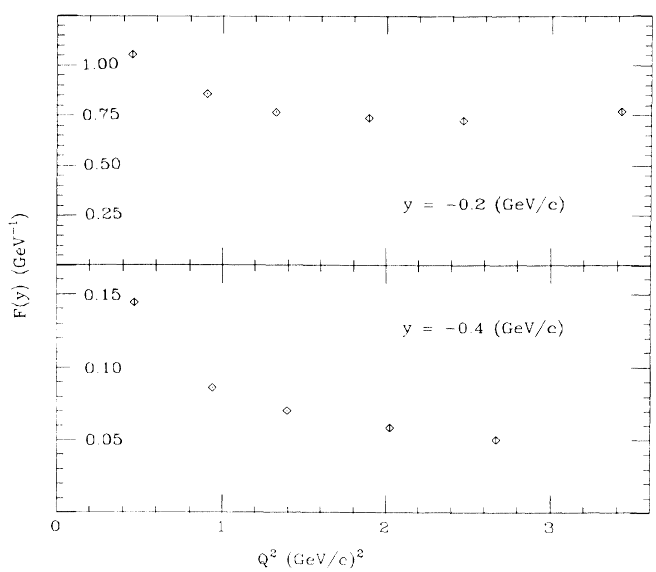

FIG. 3. The convergence of $F(y)$ for iron with Q^2 at two values of y.

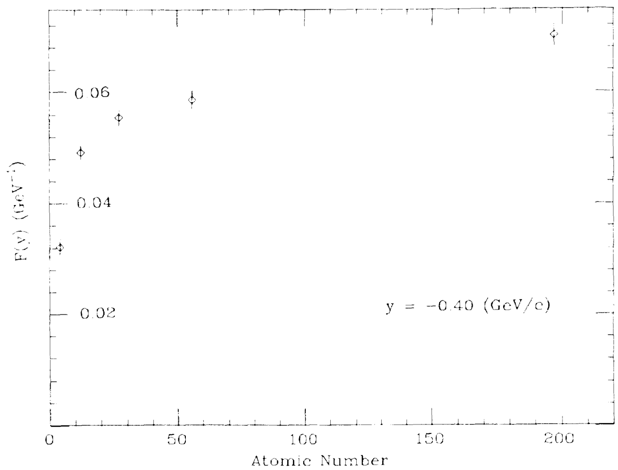

FIG. 4. The value of $F(y)$ at $y=-0.4$ vs the atomic number. The values of $F(y)$ are from the data set with $1.6 \leq Q^2 \leq 2.2$ $(\mathrm{GeV}/c)^2$.

function $F(y)$ with increasing nuclear mass number A. The nucleon-momentum distribution for large A is expected to be a universal function that depends mainly on the short-range NN interaction and little on the specific nucleus. To determine the A dependence, we use the data with $1.6 \leq Q^2 \leq 2.2$ $(\mathrm{GeV}/c)^2$, fitting $F(y)$ for each nucleus in the y range $-0.5 < y < -0.3$. The value of the fit at $y=-0.4$ is plotted in Fig. 4. The data clearly approach an asymptotic value at large A, with a rate of convergence that reflects the increase of volume-to-surface ratio with A.

The present experiment has confirmed that the inelastic electron-nucleus data at $y<0$ exhibit a scaling behavior at large q. Such an observation is very helpful in the identification of the reaction mechanism one needs to know for a quantitative understanding of the data. The approach to scaling as a function of Q^2 for heavy nuclei is less rapid than for ^{4}He, indicating either more important FSI or other inelastic processes contributing to the cross section. These inclusive measurements in the region $y<0$ can be used as a test for possible modification of the bound-nucleon properties, and a quantitative measurement of the momentum distribution in nuclei and nuclear matter.

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[a]Present address: Department of Physics, Stanford University, Stanford, CA 94305.

[b]Present address: Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, CA 91125.

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