following set: 5, 11, 13, 17, 19, 23, 29. It would be a very simple matter to extend these sets.

It is now easy to prove the following theorem: *Every group in which the total number of non-invariant operators is a prime number contains no invariant operator besides the identity.* In fact, if the total number of the non-invariant operators of the group is a prime number the order of the central quotient group diminished by unity must be equal to this prime number. The central quotient group must be simply isomorphic with the group since the number of the non-invariant operators of a group is the product of the order of the central, and the order of the central quotient group diminished by unity. The converse of this theorem is obviously not necessarily true since it is easy to find groups which contain no invariant operator besides the identity but in which the number of the non-invariant operators is not a prime number. For instance, the dihedral group of order 10 has this property.

There is one and only one group in which the total number of the non-invariant operators is one of the three prime numbers 5, 11, 13 but there are two groups in which the number of the non-invariant operators is exactly 17. One of these is the dihedral group of order 18 and the other is the generalized dihedral group of this order. When the number of the non-invariant operators contained in G is exactly 23 the symmetric group of order 24 is the only one of the 15 groups of this order which contains no invariant operator besides the identity and hence this group is characterized by the fact that it contains exactly 23 non-invariant operators. This characterization of the symmetric group of degree 4 may therefore be added to the numerous known definitions of this well-known group.

NOTE ON THE FUNCTION $F(a, b; c - n; z)$

BY H. BATEMAN*

NORMAN BRIDGE LABORATORY OF PHYSICS, CALIFORNIA INSTITUTE OF TECHNOLOGY

Communicated January 8, 1944

The generating function

$$(1 - t)^{b-1}(1 - z + zt)^{-a} = \sum_{n=0}^{\infty} (t^n/n!)(1 - b, n)F(a, b; b - n; z) \quad |t| < 1, \ |zt| < |1 - z|$$

may be used to find an estimate of $F(a, b; b - n; z)$ for large positive values of n. When the point $t = 1 - 1/z$ lies outside the circle $t = 1$ the singularity $t = 1$ may be used to find an estimate by the method of Darboux[1] and the result is

$$F(a, b;\ b - n;\ z) \sim 1.$$

When $c - b$ is an integer the foregoing result indicates that $F(a, b;\ c - n;\ z) \sim 1$ under the same restriction on z. This is the result used in previous papers[2] but the case in which the point $t = 1 - 1/z$ lies within the unit circle or on its circumference was not considered. These cases can also be treated by the method of Darboux. It may be noted, however, that when $1 - 1/z$ lies inside the unit circle the point $1 - 1/(1 - z)$ lies outside the circle. Use may then be made of the formula of Barnes[3]

$$F(a, b;\ c - n;\ z) = AF(a, b;\ 1 + a + b - c + n;\ 1 - z) +$$
$$Bz^{1 - c + n}(1 - z)^{c - a - b - n}F(1 - a, 1 - b;\ 1 - a - b + c - n;\ 1 - z)$$

where

$$A = \Gamma(c - n - a - b)\Gamma(c - n)/\Gamma(c - n - a)\Gamma(c - n - b)$$
$$B = \Gamma(a + b - c + n)\Gamma(c - n)/\Gamma(a)\Gamma(b).$$

It should be noticed that $(1 - a - b + c) - (1 - a)$ is an integer when $c - b$ is an integer so the previous result may be applied. It is probable, however, that the result holds without this restriction. The first term is $0(1)$ when n is large and so in this case we have the estimate

$$F(a, b;\ c - n;\ z) \sim n^{a + b - 1}(1 - z)^{1 - a - b}\left(\frac{z}{1 - z}\right)^{1 - c + n}\pi \div [\Gamma(a)\Gamma(b) \sin \pi(c - n)].$$

On this account restrictions are needed for the deduction $\lim_{u \to \infty} F(-m, -r;\ -n - r;\ x) = 1$. When the point $1 - 1/z$ lies on the unit circle there are two possible singularities to be taken into consideration in the method of Darboux and so unity should be added to the foregoing expression. If, however, $R(a + b) < 1$ unity is negligible in comparison with the term just mentioned and if $R(a + b) > 1$ unity is the dominant term. The case in which $R(a + b) = 1$ is exceptional because then both terms must be taken into consideration.

It is noteworthy that in the case $z = 1/2$ there are two singularities on the unit circle. For the function of Mittag-Leffler

$$g_n(z) = 2zF1 - n, 1 - z;\ 2;\ 2)$$
$$= 2^{-z}(z/, n)F(z + 1, z;\ 1 + z - n;\ 1/2)$$

the condition $R(a + b) < 1$ is satisfied when $R(z) < 0$ and in this case an estimate is

$$g_n(z) \sim 2^{-z}(z/, n).$$

When $R(z) < 0$ use may be made of the formula

$$g_n(-z) = (-)^n g_n(z),$$

so in this case

$$g_n(z) \sim (-)^n 2^z(-z/, n) = 2^z(z, n)/n.$$

For the function

$$\begin{aligned} g_n(z, r) &= (-)^n(r/, n)F(-n, -z - r, -r; 2) \\ &= (z + r/, n)2^{-z}F(z, r + z + 1; r + z - n + 1; {}^1/_2) \end{aligned}$$

the condition $R(a + b) < 1$ is satisfied when $R(2z + r) < 0$ and so the estimate

$$g_n(z, r) \sim (z + r/, n)2^{-z}$$

holds under this condition. The equation

$$g_n(-z - r, r) = (-)^n g_n(z, r)$$

indicates that when $R(2z + r) > 0$ the estimate is

$$g_n(z, r) \sim (-)^n 2^{z+r}(-z/, n) = 2^{z+r}(z, n)/n!.$$

When $R(2z + r) = 0$ the proper estimate is

$$g_n(z, r) = (z + r/, n)2^{-z} + 2^{z+r}(z, n)/n!$$

and in the case of the polynomial of Mittag-Leffler, when $R(z) = 0$

$$g_n(z) \sim 2^{-z}(z/, n) + 2^z(z, n)/n!$$

* For list of Errata to previous articles, see page 44, infra.

[1] Darboux, G., *Jour. de Math.*, (3) **6**, 1–56, 377–416 (1878).

[2] Bateman, H., *Proc. Nat. Acad. Sci.*, **26**, 491–496 (1940); **28**, 371–374 (1942).

[3] Barnes, E. W., *Proc. London Math. Soc.*, (2) **6**, 157 (1908).

INDOLE AND SERINE IN THE BIOSYNTHESIS AND BREAKDOWN OF TRYPTOPHANE*

BY E. L. TATUM AND DAVID BONNER

DEPARTMENT OF BIOLOGY, STANFORD UNIVERSITY

Communicated January 31, 1944

Indole and anthranilic acid have been suggested as intermediates in the biosynthesis of tryptophane by certain bacteria.[1, 2] Among the mutant strains of *Neurospora*, produced as described elsewhere,[3] two types of tryptophane deficient strains have been found. One strain will grow in the presence of indole, and the other in the presence of either indole or