Top-Quark Mass and Bottom-Quark Decay

Paul H. Ginsparg and Sheldon L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

Mark B. Wise
Lauritsen Laboratory of High Energy Physics, California Institute of Technology, Pasadena, California 91125
(Received 7 February 1983)

The possibility of a long B-meson lifetime is explored, in which case the weak mixing angles θ_2 and θ_3 are quite small. This allows the derivation of a *lower* bound on the top-quark mass as a function of the B-meson lifetime, by comparison of the short-distance prediction for the CP-nonconservation parameter ϵ with its experimental value. The bound is significant for $\tau_B > 4\times10^{-13}$ s.

PACS numbers: 14.80.Dq, 12.35.Eq, 12.70.+q, 14.40.Jz

In the standard six-quark model of the strong, weak and electromagnetic interactions the quarks are in left-handed doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad (1)$$

and right-handed singlets of the weak SU(2) gauge group. The primed fields are not mass eigenstates, but are related to the mass eigenstate fields d_L, s_L, and b_L by the unitary transformation[1]

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L, \qquad (2)$$

where $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$, $i \in \{1,2,3,\delta\}$. The phases of the quark fields are chosen so that θ_1, θ_2, and θ_3 lie in the first quadrant. The quadrant of δ has physical significance and cannot be fixed by convention. Experimental information from beta decay gives $s_1^2 \simeq 0.05$. The observed validity of Cabibbo universality implies the further constraint[2] $s_3 \leq 0.5$.

The B meson can have a lifetime as short as $\sim 10^{-14}$ s. On the other hand, if s_2 and s_3 are small, the lifetime can be arbitrarily long. With s_2 and s_3 small, the top quark must be heavy in order to obtain the observed degree of CP nonconservation in kaon decay. In this paper, we compute the lower limit to the top-quark mass as a function of the lifetime of the B particle.

Various constraints of the mixing angles have been derived from comparisons of the measured values of the CP-nonconservation parameter ϵ and the K_L-K_S mass difference with predictions for these quantities based on a short-distance expansion.[3] An upper bound on the top-quark mass has been derived by comparing short-distance predictions for the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate with experiment.[4] Unfortunately these predictions are not reliable, since it is difficult to justify a short-distance expansion for the K_L-K_S mass difference. Higher-dimension operators, such as the time-ordered product of two effective Hamiltonians for $\Delta s = 1$ weak nonleptonic decays, are neglected because they lack a factor of m_c^2. Matrix-element enhancements, such as those that take place in $K \to \pi\pi(I=0)$ decay,[5] probably make these higher-dimension operators more important than the short-distance piece. The same criticism does not necessarily apply to the use of a short-distance expansion for the CP-nonconservation parameter ϵ. It is likely that the higher-dimension operators do not contribute a significant imaginary part to the K^0-$\bar{K}^0$ mass matrix in the basis in which the $K^0 \to \pi\pi(I=0)$ amplitude is real.

Since the b quark is heavy compared with the scale of the strong interactions, B-meson decay can be approximated by the decay of a free b

quark. Then

$$\Gamma_B = \Gamma(b \to c) + \Gamma(b \to u), \tag{3}$$

where

$$\Gamma(b\to c) = (G_F^2 m_b^5/192\pi^3)[(c_1c_2s_3 + s_2c_3c_\delta)^2 + s_2^2c_3^2s_\delta^2]\{2f(m_c/m_b) + \varphi(m_c/m_b, m_\tau/m_b) + 3\eta_0 f(m_c/m_b)(c_1^2 + s_1^2c_3^2) + 3\eta_0 h(m_c/m_b)[s_1^2c_2^2 + (c_1c_2c_3 - s_2s_3c_\delta)^2 + s_2^2s_3^2s_\delta^2]\}, \tag{4a}$$

$$\Gamma(b\to u) = (G_F^2 m_b^5/192\pi^3)(s_1^2s_3^2)\{2 + f(m_\tau/m_b) + 3\eta_0(c_1^2 + s_1^2c_3^2) + 3\eta_0 f(m_c/m_b)[s_1^2c_2^2 + (c_1c_2c_3 - s_2s_3c_\delta)^2 + s_2^2s_3^2s_\delta^2]\}. \tag{4b}$$

In Eqs. (4) f, h, and φ are phase-space suppression factors.[6] For $m_c = 1.4$ GeV and $m_b = 4.6$ GeV they have the values $f(m_c/m_b) = 0.51$, $h(m_c/m_b) = 0.19$, $f(m_\tau/m_b) = 0.33$, and $\varphi(m_c/m_b, m_\tau/m_b) = 0.09$. The quantity η_0 takes into account strong-interaction corrections to the effective Hamiltonian for $|\Delta b| = 1$ weak nonleptonic decays, and in the leading logarithmic approximation (neglecting penguin-type contributions)

$$\eta_0 = \frac{1}{3}\left\{2\left[\frac{\alpha_s(M_W^2)}{\alpha_s(m_t^2)}\right]^{12/21}\left[\frac{\alpha_s(m_t^2)}{\alpha_s(m_b^2)}\right]^{12/23} + \left[\frac{\alpha_s(m_W^2)}{\alpha_s(m_t^2)}\right]^{-24/21}\left[\frac{\alpha_s(m_t^2)}{\alpha_s(m_b^2)}\right]^{-24/23}\right\}. \tag{5}$$

Using the quark masses mentioned previously, $M_W = 80$ GeV, and $\Lambda_{QCD} = 0.1$ GeV, we find $\eta_0 \simeq 1.1$ (note that η_0 is roughly independent of the top-quark mass).

For long B-meson lifetimes, the angles θ_2 and θ_3 are small, and to first nontrivial order in these small quantities, Eqs. (4) imply

$$(s_2^2 + s_3^2 + 2s_2s_3c_\delta) = 4.2\times10^{-3} R(b\to c)[\tau_B/(10^{-12}\text{ s})]^{-1}, \tag{6a}$$

$$s_3^2 = 3.9\times10^{-2} R(b\to u)[\tau_B/(10^{-12}\text{ s})]^{-1}, \tag{6b}$$

where R denotes branching ratio. Experimentally,[7] $R(b\to u) < 0.09$. "Higher-order" contributions to the B-meson lifetime would give even smaller angles. These effects, however, appear to be very small.[8]

The imaginary part of the K^0-$\bar{K}^0$ mass matrix violates CP conservation and can be reliably calculated by use of a short-distance expansion. Neglecting CP nonconservation from $K\to 2\pi$ decay amplitudes,[9] we find for the CP-nonconservation parameter ϵ

$$\epsilon = -\frac{s_1^2 B G_F^2 f m_K^2 m_c^2}{16\sqrt{2}\,\pi^2(m_{K_S} - m_{K_L})} c_2s_2s_3s_\delta\Big[\eta_1(-c_1c_2^2c_3 + s_2c_2s_3c_\delta) + \eta_2\left(\frac{m_t}{m_c}\right)^2(c_1s_2^2c_3 + s_2c_2s_3c_\delta) + \eta_3\ln\left(\frac{m_t^2}{m_c^2}\right)(c_1c_2^2c_3 - c_1s_2^2c_3 - 2s_2c_2s_3c_\delta)\Big]e^{i\pi/4}. \tag{7}$$

In Eq. (7), η_1, η_2, and η_3 take into account strong-interaction corrections to the effective Hamiltonian for $|\Delta s| = 2$ K^0-$\bar{K}^0$ mixing.[10] They are roughly independent of the value of the top-quark mass, and for $m_c = 1.4$ GeV, $m_b = 4.6$ GeV, $M_W = 80$ GeV, $\Lambda_{QCD} = 0.1$ GeV, and $\alpha_s(\mu^2) = 1$ have the following approximate values: $\eta_1 \simeq 0.7$, $\eta_2 \simeq 0.6$, and $\eta_3 = 0.4$. B is the factor that relates the K^0-$\bar{K}^0$ matrix element of the local operator $[\bar{s}_\alpha\gamma^\mu(1-\gamma_5)d_\alpha]\times[\bar{s}_\beta\gamma_\mu(1-\gamma_5)d_\beta]$ to fm_K^3. We use $f = 0.13$ GeV. In the soft pion and kaon limit the magnitude of B is determined in terms of the measured $K^-\to\pi^-\pi^0$ amplitude and the coefficient of the $I = \frac{3}{2}$ operator in the effective Hamiltonian for $|\Delta s| = 1$ weak nonleptonic decays.[11] With the parameters used previously, the magnitude of B is equal to 0.37. Note that the quantities η_1, η_2, and η_3 depend on the subtraction point μ as $[\alpha_s(\mu^2)]^{-2/9}$ while B depends on the subtraction point as $[\alpha_s(\mu^2)]^{2/9}$, leaving the physical quantity ϵ independent of our arbitrary choice of subtraction point.

We can derive a lower bound on the top-quark mass, m_t, by substituting the experimental values[2] $\epsilon = (2.27\times10^{-3})e^{i\pi/4}$ and $m_{K_S} - m_{K_L} = -3.5\times10^{-15}$ GeV in Eq. (7) and considering all values of the angles θ_2, θ_3, and δ allowed by Eqs. (6). In Fig. 1, we show the bounds as a function of the B-meson lifetime, assuming the current experimental limit $\Gamma(b\to u)/\Gamma(b\to c) < 0.1$ in (6). We do not consider values of m_t greater than 60 GeV

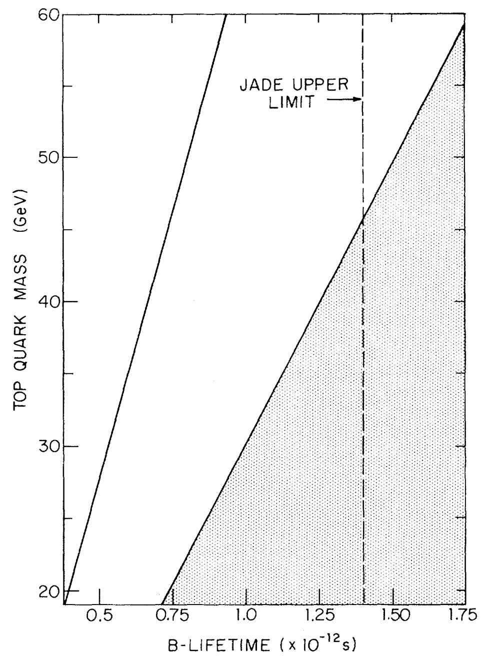

FIG. 1. Lower bounds on the top-quark mass as a function of the B-meson lifetime. The stronger bound is relevant for $\cos\delta > 0$. The shaded region is excluded whatever the sign of $\cos\delta$. These bounds assume that $\Gamma(b\to u)/\Gamma(b\to c) \leq 0.1$. Also shown is a quoted upper limit to the B lifetime (Ref. 12).

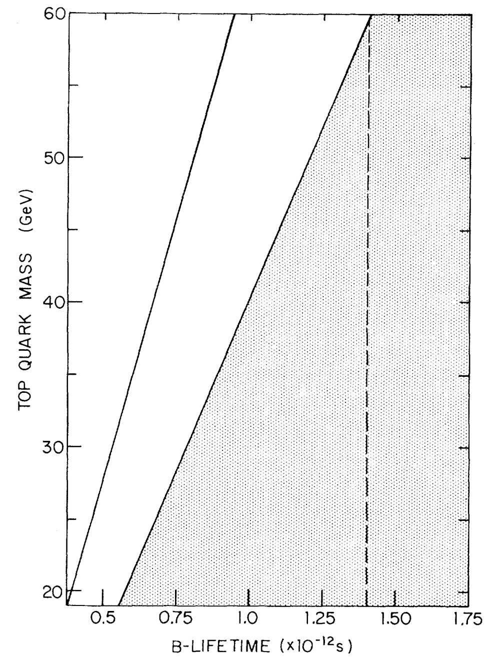

FIG. 2. Lower bounds on the top-quark mass as a function of the B-meson lifetime. Here we assume that $\Gamma(b\to u)/\Gamma(b\to c) \leq 0.05$. Such an improved result would lead to stronger bounds on the top-quark mass.

since the derivation of Eq. (7) requires m_t small compared with M_W. Bounds for $c_\delta > 0$ and $c_\delta < 0$ are plotted separately, with $c_\delta > 0$ giving a stronger bound on the top-quark mass. This is because for $c_\delta > 0$ there can be no cancellation in Eq. (6a) and the consequent smallness of s_2 and s_3 keeps the coefficient of the m_t^2/m_c^2 term in Eq. (7) small as well. If the top quark is found to have a mass consistent with the $c_\delta < 0$ bound, but not consistent with the $c_\delta > 0$ bound, then the phase δ is determined to lie in either the second or third quadrant.

We have not assumed any knowledge of the sign of B. The measured phase of ϵ implies that $Bs_\delta > 0$ for $c_\delta > 0$. However, Bs_δ can have either sign for $c_\delta < 0$ and the bound in Fig. 1 corresponds to $Bs_\delta > 0$. $Bs_\delta < 0$ implies a much more stringent constraint on the top-quark mass, corresponding to $m_t > 60$ GeV for all B lifetimes plotted in our figures.

If the experimental limit on $\Gamma(b\to u)/\Gamma(b\to c)$ is improved, then our bound on m_t is also improved. Figure 2 shows the lower bounds on m_t for $c_\delta < 0$ and $c_\delta > 0$ when we require $\Gamma(b\to u)/\Gamma(b\to c) < 0.05$.

The bounds shown in Fig. 1 become useful for $\tau_B > 4\times 10^{-13}$ s. For example, if τ_B exceeds 10^{-12} s, the top quark must be heavier than 30 GeV and toponium is inaccessible to TRISTAM. If the experimental upper limit on $\Gamma(b\to u)/\Gamma(b\to c)$ is improved by a factor of 2, the bound becomes 40 GeV. The current experimental upper limit[12] on τ_B is $\tau_B < 1.4\times 10^{-12}$ s. If the values of m_t and τ_B turn out to lie in the excluded region, new physics (like the existence of a fourth generation) is mandatory. If they lie between our bounds, we have determined that $\cos\delta < 0$, thus resolving a quadrant ambiguity of the Kobayashi-Maskawa model.

This work was supported by the National Science Foundation under Grants No. PHY77-22864 and No. PHY82-15249 and by the U. S. Department of Energy under Contract No. DE-AC03-81ER40050.

Note added.—For $\tau_B = 10^{-12}$ s, the lower bound ($c_\delta < 0$) on the top-quark mass in Fig. 1 occurs at $s_2 \simeq 0.1$, $s_3 \simeq 0.06$, and $s_\delta \simeq 0.06$. For these values, the middle term (proportional to m_t^2) in Eq. (7) contributes about 45% of ϵ. We note that the lower bound always comes from extreme values of the range of angles consistent with Eq. (6). Generic values naturally give a larger top-quark mass.

A major theoretical uncertainty in our bound is the value of B. If B were increased 25% to 0.46 by, for example, higher-momentum dependence in the amplitudes for $K\to\pi\pi(I=2)$ and for K^0-$\bar{K}^0$ mixing, then the lower bound on m_t in Fig. 1 would decrease to 24 GeV for $\tau_B = 10^{-12}$ s. Smaller values of B would give a correspondingly more stringent bound.

We thank F. Gilman for useful discussions.

[1]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 65 (1973).

[2]M. Roos *et al.* (Particle Data Group), Phys. Lett. 111B, 1 (1982).

[3]R. Shrock, S. Treiman, and L. Wang, Phys. Rev. Lett. 43, 1589 (1979); V. Barger, W. Long, and S. Pakvasa, Phys. Rev. Lett. 43, 1585 (1979); J. S. Hagelin, Phys. Rev. D 23, 119 (1981); B. Gaiser, T. Tsao, and M. Wise, Ann. Phys. (N.Y.) 132, 66 (1981); L. Chau, W. Keung, and M. Tran, Brookhaven National Laboratory Report No. BNL-31725, 1982 (to be published).

[4]A. Buras, Phys. Rev. Lett. 46, 1354 (1981).

[5]For a review, see M. K. Gaillard, in Proceedings of the Summer Institute of Particle Physics, edited by M. Zipf, SLAC Report No. 215, 1978 (unpublished), p. 397.

[6]J. Cortes, X. Pham, and A. Tounsi, Phys. Rev. D 25, 188 (1982).

[7]G. Kalmus, in Proceedings of the Twenty-First International Conference on High Energy Physics, Paris, 26–31 July 1982 (to be published).

[8]For discussions of some of these contributions, see G. Eliam, Phys. Rev. Lett. 49, 1478 (1982); M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 44, 7 (1980); B. Guberina, R. D. Peccei, and R. Rückl, Nucl. Phys. B171, 333 (1980).

[9]If the $|\Delta s| = 1$ weak nonleptonic Hamiltonian is dominated by a single operator, the contribution to the K_L-K_S mass difference from higher-dimension operators ought to be nearly real in a basis where the $K\to\pi\pi(I=0)$ amplitude is real. Then the contribution to ϵ from CP nonconservation in kaon decay amplitudes has magnitude $40|\epsilon'||\mathrm{Re}M_{12}^{\mathrm{box}}/(m_{K_L}-m_{K_S})|$, where $\mathrm{Re}M_{12}^{\mathrm{box}}$ is the short-distance contribution to the K^0-K^{-0} mass matrix element [F. J. Gilman and M. B. Wise, Phys. Lett. 83B, 83 (1979)]. For long B-meson lifetimes $|\mathrm{Re}M_{12}^{\mathrm{box}}/(m_{K_L}-m_{K_S})| \leq \frac{1}{10}$, except when m_t is enormous. In any case, the measurement of a small value for $|\epsilon'/\epsilon|$ would support the approximation.

[10]F. Gilman and M. Wise, Phys. Lett. 93B, 129 (1980); F. Gilman and M. Wise, SLAC Report No. SLAC-PUB-2940, 1982 (to be published).

[11]J. Donoghue, E. Golowich, and B. Holstein, Phys. Lett. 119B, 412 (1982).

[12]The JADE upper limit $\tau_B < 1.4\times10^{-12}$ s appears in W. Bartel *et al.*, Phys. Lett. 114B, 71 (1982). Roy Weinstein [in Proceedings of the American Physical Society Division of Particles and Fields Meeting, College Park, October 1982 (to be published)] reports $\tau_B = (1.7\pm1.0)\times10^{-12}$s.