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Direct capture cross sections at low energy

R. D. Williams and S. E. Koonin
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
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We derive simple expressions for the low-energy behavior of direct radiative capture cross sections, particularly those important in solar nucleosynthesis, such as ^{3}He$(\alpha,\gamma)^7$Be, ^{3}H$(\alpha,\gamma)^7$Li, and ^{7}Be$(p,\gamma)^8$B. Our results for the ^{3}He$(\alpha,\gamma)^7$Be reaction are inconsistent with a measurement by Rolfs *et al.*

[NUCLEAR REACTIONS ^{3}He$(\alpha,\gamma)^7$Be, ^{3}H$(\alpha,\gamma)^7$Li, $E<300$ keV, ^{7}Be$(p,\gamma)^8$B, $E<100$ keV, extrapolated S.]

There recently been much activity concerning reevaluation of the cross sections of certain solar nuclear reactions due to the unexpectedly low neutrino flux measured by Davis *et al.*[1] One of these reactions, ^{3}He$(\alpha,\gamma)^7$Be, is very difficult to measure at solar energies but has a direct bearing on the neutrino problem, since the expected neutrino flux from the sun is almost proportional to the low-energy cross section for this reaction.[2] Rolfs *et al.*[3] have remeasured the low-energy S factor (related to the cross section σ by $S=\sigma E\exp(2\pi\eta)$, where E is the entrance channel center-of-mass energy and η the usual Sommerfeld parameter) and claim it has considerably less energy dependence than was found in previous experiments[4] or predicted theoretically.[5] If this were correct, it would substantially alleviate the solar neutrino problem, but would not eliminate it entirely.[2] Our purpose in this paper is to outline relatively simple expressions for the low energy behavior of direct capture cross sections which can be used to extrapolate the high energy ($E\sim500$ keV) experimental results for ^{3}He$(\alpha,\gamma)^7$Be to solar energies ($E\sim20$ keV). We find essential agreement with previous work,[4,5] and with recent Caltech measurements,[6] but are in strong disagreement with the data of Ref. 3.

Our treatment is based on the direct capture formalism developed previously,[7] in which the entrance channel and final state are described by the relative motion of ^{3}He and ^{4}He clusters. Since only $E1$ and $E2$ transitions to the p-wave bound state from s,d and p,f continuum states, respectively, are important at low energy, the S factor for capture to a given final state are

$$S(E1)=k_\gamma^3(|I_{10}|^2+|I_{12}|^2)\ ,$$
$$S(E2)=\tfrac{75}{98}k_\gamma^5(|I_{21}|^2+\tfrac{3}{2}|I_{23}|^2)\ , \qquad (1)$$

where k is the photon wave number, and

$$I_{\lambda l}=\int_0^\infty r^2dr\left[\psi_f(r)r^\lambda\frac{\varphi_l(r)}{kr}\right]e^{\pi\eta}(2\pi\eta)^{1/2}\ . \qquad (2)$$

Here, φ_l is the continuum wave function describing the lth partial wave, while ψ_f is the radial p-wave bound-state wave function. (For ^{7}Be the final state has either total spin $j_f=\frac{3}{2}$ with binding energy relative to the entrance channel of 1.586 MeV, or $j_f=\frac{1}{2}$ with binding energy 1.157 MeV.) Note that we have omitted many (common) energy independent factors in Eqs. (1) (including a bound-state spectroscopic factor), as it is only the energy dependence of S which concerns us here.

At low energies, it is well known that the integrand in Eq. (2) peaks at radii far outside the range of the strong interactions.[7] Therefore, with good accuracy, ψ_f may be replaced with the exponentially decaying $l=1$ Coulomb wave function (related to a Whittaker function). In the same spirit, φ_l may be replaced by a Coulomb wave, modified by the strong interaction phase shift. This we take to correspond to a hard sphere of radius r_0 (see Table I), adjusted to reproduce the elastic scattering,[5] so that

$$\varphi_l(r)=F_l(kr)-\frac{F_l(kr_0)}{G_l(kr_0)}G_l(kr), \qquad (3)$$

where k is the entrance channel wave number and F_l and G_l are the usual regular and irregular Coulomb wave functions. Consistent with this description, we limit the integral in Eq. (2) to $r>r_0$.

To obtain tractable expressions for the $I_{\lambda l}$, it is useful to employ the Bessel function expansions of F_l and G_l (Ref. 8) to generate series in powers of E, each term involving E-independent radial integrals of Bessel functions, powers, and Whittaker functions. These can be readily evaluated numerically by Laguerre quadrature to ob-

TABLE I. Kinematic parameters and results.

	Binding (MeV)	Branching (%)	r_0 (fm)	a (MeV^{-1})	b (MeV^{-2})
^{3}He$(\alpha,\gamma)^7$Be					
$j_f=\frac{3}{2}$	1.586	73	2.8[a]	−0.575	−0.005
$j_f=\frac{1}{2}$	1.157	27			
^{3}H$(\alpha,\gamma)^7$Li					
$j_f=\frac{3}{2}$	2.468	73	2.8[a]	−2.034	−3.709
$j_f=\frac{1}{2}$	1.989	27			
^{7}Be$(p,\gamma)^8$B					
	0.136		4.1[b]	−2.350[c]	28.3[c]

[a] Reference 5.
[b] Reference 7.
[c] Equation (4) valid only for $E \lesssim 100$ keV.

tain the low-energy behavior of S.

We have considered the reactions ^{3}H$(\alpha,\gamma)^7$Li and ^{7}Be$(p,\gamma)^8$B as well as that of primary interest, ^{3}He$(\alpha,\gamma)^7$Be, and have calculated the low energy logarithmic derivative of S in the form

$$\frac{1}{S(E)}\frac{dS}{dE}=a+bE\,. \tag{4}$$

For the α-capture reactions, where two final states are involved, we have taken the low energy branching ratio to be energy independent and equal to the experimental value.[9] The kinematic parameters for the three reactions and our results for a and b are shown in Table I. Our value for a for ^{3}He$(\alpha,\gamma)^7$Be agrees with more involved numerical calculations[5] and is consistent with previous[4] and as yet unpublished[6] Caltech results; it is inconsistent with the data of Ref. 3. Our value for a varies by less than 5% for changes in r_0 of 10%. For ^{7}Be$(p,\gamma)^8$B, we find the previously expected[9] sharp rise in the S factor below $E=100$ keV; at energies greater than this, the expansion implied by (4) does not converge, due to the small binding energy of the final state.

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