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Measurement of electron screening in muonic lead*

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Energies of the transitions between high-lying ($n \geq 6$) states of muonic lead were accurately determined. The results are interpreted as a $\sim 2\%$ test of the electron screening. The agreement between experiment and theory is good if it is assumed that the refilling of the electron K shell is fast. The present results furthermore severely restrict possible ionization of the electron L shell.

Muonic transitions between states with high value of the principal quantum number n are only weakly affected by the nuclear finite size and by the quantum electrodynamic corrections. The deviations of the transition energies from the pure hydrogenlike values are mainly caused by the screening by atomic electrons. We recently studied low-energy ($E < 500$ keV) transitions in several muonic atoms at the Space Radiation Effects Laboratory.** The main purpose of this study was the precise determination of the transition energies strongly affected by vacuum polarization. However, as a byproduct accurate energies of transitions between higher-lying states, $n \geqslant 6$, were also obtained. In this paper we want to report the measurement and analysis of these transitions in muonic lead with particular emphasis on the effect of electron screening. The experimental details and the results of the main study will be published later. The earlier attempts to use the muonic atoms to test quantum electrodynamics and the corresponding theoretical calculations are described, for example, in the review article.[1] More recent experimental results may be found in Refs. 2 and 3.

The calculation of the screening effect for a given configuration of the atomic electrons is numerically involved but conceptually straightforward.[4-6] However, during the atomic cascade the muon ejects many Auger electrons and thus the atom can be highly ionized and consequently the electron screening reduced. In muonic lead the binding energy of the muon in state n is approximately $33/n^2$ times the total binding energy of all 82 atomic electrons. Thus, using only the energy conservation, the atom can be completely ionized when the muon is in the $n = 6$ state. Such an extreme situation is rather unlikely. Radiation is responsible for a part of the total energy loss of the muon; another part is released as kinetic energy of the Auger electrons. Moreover, in dense targets the electrons will be replenished from the surrounding atoms during the 10^{-13}–10^{-14} sec duration of the muonic cascade. Very little reliable information is available about the actual degree of ionization. A study of the Rydberg transitions is thus rather unique, because it makes it possible to estimate the number of electrons present at different times during the muonic cascade.

The muonic transitions from higher n states ($n_f \geqslant 7$) have a complex structure, because the fine structure and the less intense inner transitions ($l < n - 1$) are experimentally not resolved. Therefore correlated fits are needed to determine the transition energies and their errors. The energy of a peak was found using the known energy differences and intensity ratios between the unresolved components of a multiplet. The line shape and energy calibration functions were determined from simultaneously accumulated calibration spectra.

The energy correlation between the unresolved members of the multiplet is quite insensitive to assumed electron screening corrections, whereas the intensity ratios depend sensitively on the cascade mechanisms and on the initial population. We have assumed a statistical angular momentum distribution at $n = 20$ for the initial muon population and allowed for the K, L, and M Auger conversion. In all cases variation of the calculated intensity ratios by $\pm 20\%$ resulted in a significant increase of the χ^2 of the fit. The error due to the uncertainties in the cascade calculation, determined in this way, was quadratically added to the statistical error. In addition the fitting procedure for many unresolved peaks was estimated to add an additional 10 eV error to the measured energies. In Table II only the energies of the most intense transitions of each correlated group are listed.

The main results of our theoretical calculations are shown in Table I. The eigenvalues of the Dirac equation with corrections, column 2, include finite size effects (Fermi distribution $c = 6.64$ fm, $t = 2.35$ fm), and are corrected for the Lamb shift, relativistic mass effect, and nuclear polarization. The latter three corrections are important only in the $n = 5$ states, where the results of Ref. 1 were used. The vacuum polarization term in column 3 includes effects of the order $\alpha Z\alpha$, $\alpha^2 Z\alpha$, and $\alpha(Z\alpha)^3$. The Uehling term, $\alpha Z\alpha$, includes the finite nuclear size effects and is treated as an addition to the nuclear Coulomb potential, i.e., to all orders. The $\alpha^2 Z\alpha$ and $\alpha(Z\alpha)^3$ terms were evaluated using the tabulated potentials.[7] However, for the $n = 5$ states the vacuum polarization diagrams $\alpha(Z\alpha)^{n>3}$, calculated in Ref. 1, were added, and the finite size corrections in the $\alpha(Z\alpha)^{n\geq 3}$ diagrams from Ref. 8 were also included.

The various parts of the electron screening calculation are summarized in columns 4–6 of Table I. The calculation is based on the fact that the muon transition rates (for $n > 2$) are considerably smaller than the frequencies of the electron motion. Thus the muon + electrons system forms a stationary state, which is slightly different for different muon orbits.

The simplest estimate of the electron screening is the "$Z - 1$ approximation" (method II of Ref. 6). In it one assumes that the muon is so close to the nucleus that it simply cancels one unit of the nuclear charge. The electrons are then in the same orbits as in the normal $Z - 1$ atom. The electron charge density of the $Z - 1$ atom is therefore used to calculate the potential $V_{Z-1}^{e-\mu}(r)$ [Eq. (2), Ref. 6] acting on the muon. The quantity E^{Z-1} in column 4 of Table I is the difference of muon eigenvalues $E_\mu^{(Z-1)}$ calculated with and without the potential $V_{Z-1}^{e-\mu}(r)$, i.e., E^{Z-1} is the $Z - 1$ approximation to the electron screening correction. To conform with the tradition that the most deeply bound muons are least affected by electron screening, we have subtracted the constant $V_{Z-1}^{e-\mu}(r = 0)$ from the potential $V_{Z-1}^{e-\mu}(r)$. Such renormalization, naturally, does not affect the x-ray energies. It makes, however, the remaining screening correction look like an increase in binding energy (positive E^{Z-1}). The quantity E^{Z-1} is a useful reference point for further discussion of the electron screening.

In a most sophisticated approach[5,6] the Dirac-Hartree-Slater self-consistent program is used for the combined system electrons + muon. All quantities, particularly the electron charge density, now depend on the muon quantum numbers nlj. Let us define the relevant quantities. There are three different self-consistent potentials: $V^{e-e}(r)$ is the electron-electron potential

$$V^{e-e}(r) = \frac{4\pi e^2}{r}\int_0^r \rho_{\mathrm{el}}(t)t^2\,dt + 4\pi e^2\int_r^\infty \rho_{\mathrm{el}}(t)t\,dt - 6e^2[3\rho_{\mathrm{el}}(r)/8\pi]^{1/3}\,, \tag{1}$$

where $\rho_{\mathrm{el}}(r)$ is the radial electron density. The electron potential acting on the muon, $V^{e-\mu}(r)$, is the same as $V^{e-e}(r)$ except for the last term describing the Slater exchange which is therefore missing. Finally, $V^{\mu-e}(r)$ is the muon potential acting on the electrons,

$$V^{\mu-e}(r) = \frac{4\pi e^2}{r}\int_0^r \rho_\mu(t)t^2\,dt + 4\pi e^2\int_r^\infty \rho_\mu(t)t\,dt\,, \tag{2}$$

where $\rho_\mu(r)$ is the radial muon density. The Dirac equation for electrons contains three potentials, the nuclear Coulomb potential, V^{e-e}, and $V^{\mu-e}$, and has eigenvalues ϵ_i. The Dirac equation for the muon containing the nuclear Coulomb potential plus $V^{e-\mu}$ has eigenvalues E_μ^{sc}. The total energy of the system is

TABLE I. Theoretical total binding energies E, total vacuum polarization corrections E^{vp}, screening by the normal atom of charge $Z-1$, E^{Z-1}, corrections caused by the self-consistent treatment of the electrons and muon, ΔE^{sc}, and corrections caused by rearrangement of the electrons, ΔE^{re}. All energies in eV.

State	E	E^{vp}	E^{Z-1}	ΔE^{sc}	ΔE^{re}
$5f_{7/2}$	763 737	1808	221	−11	−3
$5g_{9/2}$	760 670	1525	178	−8	−1
$5g_{7/2}$	763 451	1550	176	−8	−1
$6f_{7/2}$	530 533	1005	407	−17	−11
$6g_{9/2}$	528 725	838	360	−15	−11
$6h_{11/2}$	527 473	699	299	−13	−6
$6h_{9/2}$	528 534	706	297	−13	−6
$7g_{9/2}$	388 778	502	600	−21	−8
$7h_{11/2}$	387 958	412	536	−20	−9
$7h_{9/2}$	388 625	416	534	−20	−9
$7i_{13/2}$	387 326	335	457	−19	−11
$7i_{11/2}$	387 799	337	455	−19	−11
$8k_{15/2}$	296 625	165	650	−23	−10
$9i_{13/2}$	235 169	140	1076	−29	−9
$9k_{15/2}$	234 884	109	984	−26	−6
$10i_{13/2}$	191 044	98	1435	−30	−10
$10k_{15/2}$	190 819	76	1348	−30	−4
$11i_{13/2}$	158 504	72	1808	−32	−5
$11l_{17/2}$	158 135	41	1634	−31	−8
$12k_{15/2}$	133 683	42	2110	−31	−5
$12l_{17/2}$	133 535	31	2025	−31	−4

$$E_{\rm tot}=\sum_i\,(\epsilon_i-\tfrac{1}{2}\langle V^{e-e}+V^{\mu-e}\rangle_i)+E_\mu^{\rm sc}-\tfrac{1}{2}\langle V^{e-\mu}\rangle_\mu\ , \tag{3}$$

where the summation is over all occupied electron states.

In the past (with a single exception[9]) the electron screening correction was calculated as the difference of muon eigenvalues $E_\mu^{\rm sc}$ obtained with and without the potential $V^{e-\mu}(r)$. In such a self-consistent treatment (method III of Ref. 6) the adjustment of the electron cloud to the muon orbit is taken into account. The resulting screening correction is smaller than in the $Z-1$ approximation. The reduction of the $Z-1$ screening

$$\Delta E^{\rm sc}=E_\mu^{(Z-1)}-E_\mu^{\rm sc} \tag{4}$$

is shown in column 5 of Table I.

The just described way of calculating the screening correction is not yet completely satisfactory, because it does not include the changes in the electron-electron interaction energy caused by changing muon orbits (rearrangement effects). To include them one has to deal with total energies (3) instead of the muon eigenvalues E_μ. Since we would like to use the $Z-1$ approximation as a reference point, we show in column 6 of Table I the correction to the $Z-1$ approximation in the more accurate theory. The correction is given by

$$\Delta E^{\rm re}=E_\mu^{(Z-1)}-[E_{\rm tot}-E_{\rm tot}^{(Z-1)}-V_{Z-1}^{e-\mu}(r=0)] \tag{5}$$

($\Delta E^{\rm re}$ is calculated as a small difference of large quantities; due to the computer round-off errors its accuracy is about ± 2 eV). The constant $E_{\rm tot}^{(Z-1)}$ is present in (5) because $E_{\rm tot}$ describes the whole system while E_μ only the muon. The last constant $V_{Z-1}^{e-\mu}(r=0)$ in (5) reflects our normalization of the eigenvalues $E_\mu^{(Z-1)}$. The negative sign of $\Delta E^{\rm re}$ means that the rearrangement effects decrease the simplest E^{Z-1} value of the screening correction and the smallness of $\Delta E^{\rm re}$ means that E^{Z-1} is an excellent approximation. The value $\Delta E^{\rm re}+E^{Z-1}$ was used in calculating the total binding energy E in column 2 of Table I and the screening correction in column 5 of Table II.

The measured and calculated transition energies are compared in Table II. Besides the corrections discussed so far, we have decreased the screening effect of the $7\to 6$ and $7\to 5$ transitions by 2 eV and of the $6\to 5$ transitions by 1 eV. This reduction is caused by the non-Lorentzian shape of the x-ray lines and was calculated in Table IV, Ref. 10. Let us stress once again that in the present calculation we have assumed that all 81 electrons are present in their ground state. The agreement between calculated and measured energies is excellent, showing that the screening correction was calculated correctly [p_i below are (2–5)%]. When all data are added together using

$$p_i=\frac{E_i^{\rm th}-E_i^{\rm exp}}{E_{\rm scr}}\ ,\quad \delta p_i=\frac{\delta E^{\rm exp}}{E_{\rm scr}}\ ,\quad p=\frac{\sum p_i/\delta p_i^2}{\sum 1/\delta p_i^2}\ , \tag{6}$$

one obtains $p=-1.7\pm 1.2\%$, suggesting an even better agreement. It is also worth noting that the χ^2 per degree of freedom for all transitions is significantly smaller than 1. That means that the errors include the systematic errors from the intensity correlations. Thus the experimental data indicate that essentially all electron shells contributing to the screening correction are occupied when the muon is in the $n=6$–12 states. This con-

TABLE II. Experimental transition energies $E^{\rm exp}$, calculated transition energies $E^{\rm th}$, their differences $E^{\rm th}-E^{\rm exp}$, total screening effect $E^{\rm scr}$ and screening effect of one $2s$ electron $\Delta(2s)$. All energies in eV.

Transition	$E^{\rm exp}$	$E^{\rm th}$	$E^{\rm th}-E^{\rm exp}$	$E^{\rm scr}$	$\Delta(2s)$
$11l_{17/2}\to 8k_{15/2}$	138 475 ±53	138 490	15 ±53	−986	−54
$7i_{13/2}\to 6h_{11/2}$	140 138 ±13	140 148	10 ±13	−153	−9
$7i_{11/2}\to 6h_{9/2}$	140 729 ±13	140 736	7 ±13	−153	−9
$7h_{11/2}\to 6g_{9/2}$	140 769 ±20	140 768	−1 ±20	−176	−11
$7g_{9/2}\to 6f_{7/2}$	141 741 ±51	141 756	15 ±51	−194	−12
$9k_{15/2}\to 7i_{13/2}$	152 446 ±14	152 442	−4 ±14	−532	−31
$12l_{17/2}\to 8k_{15/2}$	163 130 ±47	163 090	−40 ±47	−1381	−73
$10k_{15/2}\to 7i_{13/2}$	196 524 ±44	196 507	−17 ±44	−897	−51
$6h_{11/2}\to 5g_{9/2}$	233 199 ±12	233 196	−3 ±12	−115	−5
$6h_{9/2}\to 5g_{7/2}$	234 922 ±12	234 916	−6 ±12	−115	−5
$12k_{15/2}\to 7i_{13/2}$	253 626 ±66	253 643	17 ±66	−1659	−88
$9i_{13/2}\to 6h_{11/2}$	292 324 ±27	292 304	−20 ±27	−774	−42
$10i_{13/2}\to 6h_{11/2}$	336 508 ±39	336 430	−78 ±39	−1132	−64
$7h_{11/2}\to 5g_{9/2}$	372 724 ±16	372 714	−10 ±16	−348	−21
$7h_{9/2}\to 5g_{7/2}$	374 842 ±36	374 828	−14 ±36	−348	−21
$7g_{9/2}\to 5f_{7/2}$	374 947 ±36	374 960	13 ±36	−372	−23

clusion agrees with the observed shifts of the electronic x rays emitted during the muonic cascade.[11] It disagrees, however, with the suggestion[9] that only 10 electrons are present.

To make our statement about the degree of ionization more quantitative, we have to remember that the different electron states contribute to the screening differently. In heavy atoms each $1s$ electron contributes ~40%, each $2s$ electron ~(5–6)% (the $2s$ contribution is shown explicitly in Table II), each $3s$ or $2p_{1/2}$~ (1–2)% and all the remaining electrons give together (2–4)%. The ejection of $1s$ electrons becomes important only for $n \leq 7$ states; the good agreement for the $7 \to 6$, $7 \to 5$, and $6 \to 5$ transition means that our treatment of the refilling process[10] is essentially correct. The good overall agreement means that no more than one $2s$ electron or not more than two $2p_{1/2}$ or $3s$ electrons are missing on the average. Thus the present experiment clearly excludes high degree of ionization of the inner (K, L) atomic shells during that part of the muonic cascade when Auger electrons are ejected out of them. It also shows that the rate of refilling of the electron K shell is close to the rate of the normal atom, again indicating low degree of ionization. The precision of the experiment, however, does not allow us to determine the rate of refilling of the electron L shell. However, the good overall agreement between the theoretical calculations and experimental data strongly supports the reliability of the screening corrections for the lower n transitions used in deducing the negative pion mass and the vacuum polarization effects.

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