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Neutrino Opacity. II. Neutrino-Nucleon Interactions*

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The contribution of neutrino-nucleon interactions to the neutrino opacity of matter is studied, special attention being paid to possible astrophysical applications such as supernova explosions. The results of recent accelerator experiments with high-energy neutrinos are used to show that nonresonant neutrino-nucleon scattering does not make a significant contribution to the neutrino opacity for astrophysically important conditions. The results of deep-mine cosmic-ray studies are then used to show that, (a) there are no resonances in the ν_μ-nucleon and $\bar{\nu}_\mu$-nucleon systems with masses less than 60 BeV (laboratory neutrino energies $<2\times10^{+3}$ BeV), and (b) there are no resonances in the ν_e-neucleon and $\bar{\nu}_e$-nucleon systems with masses less than 7 BeV (laboratory neutrino energies <30 BeV). Neutrino absorption by bound nucleons is also discussed and a sum rule is proved for neutrino capture that is sufficiently accurate for most astrophysical applications. The effect of the exclusion principle on the capture cross sections is described and some applications to specific nuclei are presented. The accelerator experiments with high-energy neutrinos are then used to show that neutrino radioactivity, i.e., nuclear de-excitation by emission of a neutrino-antineutrino pair, is a substantially less important mechanism for stellar energy loss than was suggested by some previous estimates.

I. INTRODUCTION

ONE of the principal ways that a star loses energy in the later stages of stellar evolution is by emitting neutrinos from a hot core. Thus, one must know the neutrino opacity of matter, under appropriate astrophysical conditions, in order to predict the rate at which highly evolved stars radiate energy in the form of neutrinos. Here neutrino opacity is defined,[1] analogous to photon opacity, as

$$\kappa_\nu=(\sum_i \sigma_i n_i)/\rho\,, \tag{1}$$

where σ_i is the neutrino-interaction cross section for particles of number density n_i, and ρ is the stellar-matter density. A knowledge of the neutrino opacity of matter under extreme conditions is also required for some theories of supernova explosions.[2,3] In fact, we pay particular attention to the neutrino opacity under conditions that might be appropriate to supernova explosions.

In this paper, we investigate the contribution of neutrino-nucleon interactions to the total neutrino opacity of matter under appropriate astrophysical conditions; the contribution of neutrino-lepton interactions has been investigated in a previous paper[1] (I). In Sec. IIa, we use the results of recent accelerator experiments with high-energy neutrinos to show that nonresonant neutrino-nucleon scattering does not make a significant contribution to κ_ν for astrophysically important conditions. As a natural extension of our previous investigation[4] of the possibility of using neutrino resonances to detect neutrinos from strong radio sources, we extend to much higher energies, in Sec. IIb, our previous investigation of neutrino-nucleon resonances. Using the results of deep-mine cosmic-ray studies, we are able to conclude that (a) there are no resonances in the ν_μ-nucleon and $\bar{\nu}_\mu$-nucleon systems with masses less than 60 BeV (laboratory neutrino energies $<2\times10^{+3}$ BeV), and (b) there are no resonances in the ν_e-nucleon and $\bar{\nu}_e$-nucleon systems with masses less than 7 BeV (laboratory neurino energies <30 BeV). These results, which are also of interest in high-energy physics, could not have been obtained using currently available accelerators because of the very high energies involved. In Sec. III, we discuss neutrino absorption by bound nucleons, and prove a sum rule for neutrino capture that is sufficiently accurate for most astrophysical applications. We also describe the effect of the exclusion principle on the capture cross sections and present some applications to specific nuclei. In Sec. IV, the accelerator experiments with high-energy neutrinos are used to show that neutrino radioactivity, i.e., nuclear de-excitation by emission of a neutrino-antineutrino pair, is a substantially less important mechanism for stellar energy loss than was suggested by some previous estimates.

II. NEUTRINO-NUCLEON SCATTERING

(a) Nonresonant Scattering

In this subsection,[5] we set up an upper limit on the contribution of nonresonant neutrino-nucleon scattering to the total neutrino opacity. In order to establish the upper limit, we adopt a phenomenological interaction

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[1] J. N. Bahcall, Phys. Rev. 136, B1164 (1964). This paper contains a discussion of the contributions of neutrino-lepton interactions to the neutrino opacity and will be referred to as I.

[2] See, for example, H.-Y. Chiu, Ann. Phys. (N. Y.) 26, 364 (1964) and the numerous references contained therein.

[3] S. A. Colgate and R. H. White, *Proceedings of the International Conference on Cosmic Rays* [Jaipur, India, 1963 (to be published)]. See also, S. A. Colgate and M. H. Johnson, Phys. Rev. Letters 5, 235 (1960). A more complete exposition of this theory is being prepared for publication.

[4] J. N. Bahcall and S. C. Frautschi, Phys. Rev. 135, B788 (1964).

[5] W. A. Fowler and F. Hoyle pointed out (California Institute of Technology Seminar, 1963) the possible importance of neutrino-nucleon scattering for supernova explosions. We are grateful to Fowler and Hoyle for bringing this problem to our attention.

of the form

$$H_\beta = 2^{-1/2}G^0[(\bar{p}\theta_\alpha p)(\bar{\nu}\theta_\alpha\nu)+(\bar{n}\theta_\alpha n)(\bar{\nu}\theta_\alpha\nu)], \qquad (2)$$

where θ_α represents any of the possible Lorentz covariants. A convenient choice for θ_α is $\gamma_\alpha(1+\gamma_5)$, i.e., a pure V-A interaction. One can easily show by a Fierz transformation of interaction (2) that the cross sections for neutrino-nucleon (either neutron or proton) scattering have the same form as the cross sections given in paper I for neutrino-electron scattering; the cross sections for antineutrino-nucleon scattering likewise have the same form as the cross sections for antineutrino-electron scattering. Thus, for neutrino-neutron and neutrino-proton scattering one finds[1]:

$$\sigma_{\nu-N} = \sigma_0(n)\frac{(p\cdot\omega)^2}{1+2p\cdot\omega}, \qquad (3)$$

where p_α and ω_α are the four-momenta (in units of M_Nc^2) of the initial nucleon and neutrino, respectively, and

$$\begin{aligned}\sigma_0(N) &\equiv (G^0/G)^2(M_N/m_e)^2\sigma_0(e), \\ &\cong (G^0/G)^2\times 6\times 10^{-38}\ \mathrm{cm}^2.\end{aligned} \qquad (4)$$

Note that the characteristic number, 6×10^{-38} cm^2, for neutrino-nucleon scattering is much larger than the characteristic cross section for neutrino-electron scattering, $\sigma_0(e^-)(\approx 2\times10^{-44}$ cm$^2)$. For antineutrino-nucleon scattering, one finds

$$\sigma_{\bar{\nu}-N} = 6^{-1}\sigma_0(N)(p\cdot\omega)[1-(1+2p\cdot\omega)^{-3}]. \qquad (5)$$

For astrophysical applications, one is interested in neutrino and antineutrino energies that are much less than a BeV.[2,6] Hence, the following approximations are satisfactory for astrophysical applications:

$$\sigma_{\nu-N} \approx \sigma_{\bar{\nu}-N} \qquad (6a)$$

$$\approx (G^0/G)^2\sigma_0(e)(q/m_ec^2)^2, \qquad (6b)$$

where q is the neutrino energy. If we had chosen a different form for θ_α, e.g., a pure vector interaction, the net result would have been to change Eq. (6) by a multiplicative factor which is 2 or $\frac{1}{2}$ in typical cases. For the cross sections, our conclusions, which depend only on the ratio of measured quantities, are independent of such changes.

An upper limit on $(G^0/G)^2$ can be obtained by comparing the rates predicted from Eqs. (3) and (5) with the upper limit for neutrino-proton scattering events obtained in the CERN-neutrino experiment. In the CERN experiment, the ratio of $\nu+p\rightarrow\nu'+p'$ events to $\nu+n\rightarrow\mu^-+p$ events was less than 3%.[7] Since the reaction $\nu+n\rightarrow\mu^-+p$ has a cross section of the order of[8] 5×10^{-39} cm^2 per nucleon for energies of the order of a few BeV, we conclude that

$$(G^0/G)^2 \lesssim 10^{-2}. \qquad (7)$$

The inequality given in Eq. (7) implies that nonresonant neutrino-nucleon scattering does not make an important contribution to the neutrino opacity under most astrophysically important conditions. This conclusion is easily established by comparing Eqs. (6) and (7) with the cross sections, given in Sec. V of paper I, for neutrino scattering by a gas of electrons.

(b) Resonant Scattering

In this subsection, we use information from deep-mine cosmic-ray studies to set lower limits on the masses of any resonances that may exist in neutrino-nucleon scattering. Resonances in neutrino-nucleon scattering have been predicted, for example, by Kinoshita[9] on the basis of the theory of weak interactions proposed by Tanikawa and Watanabe,[10] in which the mediating particles posses both lepton and baryon number. Independent of any specific theoretical motivation, we think it is interesting to note (see below) that deep-mine cosmic-ray experiments enable one to set very high lower limits on the masses of any resonances in neutrino-nucleon scattering. The results obtained in this subsection support our previous conclusion[4] that the only practical way currently available to detect neutrinos from strong radio sources is by utilizing a resonance in the $\bar{\nu}_e-e^-$ system.

Typical resonant reactions we consider for neutrino-nucleon and antineutrino-nucleon systems are

$$\nu_\mu+n\rightarrow(B_\mu{}^0)\rightarrow\mu^-+p, \qquad (8a)$$

$$\bar{\nu}_\mu+p\rightarrow(B_\mu{}^+)\rightarrow\mu^++n, \qquad (8b)$$

$$\nu_e+n\rightarrow(B_e{}^0)\rightarrow e^-+p, \qquad (8c)$$

$$\bar{\nu}_e+p\rightarrow(B_e{}^+)\rightarrow e^++n. \qquad (8d)$$

In the experiments that we analyze, only the energetic lepton in the final state is normally detected, and therefore we actually consider all resonant reactions in which the final state consists of a charged lepton plus a baryon and any number of mesons.

We have shown previously,[4] by an analysis of the experiment of Danby *et al.*,[11] that any resonance in reaction (8b) must have a mass in excess of 2 BeV. A similar analysis could be carried out for reactions (8) using the CERN neutrino experiments. We shall see, however, that the deep-mine experiments provide much more information about the masses of possible resonances in reactions (8) than can be obtained from accelerator experiments at currently available accelerator energies.

[6] W. A. Fowler and F. Hoyle, Astrophys. J. Suppl. **91**, 1 (1964).
[7] J. S. Bell, J. Løvseth, and M. Veltman, International Conference on Elementary Particles, Sienna, 1963 (unpublished).
[8] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960); N. Cabibbo and R. Gatto, Nuovo Cimento **15**, 304 (1960).
[9] T. Kinoshita, Phys. Rev. Letters **4**, 307 (1960).
[10] Y. Tanikawa and S. Watanabe, Phys. Rev. **113**, 1344 (1959); S. Oneda and Y. Tanikawa, Phys. Rev. **113**, 1354 (1959).
[11] G. Danby, J.-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962).

The most informative cosmic-ray experiments with respect to neutrino-nucleon resonances have been performed by the Tata Institute Group at the Kolar Gold Fields.[12] These experiments are especially informative since they include observations at the greatest depth yet obtainable (8400 m.w.e.). At this great depth, the muons produced in the decay of cosmic-ray secondaries in the atmosphere are greatly attenuated by interactions in the earth. Thus, the muons produced in the earth's crust by interactions of neutrinos (which themselves are produced by the decay of cosmic-ray secondaries) are competitive in number with muons produced in the earth's atmosphere.

The detector used by the Tata Institute Group had[12] an (effective area)$\times$(solid angle) factor for muons from neutrino interactions of about 10^{+5} cm^2-sr. In the experiment performed at 8400 m.w.e., *no count* was registered in an operating period of $5\times10^{+6}$ sec.

The number of counts C that should have been registered if a resonance exists in any of the reactions (8) can be estimated from the following formula:

$$C=\left(\int \varphi_\nu dE\right)\sigma N t\,, \tag{9}$$

where $\int\varphi_\nu dE$ is the integral of the neutrino flux over the effective width of the Doppler-broadened cross section, σ the effective cross section near resonance, N the number of nucleons of the correct type encountered by neutrinos on the way to the detector, and t the observation time. The full width of the Doppler-broadened resonance is[1,4] about $(q_0/2)$, where q_0 is the laboratory resonance energy of the neutrino. The flux of muon neutrinos from cosmic-ray secondaries has been calculated by several authors[13–15] and is approximately

$$\begin{aligned}\varphi_{\nu_\mu}(\pi-\mu)&\approx10^{-2}q_0^{-2.8}\,, && 0\le q_0\le10\\ &\approx4\times10^{-2}q_0^{-3.2}\,, && 10\le q_0\le300\,,\end{aligned} \tag{10}$$

where q_0 is measured in BeV and φ_ν in (BeV cm^2 sec sr)$^{-1}$. The flux of high-energy electron neutrionos is much less. In the energy range of interest (K-decays unimportant), the flux of electron nuetrinos can be estimated from the relation

$$\varphi_{\nu_e}(\mu-e)=(1-e^{-(l/c\tau_\mu)})\times\varphi_{\nu_\mu}(\pi-\mu)\,, \tag{11}$$

where l is some average production height[16] ($\sim1.6\times10^{+6}$ cm) for muons, c the velocity of light, and τ_μ the decay lifetime for muons of the relevant energy. The kinematics are such that, on the average, ν_e and ν_μ get approximately the same amount of energy in the sequence $\pi^+\to\mu^++\nu_\mu\to(e^++\nu_e+\bar\nu_\mu)+\nu_\mu$; hence, the two fluxes in Eq. (11) are evaluated at the same energy. For fairly high energies q_0, one easily obtains the following approximate relation

$$\varphi_{\nu_e}(\mu-e)\sim2q_0^{-1}\text{ (BeV) }\varphi_{\nu_\mu}(\pi-\mu)\,. \tag{12}$$

The effective cross section near resonance for each of the reactions (8a)–(8d) is expected to be[1,4] about 2×10^{-31} cm^2 when the Doppler shift induced by the motion of the target nucleons is taken into account; this effective cross section is independent of the mass of the resonance.[17] The electromagnetic mean free path for the electrons and muons produced by reactions (8) is less than 8400 m.w.e.[18]; hence N, the number of nucleons that can convert neutrinos into electrons or muons capable of reaching the detector, is proportional to the electromagnetic mean free path.

For muons with energies less than or of the order of 10^{+3} BeV, ionization is the dominant mode of energy loss[18] and one finds $N\approx10^{+31}q_0$ (BeV). For energies greater than 10^{+3} BeV, radiation losses are more severe than ionization losses and the mean free path remains nearly constant and equal to its value at 10^{+3} BeV.

Inserting the above numbers in Eq. (9), using a conservative extrapolation of the flux and the conservative statement that $C\le5$ (to take account of statistical fluctuations and the efficiency of the detector), we conclude that *there are no resonances in the ν_μ-nucleon and $\bar\nu_\mu$-nucleon systems for laboratory neutrino energies less than $2\times10^{+3}$ BeV*. Since the boson masses M_B are related (for $q_0\gg1$) to the resonance energy q_0 by the simple relation[4,9]

$$M_B\approx[2q_0(\text{BeV})]^{+1/2}\,, \tag{13}$$

where M_B is also in BeV, we conclude that *the masses of $B_\mu{}^0$ and $B_\mu{}^+$ must exceed 60 BeV.*

For electrons passing through rock, radiation is the dominant mode of energy loss above 100 MeV. Thus, one can take $N\sim$several$\times10^{+30}$ for the Kolar Gold Field experiment.[12] Inserting the appropriate numbers in Eq. (9), *we conclude that there are no resonances in the ν_e-nucleon and $\bar\nu_e$-nucleon systems for laboratory neutrino energies less than 30 BeV. Hence the masses of $B_e{}^0$ and $B_e{}^+$ must exceed 7 BeV.*[19]

[12] M. G. K. Mennon, P. V. Ramana Murthy, B. V. Sreekantan, and S. Miyake, Phys. Letters **5**, 272 (1963); S. Miyake, V. S. Narasimhan, and P. V. Ramana Murthy, Proc. Phys. Soc. (Japan) **3**, 318 (1962).

[13] G. T. Zatsepin and V. A. Kuz'min, Zh. Eksperim. i Teor. Fiz. **41**, 1818 (1964) [English transl.: Soviet Phys.—JETP **14**, 1294 (1962).

[14] K. Greisen, Ann. Rev. Nucl. Sci. **10**, 63 (1950).

[15] T. D. Lee, H. Robinson, M. Schwartz and R. Cool, Phys. Rev. **132**, 1297 (1963).

[16] This corresponds to a production-depth of 100 gm/cm^2 from the top of the atmosphere. See J. Pine, R. J. Davisson, and K. Greisen, Nuovo Cimento **14**, 1181 (1959) and B. Rossi, *High-Energy Particles* (Prentice-Hall, Inc., New York, 1952), p. 545.

[17] Since the experiments of interest actually count all resonant events in which a charged high-energy lepton is produced, the experiments effectively sum over all Γ_{out} [see I, Eqs. (31)]. This is in fact a necessary condition for σ_{res} being independent of boson mass.

[18] E. Fermi, *Nuclear Physics* (University of Chicago Press, Chicago, 1950), Chap. II.

[19] The deep-mine experiments currently being carried out by F. Reines and his associates in South Africa will enable one to carry this investigation of neutrino-nucleon resonances to even higher energies. The experiment of J. C. Barton, Phil. Mag. **6**, 1271 (1961) enables one to say, using an analysis similar to that given in Sec. IIb of our paper, that the masses of $B_\mu{}^0$ and $B_\mu{}^+$ must exceed 4 BeV.

III. NEUTRINO ABSORPTION BY BOUND NUCLEONS

The absorption of neutrinos by nucleons has generally been regarded as the major term in the neutrino opacity of matter and has therefore been discussed by a number of authors.[2,3,20,21] It has almost always been assumed in these discussions that the absorption cross sections for nucleons in a nucleus were essentially the same as for free nucleons. We show in this section that the assumption that nucleons in a nucleus absorb neutrinos like free nucleons is *not correct* for neutrino energies less than 250 MeV, i.e., this assumption is not correct for neutrinos that are of interest in most astrophysical problems. We begin in subsection (a) with some introductory remarks and in subsection (b) prove a sum rule that shows the dependence of the neutrino cross sections upon neutrino energy and nuclear parameters. The relevant results and applications are summarized in subsection (c).

(a) General Remarks

The reactions in which we are interested are

$$\nu_e+(Z,A)\rightarrow(Z+1,\,A)+e^- \tag{14a}$$

and

$$\bar{\nu}_e+(Z,A)\rightarrow(Z-1,\,A)+e^+. \tag{14b}$$

The corresponding reactions for ν_μ and $\bar{\nu}_\mu$ are possible only for neutrinos with enough energy (>100 MeV) to create muons. Thus, matter is almost completely transparent under ordinary conditions to muon-neutrinos of energies less than 100 MeV.

The qualitative behavior of the neutrino cross sections for processes (14) depends upon which of three domains the laboratory neutrino energy q falls into. The three domains are, roughly speaking: (i) low energy ($q\lesssim 15$ MeV); (ii) high energy ($q\gtrsim 500$ MeV); and (iii) intermediate energy (15 MeV$<q<$500 MeV). In the low-energy domain, the neutrinos do not have sufficient energy to excite some of the important levels of the final nucleus. Thus, the neutrino cross sections in the low-energy domain depend strongly on the precise value of q and on how high in excitation the states in the final nucleus lie that have the largest matrix elements for neutrino capture. The most important example of neutrino absorption in the low-energy range is the capture of solar neutrinos from B^8 (endpoint energy 14 MeV) via the reaction $Cl^{37}(\nu_{\rm solar},e^-)A^{37}$. A detailed theoretical analysis of the nuclear structure of mass 37 has been necessary in order to predict the average cross section[21] for $Cl^{37}(\nu_{\rm solar},e^-)A^{37}$; more experimental and theoretical work on the structure of the mass-37 levels is in progress in order to obtain really accurate values for this important cross section.

Calculations of neutrino absorption cross sections in the low-energy domain are dependent upon the nuclear model adopted and hence are rather uncertain. Cross sections for typical allowed transitions are found to be of the order of $\sigma(e)\approx10^{-44}$ cm^2.

The neutrino absorption cross sections at high energies have been studied theoretically by a number of authors.[8,22] There are basic uncertainties in the results at present, due primarily to our lack of knowledge about the weak-interaction form factors and the existence of possible mediating bosons. Nevertheless, at very high energies nucleons in a nucleus almost certainly behave in absorption like free nucleons. Thus one can take $\sigma_{\rm total}=$ (number of nucleons)$\times\sigma_{\rm single\ nucleon}$, at high energies, where $\sigma_{\rm single\ nucleon}\sim10^{-37\pm1}$ cm^2.

The most interesting domain from the point of view of many astrophysical applications[1–3,6] is the intermediate-energy region. This region, fortunately, is particularly appropriate for a sum rule since practically all nuclear states are energetically accessible and the unknown properties of weak interactions at high energies do not enter strongly into the results.

(b) Derivation of Sum Rule

The sum rule for reactions (14) can be derived in much the same way as one derives[23] a sum rule to estimate the effect of atomic overlap and exchange effects in ordinary beta decay. We only outline briefly the relevant steps here; the reader is referred to Ref. 23 for a more detailed justification of some of the procedures.

The total absorption cross section for reaction (14a) is

$$\sigma=2\pi\sum\nolimits_{A':\ \text{lepton spins}}\int\int d^3n_{A'}d^3n_{p_e}$$
$$\times|\langle A';p_e|H_\beta|A;q_\nu\rangle|^2\delta(E_i-E_f)\,, \tag{15}$$

where A, A' are the initial and final nuclear states, respectively; q, p_e are the neutrino and electron momenta, respectively; and H_β is the usual[24] $V-A$ interaction. The argument of the delta function is[25]

$$E_i-E_f=q_\nu+\Delta W_0(A')-p_e-(P^2/2M_A)\,, \tag{16}$$

where $\Delta W_0(A')$ is the difference between initial and final nuclear energies and $\mathbf{P}$ is the momentum with which the center of mass of the nucleons recoils. We have set $\hbar=m_e=c=1$ in Eqs. (15) and (16) and have chosen a unit volume of quantization.

In order to perform the summation over A', we replace for the moment $\Delta W(A')$ in Eq. (16) by some guessed average value, ΔW^{gu}, and also neglect the

[20] R. W. Euwema, Phys. Rev. **133**, B1046 (1964).

[21] J. N. Bahcall, Phys. Rev. **135**, B137 (1964).

[22] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, 1117 (1960).

[23] J. N. Bahcall, Phys. Rev. **129**, 2583 (1963). See particularly Sec. III.

[24] E. J. Konopinski, Ann. Rev. Nucl. Sci. **9**, 99 (1959).

[25] If statistical equilibrium obtains, a term of the order of kT should be added to the right-hand side of Eq. (16) to take account of the fact that the initial nuclei will have an average excitation energy of the order of kT.

numerically unimportant term, $P^2/2M_A$. Then

$$E_i^0-E_f^0\equiv q_\nu+\Delta W^{gu}-p_e. \tag{17}$$

The conservation of momentum is most conveniently included by writing

$$|A'\rangle=|A'_{\text{internal}}\rangle \exp+i(\mathbf{P}\cdot\mathbf{x}). \tag{18}$$

Inserting relations (17) and (18) in Eq. (15) and using closure to sum over all $|A'\rangle$, one finds

$$\sigma^0=\pi G^2\int\int d^3n_P d^3n_{p_e} C_{\alpha\beta}\delta(E_i^0-E_f^0)$$
$$\times\langle A;q_\nu|\int\int d^3x'd^3x''N_\alpha(x')N_\beta^*(x'')$$
$$\times\exp[+i(\mathbf{q}-\mathbf{p}-\mathbf{P})\cdot(\mathbf{x}'-\mathbf{x}'')]|A;q_\nu\rangle, \tag{19}$$

where $C_{\alpha\beta}$ is a trace involving only lepton variables,

$$C_{\alpha\beta}\equiv-2(p_eq_\nu)^{-1}[q_\alpha p_\beta+q_\beta p_\alpha-(p\cdot q)\delta_{\alpha\beta}+\epsilon_{\rho\alpha\sigma\beta}q_\rho p_\sigma], \tag{20}$$

and $N_\alpha(x')$ and $N_\beta^*(x'')$ are combinations of neutron and proton field operators:

$$N_\alpha(x')\equiv\bar{\Psi}_n\gamma_\alpha(C_V-C_A\gamma_5)\psi_p, \tag{21a}$$

$$N_\beta^*(x'')\equiv\bar{\Psi}_p\gamma_\beta(C_V-C_A\gamma_5)\psi_n. \tag{21b}$$

Equation (19) can be greatly simplified by first integrating over all $\mathbf{P}$ using the relation

$$\int d^3n_P\exp[+i\mathbf{P}\cdot(\mathbf{x}'-\mathbf{x}'')]=\delta^{(3)}(\mathbf{x}'-\mathbf{x}''). \tag{22}$$

The integral over $\mathbf{x}''$ is then trivial.[26] One finally finds

$$\sigma^0=G^2(q_\nu+\Delta W^{gu})^2\pi^{-1}M, \tag{23}$$

where

$$M\equiv\left[2\langle A|\int d^3x\,N_4(x)N_4^*(x)|A\rangle-\langle A|\int d^3xN_\alpha(x)N_\alpha^*(x)|A\rangle\right]. \tag{24}$$

In writing Eq. (24) for M, we have made use of the fact that terms like $\langle A|\int d^3xN_3(x)N_4(x)|A\rangle$ vanish when one averages over the direction of the initial nuclear spin.

Making the usual[24] nonrelativistic approximation for the nucleon operators that appear in Eq. (24), one finds

$$M\approx(C_V^2+3C_A^2)\langle A|\psi_n^+\psi_p\psi_p^+\psi_n|A\rangle. \tag{25}$$

If one ignores the fact that some of the proton states are initially occupied, then one can use closure on the proton variables. In this case, $M\approx(C_V^2+3C_A^2)\times\langle A|\psi_n^+\psi_n|A\rangle\equiv N(C_V^2+3C_A^2)$, where N is the number of neutrons in the initial nucleus A. In this case, the cross section given by Eq. (23) is just N times the single-particle cross section when the single-particle value is corrected for the average binding of the neutrons in the nucleus.

In order to take account of the inhibiting effect of the exclusion principle, we introduce a quantity $\mathfrak{J}$ defined by the relation

$$M\equiv(C_V^2+3C_A^2)N(\mathfrak{J}), \tag{26}$$

and calculate $\mathfrak{J}$ on the basis of a simple model. Since $\mathfrak{J}$ is the average number of nucleons that can participate in an absorption process induced by a neutrino of momentum q, we have

$$\mathfrak{J}\cong\left(\int\frac{d\sigma}{dt}\delta_N(t)dt\right)\Big/\left(\int\frac{d\sigma}{dt}dt\right), \tag{27}$$

where the momentum transfer t is defined by

$$t\equiv|\mathbf{q}_\nu-\mathbf{p}_e|, \tag{28a}$$

and for a single nucleon at rest.[27]

$$(d\sigma/dt)dt\cong\tfrac{1}{2}\sigma_0W_e^2(tdt/q^2)[1+(0.04t^2/q^2)], \tag{28b}$$

where W_e is the energy of the electron that is produced. We have again neglected nucleon recoil in obtaining Eq. (28b); in what follows, we shall also set the bracketed term in Eq. (28b) equal to unity. The quantity $\delta_N(t)$ is defined to be the fraction of neutrons that can participate in the reaction (14a) for a given momentum transfer t.

We estimate $\delta_N(t)$, following Berman,[28] by assuming that the nucleons in the nucleus constitute a degenerate gas of noninteracting fermions. Then

$$\delta_N(t)=\left(\int_{|\mathbf{t}+\mathbf{p}|\geq P_F(p)}d^3p\right)\Big/\left(\int d^3p\right), \tag{29}$$

where the integral in the denominator of Eq. (29) is taken over the entire Fermi sea and $P_F(p)$ is the proton Fermi momentum. Berman[28] has estimated $\delta_N(t)$ taking account of the fact that for heavy nuclei the neutron Fermi energy is appreciably larger than the proton Fermi energy. The difference between neutron and proton Fermi energies is not important for most of the cases we consider (light nuclei), since it gives rise to terms of the order of $(N-Z)/2N$. Assuming $P_F(p)$

[26] It would be interesting to apply the approach outlined here [including conservation of momentum via Eqs. (18) and (22)] to the problem of muon capture in nuclei. The nucleon-nucleon spatial correlation functions, $F^{\pm}(\mathbf{r},\mathbf{r}')$, introduced by H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959) might then be replaced by a Gaussian distribution in momentum space that could be approximately obtained from electron scattering experiments.

[27] E. J. Konopinski (manuscript on the theory of nuclear beta decay to be published by Oxford University Press).

[28] S. M. Berman, *International Conference on Theoretical Aspects of Very High Energy Phenomena* (CERN, Geneva, Switzerland, 1961), p. 9. See also J. S. Bell and J. Løvseth, Nuovo Cimento **32**, 433 (1964).

TABLE I. Some values of ΔW^{gu}.

Isotopes	ΔW^{gu} (in MeV)
He^4	$\sim$25–30
Li^7	$\sim$ 2
C^{12}	$\sim$25 to N^{12} and $\sim$20 to B^{12}
O^{16}	$\sim$25 to F^{16} and $\sim$20 to N^{16}
Ne^{20}	$\sim$15 to F^{20} and $\sim$25 to Na^{20}
Fe^{56}	$\sim$10

$\cong P_F(n)$, we find

$$\delta_N(t)=(n/4)+(n^3/2)(1-n^4/16)+(3/4)n(1-n^2)\times(1-n^2/4)-(n/4)(1-n^2+n^4/4)^{3/2} \quad (30a)$$

$$\cong(3/4)n-(1/16)n^3, \quad (30b)$$

where $n\equiv(t/P_F)$. Equation (30b) is in agreement with the result of Berman.

Inserting Eqs. (30b) and (28b) in Eq. (27), we find

$$\mathfrak{J}\cong(q/P_F)-\tfrac{1}{16}(q/P_F)^3, \quad q\ll P_F. \quad (31)$$

In general, $\mathfrak{J}$ is a monotonically increasing function of (q/P_F) for $(q/P_F)<2$ and $\mathfrak{J}$ is equal to unity for all values of $(q/P_F)>2$.

(c) Results and Summary

The results of the previous section for neutrino absorption [reaction (14a)] can be written

$$\sigma=\sigma^0(e)\left(\frac{q_\nu+\Delta W^{gu}}{m_ec^2}\right)^2 N\mathfrak{J}, \quad (32)$$

where ΔW^{gu} is some guessed average value for the difference between initial and final nuclear energies, N is the number of neutrons in the initial nucleus, and $\mathfrak{J}$ is a factor defined in Eq. (27) that approximately takes account of the exclusion principle. A similar formula obtains, of course, for antineutrino absorption, the only difference being that N is replaced by Z in Eq. (32).

The principal assumptions made in deriving Eq. (32) were: (1) $q_\nu\gg\Delta W^{g\mu}$; (2) nucleons in the nucleus may be treated nonrelativistically. Assumption (1) is, of course, necessary in order that the cross section be independent of nuclear-structure details; assumption (2) is standard in the theory of beta decay and can lead to errors in the total cross section that are typically of the order of 25%. The errors in the first order formula, Eq. (32), due to assuming a constant value of $\Delta W(A')\times(\cong\Delta W^{gu})$ can easily be estimated[23] by expanding the delta function, $\delta(E_i-E_f)$, in a power series. In this way, one can readily show that the next highest term in the closure approximation is of order $(\Delta W^{av}-\Delta W^{gu})/W_e$, where ΔW^{av} is the average value of the difference between initial and final nuclear energies weighted according to their transition probabilities. In practical cases, this correction to the closure approximation should amount to less than 25% whenever closure is a valid approximation.

Values of ΔW^{gu} are given in Table I for some of the most important cases from the point of view of astrophysical applications. Note that Eqs. (27), (31), and (32) show that the actual neutrino-absorption cross sections are much less than N times the single-particle values for neutrino energies less than $cP_F(\sim 250$ MeV). This result shows that the (single-particle) values for neutrino-absorption cross sections used by some other authors in previous astrophysical applications are too large.[29]

IV. NEUTRINO RADIOACTIVITY

Several authors,[30,31] following Pontecorvo,[32] have discussed the role in highly evolved stars of neutrino radioactivity, i.e., nuclear de-excitation by emission of a neutrino-antineutrino pair. These authors have calculated the effects of neutrino radioactivity by using an interaction of the form given in our Eq. (2) of Sec. IIa. They have found, assuming the coupling constant for neutral decays G^0 equal to the coupling constant for charge changing decays G that neutrino radioactivity is a major energy loss mechanism in, for example, supernova explosions. The result given in Eq. (7) of Sec. IIa shows that these previous authors have overestimated the effects of neutrino radioactivity by at least two orders of magnitude. The results of Sakashita and Nishida[31] in particular suggest, when Eq. (7) is taken into account, that neutrino radioactivity is not a dominant energy loss mechanism in stars.[33]

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[29] A similar conclusion has been reached by G. Fraley (to be published) who has made detailed numerical calculations using a specific nuclear model.

[30] See, for example, V. N. Baier and I. B. Khriplovich, Astron. Zh. **40**, 785 (1963) [English transl. Soviet Astron.—AJ **7**, 599 (1964)]; Zh. Eksperim. i Teor. Fiz. **39**, 1374 (1960) [English transl.: Soviet Phys.—JETP **12**, 959 (1963)].

[31] S. Sakashita and M. Nishida (to be published).

[32] B. Pontecorvo, Phys. Letters **1**, 287 (1962); Usp. Fiz. Nauk **79**, 3 (1963) [English transl.: Soviet Phys.—Usp. **6**, 1 (1963)].

[33] The limit on G^0/G expressed in Eq. (7) also rules out the weak interaction theory of S. Bludman, Nuovo Cimento **9**, 433 (1958), in which neutral currents play a vital role. This theory was of some astrophysical interest because it predicts that the coupling constant for the interaction $(\bar{e}\nu)(\bar{\nu}e)$ is in fact zero.