FACTORIZATION OF COUPLING TO REGGE POLES*

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In the application[1] of the Regge-pole hypothesis[2-4] to relativistic two-body scattering processes, we consider[1] the position $\alpha(t)$ of the pole in the complex angular momentum plane (as a function of the crossed energy variable t) and also the coupling strength $b(t)$ of the Regge trajectory to the particles involved. Suppose we are treating high energy (large s) for the reaction $a+b\to c+d$; the t reaction is $a+\bar{c}\to\bar{b}+d$ and we keep t fixed. For simplicity, we consider spinless particles a,b,c,d.

Now the position $\alpha(t)$ of each Regge pole is a characteristic of the quantum numbers of the t reaction. In another process, for which the t reaction is $a'+\bar{c}'\to\bar{b}'+d'$, but the conserved quantum numbers have the same values, each $\alpha(t)$ will be the same as before. However, the coupling strength $b(t)$ of a given Regge pole will be different in the new reaction. The situation is analogous to that of usual resonance theory. The complex energy of a resonance is independent of the reaction in which it occurs, but the strength depends on the reaction.

Now at certain values of t, where $\mathrm{Re}\,\alpha(t)$ equals an even integer (for positive signature) or an odd integer (for negative signature), we are dealing with an exchanged particle A, which is a bound state for $\mathrm{Im}\,\alpha=0$ and a resonance for $\mathrm{Im}\,\alpha$ small. The quantity $b(t)[\pi\,\mathrm{Re}\,\alpha'(t)]^{-1}$ is then just the product[1] of the coupling parameters g_{abA} and g_{cdA} to the particle A. Thus $b(t)$ has the property of factoring into a number characteristic of a and b and a number characteristic of c and d.

We conjecture that this factorization is valid all along the Regge trajectory. Since proofs of Regge pole properties are available only for the Schrödinger equation, let us demonstrate the factoring for a simple case involving the Schrödinger equation. We consider a number N of coupled Schrödinger equations with common angular momentum l:

$$\left(\frac{d^2}{dr^2}+k_i^{\,2}-\frac{l(l+1)}{r^2}\right)\psi_i(r)=\sum_{j=1}^{N}V_{ij}(r)\psi_j(r). \qquad (1)$$

The S matrix S_{ij} is defined by solutions "regular" at the origin in all channels ($\psi_i\propto r_i^{\,l+1}$) and asymptotic to

$$e^{-ik_ir}-e^{ik_ir}e^{-il\pi}S_{ii} \text{ in channel } i,$$

$$-e^{ik_jr}e^{-il\pi}S_{ij}(k_i/k_j)^{1/2} \text{ in channels } j\neq i. \qquad (2)$$

A Regge pole corresponds to a value of l (in general, complex) for which some or all of the S_{ij} are infinite. The infinity arises because there exists a wave function (for that l) asymptotic to

$$e^{ik_jr}f_jk_j^{\,-1/2} \text{ in all channels.} \qquad (3)$$

Except accidentally, not more than one such function will exist for that l and fixed energy. Near the pole, then, we have

$$f_i/f_j=S_{1i}/S_{1j}=S_{2i}/S_{2j}=\cdots, \qquad (4)$$

and S_{ij} factors into a number depending on i times one depending on j. Stated differently, the point is that not more than one "eigenstate" of the S matrix is likely to have a pole at given values of l and energy.

The application of the factoring principle to the relativistic problem of strong interactions should simplify it enormously. We need deal with Regge trajectories only in terms of their vertices (couplings to a and b, c, and d, etc.) instead of their occurrence in complete two-body scattering amplitudes.

A striking consequence of the factoring appears when we consider the couplings to the Pomeranchuk Regge trajectory P. For elastic scattering of particles a and c, we have[1]

$$\sigma_{\mathrm{tot}}(a,c)\to b_{aaPcc}(0) \text{ as } s\to\infty. \qquad (5)$$

But since $b(0)$ factors, so does the asymptotic value of σ_{tot}. We thus have, for the asymptotic values of total cross sections, relations like this one:

$$\sigma_{\pi\pi}=(\sigma_{\pi N})^2(\sigma_{NN})^{-1}. \qquad (6)$$

Using $\sigma_{\pi N} \approx 25$ mb and $\sigma_{NN} \approx 40$ mb, we obtain $\sigma_{\pi\pi} \approx 15$ mb.

The Regge pole formalism, which rests entirely on conjecture for the relativistic problem, may require some modification for systems with important anomalous thresholds, like nuclei. We therefore restrict our present work, for the time being, to the case with no anomalous thresholds. The interesting question of asymptotic nuclear cross sections will be discussed elsewhere.[5]

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