DECAY RATES OF NEUTRAL MESONS*

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The partial width for the observed decay mode $\omega\to\pi^++\pi^-+\pi^0$ can be estimated by assuming, in the sense of dispersion theory, that the process is dominated by $\omega\to\rho+\pi$ followed by $\rho\to 2\pi$. We treat ρ as a nearly stable particle and define[1] a coupling parameter $f_{\omega\rho\pi}$. Then the width comes out to be

$$\Gamma(\omega\to 3\pi)=(m_\omega-3m_\pi)^4(m_\rho^2-4m_\pi^2)^{-2}m_\omega m_\pi^2 3^{-3/2}$$
$$\times(\gamma_{\rho\pi\pi}^2/4\pi)(f_{\rho\omega\pi}^2/4\pi)W(m_\omega), \qquad (1)$$

where $\gamma_{\rho\pi\pi}^2/4\pi$ is the coupling constant[2-4] for ρ decay into 2π, in terms of which we have

$$\Gamma(\rho\to 2\pi)=\tfrac{1}{3}(\gamma_{\rho\pi\pi}^2/4\pi)(m_\rho^2-4m_\pi^2)^{3/2}m_\rho^{-2}. \qquad (2)$$

The factor $W(m_\omega)$ is a correction factor for relativistic kinematics and approaches unity as $m_\omega\to 3m_\pi$. For $m_\omega=787$ Mev, we find numerically that $W=3.56$.

A virtual photon can turn into ρ^0; at such a vertex in a diagram we insert[3,4] the constant $em_\rho^2/2\gamma_\rho$. Then the strength of the ρ resonance $m_\rho^2(m_\rho^2-t)^{-1}$ in the electric form factor of the pion is just $\gamma_{\rho\pi\pi}/\gamma_\rho$. If the ρ resonance dominates the form factor, then $\gamma_{\rho\pi\pi}/\gamma_\rho$ is of the order of unity. We may now estimate the rate of the decay $\omega\to\pi^0+\gamma$ assuming that it is dominated, in the sense of dispersion theory, by $\omega\to\pi^0+\rho^0$ followed by $\rho^0\to\gamma$. As in reference 3, we obtain

$$\Gamma(\omega\to\pi^0+\gamma)=\alpha(\gamma_\rho^2/4\pi)^{-1}(m_\omega^2-m_\pi^2)^3m_\omega^{-3}$$
$$\times(96)^{-1}(f_{\rho\omega\pi}^2/4\pi). \qquad (3)$$

If we take a width of 100 Mev for ρ, we have $\gamma_{\rho\pi\pi}^2/4\pi=\frac{1}{2}$; assuming that $\gamma_\rho^2/4\pi$ has the same value, we find for the branching ratio

$$\Gamma(\omega\to\pi^0+\gamma)/\Gamma(\omega\to 3\pi)=0.17. \qquad (4)$$

Corrections for $\gamma_{\rho\pi\pi}/\gamma_\rho\neq 1$ and for different ρ widths can easily be made.

We may also try to interpret the π^0 decay as dominated by the vertex $\pi^0\to\rho^0+\omega^0$ followed by $\rho^0\to\gamma$, $\omega^0\to\gamma$. At the γ-ω vertex, we insert the constant $em_\omega^2/(2\sqrt{3}\gamma_\omega)$. If we define the vector coupling of ω to nucleons, for example, to have the strength $\sqrt{3}\gamma_{\omega NN}$, then $\gamma_{\omega NN}/\gamma_\omega$ gives the strength of the ω resonance $m_\omega^2(m_\omega^2-t)^{-1}$ in the electric isoscalar form factor of the nucleon. The factor $\sqrt{3}$ is used so that in the limit of unitary symmetry[2,4] we have $\gamma_\omega\to\gamma_\rho$. The π^0 decay rate then comes out to be

$$\Gamma(\pi^0\to 2\gamma)=\alpha^2(\gamma_\rho^2/4\pi)^{-1}(\gamma_\omega^2/4\pi)^{-1}(192)^{-1}$$
$$\times m_\pi^3(f_{\rho\omega\pi}^2/4\pi). \qquad (5)$$

[The ratio $\Gamma(\omega^0\to\pi^0+\gamma)/\Gamma(\pi^0\to 2\gamma)$ in reference 3 is too small by a factor of 4.] From the measured π^0 decay width of ~3 ev, we can now estimate $f_{\rho\omega\pi}^2/4\pi$ if $\gamma_\rho^2/4\pi$ and $\gamma_\omega^2/4\pi$ are known.

Direct measurement of $\gamma_\rho^2/4\pi$ and $\gamma_\omega^2/4\pi$ is possible by means of the direct decay of the neutral vector meson into an electron or muon pair. If m_l is the lepton mass, then the decay rates are[5]:

$$\Gamma(\omega\to l^++l^-)=\alpha^2(\gamma_\omega^2/4\pi)^{-1}(m_\omega/36)(1-4m_l^2/m_\omega^2)^{1/2}$$
$$\times(1+2m_l^2/m_\omega^2), \qquad (6)$$

$$\Gamma(\rho^0\to l^++l^-)=\alpha^2(\gamma_\rho^2/4\pi)^{-1}(m_\rho/12)(1-4m_l^2/m_\rho^2)^{1/2}$$
$$\times(1+2m_l^2/m_\rho^2). \qquad (7)$$

In the absence of such information, we crudely estimate $\gamma_{\rho\pi\pi}^2\approx\gamma_\rho^2$ as above and take $\gamma_\omega^2\approx\gamma_\rho^2$ on the basis of conjectured approximate unitary symmetry. The estimated partial widths for the various modes of ω decay are then given as in Table I, for $\gamma_\omega^2/4\pi=\gamma_\rho^2/4\pi=\gamma_{\rho\pi\pi}^2/4\pi=\frac{1}{2}$. The dependences on γ_ρ^2, $\gamma_{\rho\pi\pi}^2$, and γ_ω^2 indicated in the table are for fixed π^0 lifetime. Other values of these parameters can be inserted at will.

Since the total width of ω is expected to be <1 Mev, exotic decay modes occur with considerable branching fractions. An important one is $\omega\to\pi^++\pi^-$,

Table I. Estimated partial widths of ω^0 decay.

Mode	Partial width (kev)	Dependence on γ_ρ^2, $\gamma_{\rho\pi\pi}^2$, γ_ω^2
$\omega \to \pi^+ + \pi^- + \pi^0$	395	$\gamma_\rho^2 \gamma_{\rho\pi\pi}^2 \gamma_\omega^2$
$\omega \to \pi^0 + \gamma$	69	γ_ω^2
$\omega \to \pi^+ + \pi^-$	17	Roughly $\gamma_{\rho\pi\pi}^2 \gamma_\rho^{-2} \gamma_\omega^{-2}$
$\omega \to e^+ + e^-$	2.3	γ_ω^{-2}
$\omega \to \mu^+ + \mu^-$	2.3	γ_ω^{-2}

which is caused by a small electromagnetic mixing of ω and ρ^0, as noted by Glashow.[6] If mixing occurs with amplitude y, then we have $\Gamma(\omega \to \pi^+ + \pi^-) = y^2 \times 100$ Mev, and $y^2 \sim 1/5000$ is sufficient to give a branching fraction of several percent. A very crude calculation of the mixing by means of $\omega \to \gamma \to \rho^0$ gives the entry in Table I, in agreement with reference 5.

The recently discovered neutral meson[7,8] at about 550 Mev may be pseudoscalar with $G = +1$; if so, we call it χ as in references 2 and 4. The forbidden decay rates into $3\pi^0$ and $\pi^+ + \pi^- + \pi^0$ are difficult to estimate, except that $3\pi^0/(\pi^+ + \pi^- + \pi^0) \leq \frac{3}{2}$. The remaining neutral decays are expected, however, to represent $\chi \to 2\gamma$. The decay $\chi \to 2\gamma$ may be described roughly on the assumption that the important intermediate steps are $\chi \to 2\rho^0$ (followed by $\rho^0 \to \gamma$, $\rho^0 \to \gamma$) and $\chi \to 2\omega$ (followed by $\omega \to \gamma$, $\omega \to \gamma$). We now wish to estimate the ratio of this rate to that of the hitherto unobserved charged decay mode $\chi \to \pi^+ + \pi^- + \gamma$, which should be dominated by $\chi \to 2\rho^0$, followed by $\rho^0 \to \gamma$, $\rho^0 \to \pi^+ + \pi^-$. First we ignore the dissociation of $\chi \to 2\omega$. The ratio is then easy to compute numerically and comes out

$$\Gamma(\chi \to \pi^+ + \pi^- + \gamma)/\Gamma(\chi \to 2\gamma) \approx \tfrac{1}{2}(\gamma_{\rho\pi\pi}^2/4\pi)(\gamma_\rho^2/4\pi). \quad (8)$$

If we make use of unitary symmetry, we can estimate the correction factor to be applied to (8) for the inclusion of $\chi^0 \to 2\omega \to 2\gamma$, namely $\frac{9}{4}$. We see that the mode $\pi^+ + \pi^- + \gamma$, although one order lower in α than the 2γ mode, is expected to be rarer. The actual estimate is not in violent disagreement with experiment.[8]

Some other discussions of the ω decay have recently appeared.[9] The distinctive feature of our treatment and that of reference 5 is that we express our estimates in terms of measured or measurable matrix elements involving low-mass intermediate states that we guess to be dominant.

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[1] $f_{\omega\rho\pi}$ in a diagram multiples the quantity $e_{\kappa\lambda\mu\nu} k_\kappa{}^\omega e_\lambda{}^\omega k_\mu{}^\rho e_\nu{}^\rho$.

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