Mass Splittings and Ghost Tadpoles*

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The neutron-proton mass difference is expressed, on the assumption that it is purely electromagnetic in origin, in terms of partial-wave amplitudes for the process $N\bar{N} \rightarrow \gamma\gamma$ at zero total energy, and also in terms of the single $n=0$, $M=0$ $O(4)$ amplitude for the same process. This representation allows quite directly a qualitative understanding of the sign of the n - p mass difference, and also allows, with more drastic assumptions, an approximate numerical evaluation of the mass difference based on experimentally determined parameters from the A_2 Regge trajectory. This numerical estimate is in reasonable agreement with experiment. The approach is easily generalized to other electromagnetic mass differences, and is shown to yield the tadpole model.

I. INTRODUCTION

THE difference in sign between the experimental neutron-proton mass difference and the value calculated on the assumption of point Dirac nucleons demonstrates unequivocally the importance of effects due to the strong interactions of the nucleons, if one is to ascribe the mass splittings solely to electromagnetism. The first attempt to estimate some of these effects was by Feynman and Speisman in 1956.¹ They demonstrated that the inclusion of electromagnetic form factors (presumably present because of the strong interactions) in the coupling of photons to nucleons could lead to the correct sign for the mass difference, depending on the detailed behavior of the form factors. As experimental information on the form factors became available, however, it became clear that in fact the form factors did not behave in such a way as to produce

a sign change.² It therefore became necessary to look at other consequences of the strong interactions.

As a first step in this direction, it was observed by Harari³ that the existence of the Regge trajectory associated with the A_2 meson made it extremely unlikely that the simple Feynman-Speisman-type calculation of the n - p mass difference could have turned out to be correct. This is in marked contrast to $I=2$ mass differences, where such a calculation does work fairly well, and where there is no analog of the A_2 trajectory. It was further noticed that, qualitatively, one might expect the existence of the A_2 to reflect an unusually strong interaction in the $J=0^+$, $I=1$ states of nucleon and antinucleon, and that this strong force could produce a sign change in the mass difference.⁴ The physical processes which seem to be important are illustrated in Fig. 1. The sign change is associated with the existence of an extinct bound state (that is, a bound state with zero residue⁵) of negative mass squared, with $J=0^+$, $I=1$, which is produced by the very strong force. This extinct bound state can be thought of as the "tadpole" which has been invoked as a more or less phenomenological way of obtaining the correct mass shifts.⁶ [Since the A_2 is a member of an $SU(3)$ octet,

FIG. 1. The classes of diagrams which are of importance for the mass difference. Diagram (a) is the usual Feynman-Speisman diagram, with experimental form factors included at the photon-nucleon vertices. Diagram (b) represents the additional effects due to the strong interaction in the $J^P=0^+$ channel of $N\bar{N}$ scattering.

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¹ R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

² M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2**, 7 (1959).

³ H. Harari, Phys. Rev. Letters **17**, 1303 (1966).

⁴ M. Suzuki and F. Zachariasen, Phys. Rev. Letters **17**, 1033 (1966); H. Fried and T. Truong, *ibid.* **16**, 559 (1966); H. Pagels, Phys. Rev. **144**, 1261 (1966); M. Suzuki, *ibid.* **171**, 1791 (1968).

⁵ It should be noted that the ghost-elimination mechanism cannot be that suggested by M. Gell-Mann [in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 539] or the "no-compensation mechanism" [S. Y. Chu, C. B. Chiu, and L. L. Wang, Phys. Rev. **161**, 1563 (1967)] but must rather be that of G. F. Chew [Phys. Rev. Letters **16**, 60 (1966)]. The reason for this is that we require a zero of the *sense D* function where the A_2 trajectory goes through zero, and the Gell-Mann and no-compensation mechanisms produce a zero only in the nonsense *D* function. The Chew mechanism, on the other hand, has a zero in the *sense D* function. Fortunately, the Chew mechanism in fact seems to be consistent with fits to the A_2 . See Ref. 10.

⁶ S. Coleman and S. Glashow, Phys. Rev. **134**, B671 (1964).

the extinct bound state, or tadpole, presumably is also; therefore, all predictions of the tadpole model are obtained.] The "tadpole" is thus a consequence of the strong-interaction dynamics rather than a new elementary particle.

It is our intent here to explore further and to elaborate on this physical picture of what is important for the $I=1$ mass shifts. To this end, we should like to express the mass differences in terms of the physics of the $N\bar{N}$ channel rather than in terms of virtual Compton scattering as is usually done. Our first step, then, will be to derive from the usual Cottingham formula⁷ an expression for δM in terms of the partial-wave amplitudes for $N\bar{N} \rightarrow \gamma\gamma$ for virtual photons of equal mass. This is done in Sec. II. The process evidently is of interest at zero total energy; $O(4)$ symmetry thus applies and we also express δM in terms of the $n=0$, $M=0$, $O(4)$ amplitude for $N\bar{N} \rightarrow \gamma\gamma$.

The formula obtained in Sec. II is specialized in Sec. III to two-particle unitarity, including any number of two-body channels as intermediate states in the $N\bar{N} \rightarrow \gamma\gamma$ amplitudes. With the assumption that these amplitudes satisfy unsubtracted dispersion relations, it is shown, qualitatively, how the sign change produced by the existence of the A_2 can come about. We also outline, in this section, a possible calculation based on $O(4)$ symmetry and the Bethe-Salpeter equation which includes the same basic physics, and which may permit the mass shift to be related more directly to experimentally determined A_2 trajectory parameters.

In Sec. IV, we make some simple numerical estimates, with the intent of showing that the physical ideas we are using not only can provide the correct sign but may even give a result which is quantitatively not too unreasonable, though it is important to emphasize that numerical estimates require far more drastic assumptions than are necessary for understanding the sign.

Finally, in Sec. V we comment on the relation of our results to the tadpole model, and in Sec. VI we give a brief summary of what we believe to be the basic conclusions.

II. RELATION OF δM TO THE PROCESS $N\bar{N} \rightarrow \gamma\gamma$

Our starting point is the usual formula for the electromagnetic mass shift of a nucleon to first order in α , the fine structure constant, which we write as

$$\delta M = \int \frac{d^4 q}{(2\pi)^4} \left(\frac{-i}{q^2} \right) \delta_{\mu\nu} T_{\mu\nu}(q, -q; p, -p), \quad (2.1)$$

where $T_{\mu\nu}(q, \bar{q}; p, \bar{p})$ is the Feynman amplitude for the process $N\bar{N} \rightarrow \gamma\gamma$, for virtual photons of mass q^2 , where p and $\bar{p}$ are the nucleon and antinucleon four-

momenta, q and $\bar{q}$ are the photon four-momenta (we take $q^2 = \bar{q}^2$), and where μ and ν are the photon polarizations.⁸ The nucleon and antinucleon are, of course, on the mass shell.

We may place Eq. (2.1) between the spinors $\bar{v}_{\bar{p}, -\sigma}$ and $u_{p, \sigma}$, sum over spins and set $\bar{p}$ equal to $-p$. Note that the relation

$$\sum_{\sigma} (\bar{v}_{\bar{p}, -\sigma} \delta_{\mu\nu} T_{\mu\nu}(q, \bar{q}; p, \bar{p}) u_{p, \sigma}) = - \sum_{\lambda=-1}^1 \sum_{\mu=-1/2}^{1/2} T_{\lambda\lambda, \mu\mu}(t, \nu; q^2) \quad (2.2)$$

holds, where $T_{\lambda\lambda, \mu\mu'}$ are (essentially) helicity amplitudes for $N\bar{N} \rightarrow \gamma\gamma$, and where we define

$$t = (q + \bar{q})^2 = (p + \bar{p})^2$$

and

$$\nu = q \cdot p / M.$$

As a result, we may rewrite Eq. (2.1) in the form

$$\delta M^2 = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} T(t, \nu; q^2) \Big|_{t=0}, \quad (2.3)$$

where we define

$$T(t, \nu; q^2) = - \frac{|\mathbf{p}|}{M} \frac{1}{2} \sum_{\lambda, \mu} T_{\lambda\lambda, \mu\mu}(t, \nu; q^2) \quad (2.4)$$

and, of course, $|\mathbf{p}| = (\frac{1}{4}t - M^2)^{1/2}$. [Precisely the same result, Eq. (2.3), obtains for a boson mass shift, where $T(t, \nu; q^2)$ is the amplitude for $BB \rightarrow \gamma\gamma$, summed over (equal) helicities of the two photons.]

We may also apply the reasoning invented by Cottingham⁷ to Eq. (2.3). This yields, in a straightforward manner, the equation

$$\delta M^2 = - \frac{1}{8\pi^3} \int_{-\infty}^0 \frac{dq^2}{q^2} \int_{-(-q^2)^{1/2}}^{(-q^2)^{1/2}} d\nu (-q^2 - \nu^2)^{1/2} \times T(0, i\nu; q^2). \quad (2.5)$$

Now the helicity amplitudes $T_{\lambda\lambda, \mu\mu}$ have a very simple partial-wave expansion, namely:

$$T_{\lambda\lambda, \mu\mu}(t, \nu; q^2) = \sum_J (2J+1) P_J(z) T_{\lambda\lambda, \mu\mu}^J(t; q^2), \quad (2.6)$$

where z is the cosine of the center-of-mass scattering angle for $N\bar{N} \rightarrow \gamma\gamma$. Thus we have, in the physical region for the process, the relation

$$\nu = t/4M - [(t-4M^2)^{1/2}(t-4q^2)^{1/2}/4M]z. \quad (2.7)$$

The same partial-wave expansion evidently holds

⁸ Our normalization is such that the S matrix is related to $T_{\mu\nu}$ by $S = 1 - i(2\pi)^4 \delta^4(q + \bar{q} - p - \bar{p}) (\bar{v}_{\bar{p}} | T_{\mu\nu} | u_p) e_{\mu} e_{\nu} / (16E\bar{E}q_0\bar{q}_0)^{1/2}$; otherwise we use the Feynman conventions throughout.

⁷ W. H. Cottingham, Ann. Phys. (N. Y.) 25, 424 (1963).

for our amplitude $T(t, \nu; q^2)$, with

$$T^J(t; q^2) \equiv -\frac{|\mathbf{p}|}{M} \frac{1}{2} \sum_{\lambda, \mu} T_{\lambda\lambda, \mu\mu}^J(t; q^2). \quad (2.8)$$

Let us next calculate, still in the physical region for $N\bar{N} \rightarrow \gamma\gamma$, the quantity

$$X(t; q^2) \equiv \int_{-1}^1 dz (1-z^2)^{1/2} T(t, \nu; q^2). \quad (2.9)$$

We can change the variable of integration from z to ν and write

$$X(t; q^2) = \int_{\nu_-}^{\nu_+} \frac{4M d\nu}{(t-4M^2)^{1/2}(t-4q^2)^{1/2}} (1-z^2)^{1/2} \times T(t, \nu; q^2), \quad (2.10)$$

where

$$\nu_{\pm} = t/4M \pm (t-4M^2)^{1/2}(t-4q^2)^{1/2}/4M,$$

and

$$1-z^2 = \frac{(t-4M^2)(t-4q^2) - 16M^2(\nu-t/4M)^2}{(t-4M^2)(t-4q^2)}. \quad (2.11)$$

Now suppose we analytically continue the quantity $X(t; q^2)$ from the physical region ($t > 4M^2$, $t > 4q^2$, $q^2 > 0$) to $t=0$ and $q^2 < 0$. Then

$$X(0; q^2) = -\frac{1}{q^2} \int_{-\sqrt{-q^2}}^{\sqrt{-q^2}} d\nu (-q^2 - \nu^2)^{1/2} \times T(0, i\nu; q^2), \quad (2.12)$$

which is precisely the integral appearing in the Cottingham formula, Eq. (2.5). Thus we have

$$\delta M^2 = \frac{1}{8\pi^3} \int_{-\infty}^0 dq^2 X(0; q^2) \quad (2.13)$$

and, replacing X by T^J through the partial wave expansion and the definition of X , we finally obtain

$$\delta M^2 = \frac{1}{8\pi^3} \sum_J C_J \int_{-\infty}^0 dq^2 T^J(0; q^2), \quad (2.14)$$

where we define

$$C_J = (2J+1) \int_{-1}^1 dz (1-z^2)^{1/2} P_J(z). \quad (2.15)$$

The coefficients C_J are readily evaluated; we find, for J even,

$$C_J = \frac{2J+1}{1-J} \frac{2\pi}{(\frac{1}{2}J)! (\frac{1}{2}J+1)!} \times (\frac{1}{2} - \frac{1}{2}J)^2 (\frac{3}{2} - \frac{1}{2}J)^2 \cdots (\frac{1}{2})^2, \quad (2.16)$$

while for J odd, C_J vanishes. The first few C_J are the following:

$$\begin{aligned} C_0 &= \frac{1}{2}\pi = 1.57, \\ C_2 &= -5\pi/16 = -0.98, \\ C_4 &= -9\pi/64 = -0.44, \\ C_6 &= -65\pi/2048 = -0.10, \\ &\text{etc.} \end{aligned}$$

Hence, only the first few J values in Eq. (2.14) are likely to be significant.

Equation (2.14) is our desired result. It expresses the mass difference in terms of the partial-wave amplitudes for the process $N\bar{N} \rightarrow \gamma\gamma$ for virtual photons, continued to zero total energy and negative photon mass. With this equation, we should be able to implement our physical idea of relating the mass splittings to phenomena taking place in $N\bar{N}$ scattering, and in particular to whatever phenomena control this at zero energy.

We should also emphasize that Eq. (2.14) holds for any mass shift, fermion or boson, where T^J is simply summed over final (equal) helicities and averaged over initial (equal) helicities.

Finally, to calculate the mass difference between proton and neutron, we must write Eq. (2.14) for a proton and for a neutron and subtract the two. Only the $I=1$ amplitudes T^J then survive on the right-hand side.

Let us now return to Eq. (2.9), and write down the $O(4)$ expansion of $T_{\lambda\lambda, \mu\mu}(0, \nu; q^2)$. Following Freedman and Wang,⁹ we represent a state of definite equal helicities μ in terms of states with definite s and λ :

$$\langle p' \mu \mu | = \sum_{s\lambda} \langle p' \mu \mu | p s \lambda \rangle \langle p s \lambda |,$$

where

$$\langle p' \mu \mu | p s \lambda \rangle = \delta^3(\hat{p}' - \hat{p}) \delta_{\lambda 0} C(s_1 s_1 s; \mu; -\mu) (-)^{s_1 - \mu}.$$

(The three-dimensional delta function simply keeps $\mathbf{p}'$ in the direction of $\mathbf{p}$.) If we now sum over μ to obtain the particular combination of helicities needed in Eq. (2.9), and note that

$$(2s_1+1)^{1/2} C(s_1 s_1 0; \mu; -\mu) = (-)^{s_1 - \mu},$$

we obtain

$$\sum_{\mu} \langle p' \mu \mu | = \sum_{s, \lambda} \delta_{s0} \delta_{\lambda 0} (2s_1+1)^{1/2} \langle p' s \lambda |, \quad (2.17)$$

which contains only $s=\lambda=0$. Finally, then, we may write

$$T(0, \nu; q^2) = -\frac{|\mathbf{p}|}{2M} \sum_{\lambda, \mu} T_{\lambda\lambda, \mu\mu}(0, \nu; q^2) = \sqrt{3} \times \sqrt{2} \times T_{000}(\delta),$$

where $T_{ss'\lambda}(\delta)$ is essentially the quantity defined in Eq. (40) of Ref. 9. The $O(4)$ projection of this ampli-

⁹ D. Z. Freedman and J. M. Wang, Phys. Rev. **160**, 1560 (1967).

tude is

$$T_{0,0}^{(n,M)}(0; q^2) = \int_0^\pi d\delta \sin^2 \delta T_{000}(\delta) d_{000}^{(n,M)}(\delta). \quad (2.18)$$

Now

$$d_{000}^{(0,0)}(\delta) = 1;$$

thus we find that Eq. (2.9) yields simply

$$X(0; q^2) = T_{00}^{(0,0)}(0; q^2) \quad (2.19)$$

and hence we have

$$\delta M^2 = \frac{1}{8\pi^3} \int_{-\infty}^0 dq^2 T_{00}^{(0,0)}(0; q^2). \quad (2.20)$$

The mass shifts can therefore be expressed as an integral over a single $O(4)$ amplitude for $N\bar{N} \rightarrow \gamma\gamma$, as well as a sum over an infinite number of partial-wave amplitudes through Eq. (2.14). This observation may be of value if the A_2 trajectory at $t=0$ is associated with the same $O(4)$ representation, since then the A_2 parameters should by themselves suffice to give us an approximate expression for the mass shift. We shall belabor this point more fully in the following section.

III. TWO-PARTICLE APPROXIMATIONS

To implement Eq. (2.14) requires knowledge of the $N\bar{N} \rightarrow \gamma\gamma$ partial-wave amplitude at zero energy. To obtain this knowledge, let us invoke dispersion relations and unitarity. If we restrict ourselves, for the moment, to only $N\bar{N}$ intermediate states (this restriction will be lifted shortly to allow any two-particle intermediate state) in the unitarity relation, then we have for the $J=0$ partial-wave amplitude:

$$\text{Im} T^0(t; q^2) = T^0(t; q^2) \rho(t) h_{10}^0(t), \quad (3.1)$$

where $h_{10}^0(t)$ is the $N\bar{N} \rightarrow N\bar{N}$ analytic partial-wave amplitude defined by Scotti and Wong¹⁰ for the 3P_0 state and, for the p - n mass difference, with isospin $I=1$. The phase-space factor $\rho(t)$ is given by

$$\frac{1}{2} M [(t-4M^2)/t]^{1/2}.$$

For other values of J , the unitarity relation is somewhat more complicated.

If we assume that $T^0(t; q^2)$ vanishes as $t \rightarrow \infty$, the solution to Eq. (3.1) may be written

$$T^0(t; q^2) = \frac{1}{D_0(t)} \frac{1}{\pi} \int_{-\infty}^{4q^2 - q^4/M^2} \frac{\text{Im} T^0(t'; q^2) D_0(t')}{t' - t} dt', \quad (3.2)$$

where $D_0(t)$ is the conventional D function for $N\bar{N}$ scattering with $J^P=0^+$ and $I=1$. The integral is over the left-hand cut of the $N\bar{N} \rightarrow \gamma\gamma$ amplitude; the cut, as indicated, runs from $-\infty$ to $4q^2 - q^4/M^2$.

¹⁰ A. Scotti and D. Wong, Phys. Rev. **138**, B145 (1965).

The D function in Eq. (3.2) is the source of the right hand cut in the partial wave amplitude; its presence permits the inclusion of rescattering effects in the $N\bar{N} \rightarrow \gamma\gamma$ process as illustrated in Fig. 3. If it were absent, then $T^0(t; q^2)$ would be calculated simply from exchanges, of which the nucleon exchange shown in Fig. 2 is the simplest. (We recall that nucleon exchange alone—in other words, the Feynman-Speisman approximation—gives the wrong sign for δM .)

For the purpose of calculating δM , we are interested only in $T^0(t; q^2)$ at $t=0$. The ratio $D^0(t)/D^0(0)$, then, is important only if D varies significantly over the range of t' of interest. Now we know that the Regge trajectory associated with the A_2 meson passes through zero at $t=t_0 \approx -0.5 \text{ BeV}^2$.¹¹ If the trajectory chooses sense, and the corresponding ghost is eliminated by the Chew mechanism,⁵ then $D^0(t_0)=0$. [If, on the other hand, the trajectory chooses nonsense, as in the Gell-Mann or no-compensation mechanism,⁵ then nothing much happens to $D(t)$ at $t=t_0$.] Assuming that $D^0(t_0)$ does indeed vanish, we can expect the ratio $D^0(t)/D^0(0)$ occurring in Eq. (3.2) to be essentially negative for most of the interesting range. This is because the upper limit of the integral is at $4q^2 - q^4/M^2$, and because a characteristic value of q^2 is likely to be $q^2 = -q_0^2 = -0.72 \text{ BeV}^2$, the parameter appearing in the fits to the electromagnetic form factors.¹² Thus we have, generally, $t' \lesssim 4q_0^2 < t_0 < 0$ in the integral. Hence $D^0(0)$ is negative and $D^0(t)$ is predominantly positive.

We may conclude that if the ghost on the A_2 trajectory is eliminated by the Chew mechanism, the rescattering corrections will change the sign of $T^0(0; q^2)$ from the value it would have when calculated from exchanges only. Since the $J=0$ contribution to δM in Eq. (2.14) is much the largest, this means that rescattering corrections are likely to change the sign of δM too.

It is of interest to compare this result with the formula for δM obtained by separating out the “mass feedback” phenomenon,⁴ namely,

$$\delta M = \delta M' / D(0), \quad (3.3)$$

where D is the same $J=0^+$ D function we have been discussing above. Here, $\delta M'$ is the mass shift which would have been calculated ignoring feedback, and the simplest contribution to this is again just the usual Feynman-Speisman calculation. The conclusion is the same: A negative D function (at $t=0$) can change the sign of the mass shift.⁴

So much for qualitative features; can we also find a numerically reasonable approximation to the partial-wave amplitudes $T^J(0; q^2)$ which will permit us to make quantitative predictions?

¹¹ S. Y. Chu and D. P. Roy, Phys. Rev. Letters **20**, 958 (1968). These authors in an Erratum [Phys. Rev. Letters **21**, 57 (1968)] quote a corrected value of t_0 , namely -0.7 BeV^2 . This would give us a mass shift of -0.4 MeV instead of -1.0 MeV , which perhaps makes our crude estimate look a bit less contrived.

¹² D. H. Coward *et al.*, Phys. Rev. Letters **20**, 292 (1968).

FIG. 2. The "Born approximation" diagram to the $NN \rightarrow \gamma\gamma$ amplitude.

For the $J \neq 0$ amplitudes, the obvious thing to do is simply to write $T^J(0; q^2) = B^J(0; q^2)$, where B^J is the "Born approximation" (shown in Fig. 2) to the $NN \rightarrow \gamma\gamma$ amplitude. That is to say, B^J is the contribution of nucleon exchange (including form factors) to T^J . Replacement of all T^J (including $J=0$) by B^J in Eq. (2.14) reproduces the conventional Feynman-Speisman result for δM .

The use of B^J as an approximate expression for T^J when $J \neq 0$ is not unreasonable for three reasons. First, $\text{Im} T^J = \text{Im} B^J$ for small values of t' , near the right-hand edge of the left-hand cut. Second, there is no known structure, such as that evidenced by the A_2 for the $J=0^+$ channel, to suggest that $D_J(t)$, for $J \neq 0$, is particularly rapidly varying. Third, the $J \neq 0$ contributions to δM are rather small anyway.

Thus we may write

$$\delta M^2 = \delta \bar{M}^2 + \delta M^2_{J=0} - \delta \bar{M}^2_{J=0}, \quad (3.4)$$

where $\delta \bar{M}$ is the Feynman-Speisman, or Born approximation, result, and where

$$\delta M^2_J = \frac{1}{8\pi^3} C_J \int_{-\infty}^0 dq^2 T^J(0; q^2), \quad (3.5)$$

$$\delta \bar{M}^2_J = \frac{1}{8\pi^3} C_J \int_{-\infty}^0 dq^2 B^J(0; q^2), \quad (3.6)$$

and

$$\delta M^2 = \sum_J \delta M^2_J, \quad \delta \bar{M}^2 = \sum_J \delta \bar{M}^2_J. \quad (3.7)$$

It remains, then, to approximate $\delta M^2_{J=0}$.

Before making a specific approximation to $T^0(0; q^2)$ a few remarks about the Chew ghost-elimination mechanism are in order. The requirement that the ghost be eliminated ordinarily comes from the fact that the ghost pole would lead to a singular cross section for the crossed reaction. In our case the crossed reaction is nucleon Compton scattering with spacelike virtual photons, calculated to lowest order in the electric charge e . Nucleon Compton scattering in itself is a physically measurable process at $q^2=0$, so we certainly cannot tolerate a ghost pole in the exact amplitude for $q^2=0$, and it seems plausible that the ghost is removed for $q^2 \neq 0$ as well. Whether or not the ghost is removed in these amplitudes when calculated

only to lowest order in e is, however, another matter, and in fact it is quite possible that it is only higher-order in e corrections that remove the pole. In this event, $T^0(t; q^2)$ will indeed be singular at $t=t_0$, and we may calculate $T^0(t; q^2)$ ignoring whatever ghost-elimination mechanism operates when all orders in e are included.

As was the case for the $J \neq 0$ amplitudes, then, the most direct thing to do is to approximate $\text{Im} T^0(t'; q^2)$ in Eq. (3.2) by $\text{Im} B^0(t'; q^2)$, though because of the existence of the A_2 we cannot now replace $D^0(t')$ by a constant. This approximation is reasonable, we emphasize, provided only small values of t' and q^2 are relevant in Eqs. (3.2) and (2.14).

Our numerical estimates, described in the following section, will be based on the assumptions and approximations outlined above.

The $O(4)$ formula for δM^2 derived in Sec. II, namely, Eq. (2.20), may also be used as the basis for an approximate calculation. This form has the advantage that only a single $O(4)$ representation is involved, in contrast to Eq. (2.14) which contains an infinite number of angular momentum representations.

The amplitude $T_{00}^{(0,0)}(0; q^2)$ cannot itself be measured, but it can be (approximately) related to the corresponding $O(4)$ amplitude for NN scattering. As illustrated in Fig. 3, we may express it as a single integral over the amplitude $t_{00}^{(0,0)}(0; p'^2, \bar{p}'^2)$ for $NN \rightarrow NN$, with the two final nucleons off the mass shell, times the Born-approximation amplitude $B_{00}^{(0,0)}(0; q^2, p'^2, \bar{p}'^2)$ for the two off-shell nucleons becoming two off-shell photons. What is required, then, is a knowledge of the behavior of the NN amplitude as a function of the masses of the two final nucleons. This can, in principle, and to some extent in practice, be determined from the Bethe-Salpeter equation for $NN \rightarrow NN$.

We shall not pursue this approach further here; suffice it to say that a calculation along these lines does not seem insuperably difficult, and it may give relatively reliable results.

To conclude this section, let us return to the two-particle approximation to the unitarity condition for the partial-wave amplitudes $T^J(t; q^2)$, and extend it to include any number of two-particle channels. Let us label each channel by ij , where i and j run from 1 to N . Thus channel ij is considered to contain particle i and particle j . The channels in question have, of course, baryon number zero, isospin one, and strangeness zero. The particles may be two mesons or a baryon-anti-baryon pair. Generally, i and j will refer to the same particle, as in NN , $K\bar{K}$, for example, but there may also be channels such as $\pi\eta$ which contain two different particles. The mass shift thus becomes a matrix δM_{ij}^2 , which will contain off-diagonal elements mixing π and η , for example, and whose eigenvalues are the actual masses.

FIG. 3. Diagram which could be used to calculate $T_{00}^{(0,0)}(0; q^2)$.

The generalization of the equations given at the beginning of this section are obvious, and we may evidently write

$$\delta M_{ij}^2 = \frac{1}{8\pi^3} \sum_J C_J \int_{-\infty}^0 dq^2 T_{ij}^J(0; q^2), \quad (3.8)$$

where

$$T_{ij}^J(t; q^2) = \sum_{klmn} \frac{1}{\pi} \int_{-\infty}^{t_{kl}} \frac{\text{Im} T_{kl}^0(t'; q^2) D_{kl, mn}^0(t')}{t' - t} \times [(D^0(t))^{-1}]_{mn, ij}. \quad (3.9)$$

Here $D_{ij, kl}^0(t)$ is the angular momentum 0, $I=1$, $B=0$, $S=0$, etc., D function for the multichannel problem. (An analogous formula could obviously be written for the $I=2$ mass shifts as well, if we so desired.)

As in the single-channel problem, we expect that for $J=0$, the determinant of $D_{ij, kl}^{J=0}(t)$, which we call $\Delta(t)$, has a zero at $t=t_0 \approx -0.5 \text{ BeV}^2$ corresponding to the crossing of zero by the A_2 trajectory. The $J=0$ contribution in Eq. (3.8) will therefore again be enhanced.

We shall return to this topic in connection with the tadpole model in Sec. V.

IV. NUMERICAL ESTIMATES

The first step in attempting to use Eqs. (3.4) to (3.7) for quantitative estimates is to compute the Born approximation illustrated in Fig. 2. A direct calculation yields the result

$$B(t, \nu; q^2) = -\frac{8M^4}{(q^2 - t/2)^2 - 4\mathbf{p}^2 q^2 z^2} \left(\frac{2q^2 - t}{4M^2} \right) \times \left\{ \left(1 + \frac{1}{2} \frac{q^2}{M^2} - \frac{1}{2} \frac{t}{M^2} \right) [(F_1^p(q^2))^2 - (F_1^n(q^2))^2] \right. \\ + (3q^2/M - t/M - q^2/4M^3) \\ \times [F_1^p(q^2)F_2^p(q^2) - F_1^n(q^2)F_2^n(q^2)] \\ + (2q^2 + q^4/4M^2 + t^2/16M^2 - t - q^2/4M^2) \\ \left. \times [(F_2^p(q^2))^2 - (F_2^n(q^2))^2] \right\}, \quad (4.1)$$

where

$$B(t, \nu; q^2) = -\frac{1}{2} \frac{|\mathbf{p}|}{M} \sum_{\lambda=-1}^1 \sum_{\mu=-1/2}^{1/2} B_{\lambda\lambda, \mu\mu}(t, \nu; q^2). \quad (4.2)$$

In deriving this result, we have written the photon-proton vertex as

$$F_1^p(q^2)\gamma_\mu + iF_2^p(q^2)\sigma_{\mu\nu}q_\nu$$

with a corresponding form for the photon-neutron vertex. We have also thrown away a nonpole (in z) contribution in Eq. (4.1) arising from the fact that the direct Feynman-diagram calculation gives a first-degree polynomial in z^2 in the numerator. This nonpole term, fortunately, makes a small contribution (less than 0.2 MeV) to the mass difference.

Our result, Eq. (4.1), of course agrees (at $t=0$) with the pole contribution to the Born approximation-result quoted by Harari.³ However, Harari also has a nonpole contribution, different from our nonpole term,¹³ in his result. His nonpole term is a consequence of keeping only pure pole contributions to the functions called $t_1(q^2; \nu)$ and $t_2(q^2; \nu)$.^{3,7} This would be a reasonable thing to do if one believed that t_1 and t_2 satisfied unsubtracted dispersion relations in ν^2 . However, as Harari points out, t_1 probably does not satisfy such a relation. The nonpole contributions he keeps are therefore also ambiguous. But again the numerical effect of Harari's nonpole term is small, so that whether or not one keeps it is more or less irrelevant.

We emphasize, incidentally, that if it were not the case that these nonpole contributions were small, then it would be difficult to arrive at an unambiguous answer for δM . This ambiguity is tied up with, in the Feynman-diagram language, different ways of writing the γN vertex, or in the dispersion language, with subtractions.

We may now break up the Born amplitude into partial waves according to

$$B(t, \nu; q^2) = \sum_J (2J+1) P_J(z) B^J(t; q^2); \quad (4.3)$$

thus we arrive at

$$B^J(t; q^2) = -\frac{2M^2}{|\mathbf{p}| |\mathbf{q}|} Q_J \left(\frac{q^2 - t/2}{2|\mathbf{p}| |\mathbf{q}|} \right) \mathcal{F}(t; q^2), \quad (4.4)$$

where $\mathcal{F}$ is the same combination of form factors enclosed in the curly brackets in Eq. (4.1).

To begin with, we calculate the $\delta \bar{M}_J$ using the definition (3.6). A trivial numerical integration gives

$$\begin{aligned} \delta \bar{M}_{J=0} &= 0.40 \text{ MeV}, \\ \delta \bar{M}_{J=2} &= 0.12 \text{ MeV}, \\ \delta \bar{M}_{J=4} &= -0.036 \text{ MeV}, \end{aligned} \quad (4.5)$$

¹³ The difference in nonpole terms arises from different ways of defining the γN vertex. These definitions all agree for on-shell nucleons, but differ when one nucleon is off-shell, as is the case in Fig. 2. The numerical differences, however, are small.

and

$$\delta\bar{M} = \sum_J \delta\bar{M}_J = 0.50 \text{ MeV}.$$

Now, as indicated in Sec. III, we expect there to be a sizable deviation from, or enhancement of, the Born result in $\delta M_{J=0}$, so let us attempt to estimate this using Eq. (3.2). Two basic assumptions are, as will become clear shortly, essential if our approximations are to be reasonable. First, it is necessary to believe that $\text{Im}T^0(t'; q^2)$ vanishes sufficiently rapidly as $t' \rightarrow -\infty$ so that in the integral over the left-hand cut, most of the contribution comes from rather small values of t' . Thus we can approximate $\text{Im}T^0(t'; q^2)$ by $\text{Im}B^0(t'; q^2)$ under the integral. Second, it is essential to believe that in Eq. (3.5), $T^J(0; q^2)$ dies off rapidly for large q^2 , and only relatively small q^2 values are significant.

Now let us proceed. From Eq. (4.4), we see that on the left-hand cut,

$$\text{Im}B^0(t; q^2) = -\frac{\pi M}{|\mathbf{p}||\mathbf{q}|} \mathcal{F}(t; q^2). \quad (4.6)$$

The function $\mathcal{F}(t; q^2)$ is a polynomial in t . The remaining t dependence in $\text{Im}B^0$ is only in the factor

$$|\mathbf{p}||\mathbf{q}| = \frac{1}{4}(4M^2 - t)^{1/2}(4q^2 - t)^{1/2}. \quad (4.7)$$

We recall that the right-hand edge of the left-hand cut is at $t = 4q^2 - q^4/M^2$; thus at this point

$$(|\mathbf{p}||\mathbf{q}|)^{-1} = 4(4M^2 - 4q^2 + q^4/M^2)^{-1/2}(M/q^2).$$

This factor, therefore, peaks very sharply at threshold, if $|q^2| \lesssim M^2$.

Let us next approximate $D^0(t)$ by a straight line,

$$D(t) = C(t - t_0). \quad (4.8)$$

We hope that, since we assume only small values of t count, this linear approximation is not too bad.

The entire integrand in Eq. (3.2) is then a polynomial in t times the sharply peaked function $(|\mathbf{p}||\mathbf{q}|)^{-1}$. We may hope, therefore, that a not totally absurd approximation is to take out the polynomial from the integral and evaluate it at the peak value of the rest of the integrand—that is, at $t = 4q^2 - q^4/M^2$. If we do this, we obtain the approximate result

$$T^0(0; q^2) = \frac{D(4q^2 - q^4/M^2)}{D(0)} \frac{\mathcal{F}(4q^2 - q^4/M^2; q^2)}{\mathcal{F}(0; q^2)} \times B^0(0; q^2), \quad (4.9)$$

where, as we said, we take

$$\frac{D(4q^2 - q^4/M^2)}{D(0)} = 1 - \frac{4q^2 - q^4/M^2}{t_0}. \quad (4.10)$$

Let us emphasize that we believe the linear approximation for $D(t)$ only for small t —thus the use of Eq.

(4.10) is incorrect beyond small values of q^2 . Furthermore, the sharp peak in Eq. (4.7) washes out for large q^2 ; thus the factor $\mathcal{F}(4q^2 - q^4/M^2; q^2)$ also becomes a poor approximation for large q^2 . The right-hand side of Eq. (4.9) is then a reasonable approximation to $T^0(0; q^2)$ only if q^2 is small, say $q^2 \lesssim M^2$. We must assume, as mentioned earlier, that the true $T^0(0; q^2)$ disappears quickly enough as $q^2 \rightarrow 0$ so that only small q^2 matters and we can replace, for small q^2 only, $T^0(0; q^2)$ by Eq. (4.9). For small q^2 , however, we may drop the higher powers of q^2 in Eqs. (4.9) and (4.10); thus we have from Eq. (3.5) the result

$$\mathcal{F}(4q^2 - q^4/M^2; q^2) \approx \mathcal{F}(4q^2; q^2) \approx -32\pi\alpha M^3 \left(\frac{1}{1 - q^2/q_0^2} \right)^4 \left(1 - 2.17 \frac{q^2}{M^2} \right). \quad (4.11)$$

This expression is obtained from the fits¹⁴

$$F_1^v(q^2) = e \left(\frac{1}{1 - q^2/q_0^2} \right)^2,$$

$$F_1^n(q^2) = 0,$$

$$F_2^v(q^2) = 1.79 \frac{e}{2M} \left(\frac{1}{1 - q^2/q_0^2} \right)^2,$$

$$F_2^n(q^2) = -1.91 \frac{e}{2M} \left(\frac{1}{1 - q^2/q_0^2} \right)^2.$$

Empirically, $q_0^2 = 0.72 \text{ BeV}^2$.¹²

We shall also write $D(4q^2 - q^4/M^2) \approx D(4q^2)$, and thus we obtain our final expression:

$$\delta M_{J=0} = \frac{\alpha M}{2\pi} \int_{-\infty}^0 \frac{dq^2}{(-q^2)^{1/2}} \tan^{-1} \left(\frac{2M}{(-q^2)^{1/2}} \right) \times \left(\frac{1}{1 - q^2/q_0^2} \right)^4 \left[1 + \left(-\frac{4M^2}{t_0} - 2.17 \right) \frac{q^2}{M^2} + \left(8.7 \frac{M^2}{t_0} \right) \frac{q^4}{M^4} \right]. \quad (4.12)$$

Numerically integrating, this gives

$$\delta M_{J=0} = (0.92 + 1.02/t_0) \text{ MeV}, \quad (4.13)$$

where t_0 is measured in BeV^2 .

If we take the value $t_0 \approx -0.5 \text{ BeV}^2$ quoted in Ref. 11, we find

$$\delta M_{J=0} = -1.12 \text{ MeV}.$$

¹⁴ These fits differ slightly from the fits used by Harari (Ref. 3), which are

$$G_E^P = G_M^P/2.79 = -G_M^N/1.91 = (1 - q^2/q_0^2)^{-2}$$

and $G_E^N = 0$. The differences are, however, of order $q^2/4M^2$ and are small if it is indeed true, as we assume, that only rather small q^2 values are important. Both our choice and Harari's agree with the experimental form factors.

Combining this, through Eq. (3.4), with the other terms gives, finally,

$$\delta M = +0.50 - 1.12 - 0.40 = -1.02 \text{ MeV},$$

which, by some coincidence, happens to agree rather well with the experimental value.

V. TADPOLES

We recall Eqs. (3.8) and (3.9). To make use of these, we need a model for the matrix $D^0(t)$, which has the following properties: (i) $\Delta(t) = \det D^0(t)$ has a zero at $t = t_0$. (ii) $t^0(t)$, the $J^J = 0^+$ scattering matrix, looks like a Regge pole with factorizable residues which vanish at $t = t_0$ so that the trajectory does not give rise to a ghost there.

Such a model is readily written down (we now use indices $\alpha, \beta, \dots$, as channel indices. Thus $\alpha, \beta, \dots$, stand for the particle pairs $ij, kl, \dots$). Take

$$t_{\alpha\beta}^0(t) = \beta_\alpha \beta_\beta f(t) / \Delta(t), \quad (5.1)$$

$$N_{\alpha\beta}^0(t) = \beta_\alpha \beta_\beta f(t), \quad (5.2)$$

$$D_{\alpha\beta}^0(t) = \delta_{\alpha\beta} - R_\alpha(t) \beta_\beta / \Delta(t), \quad (5.3)$$

$$R_\alpha(t) = - \int \frac{f(t') \rho(t') dt'}{\pi (t' - t)}, \quad (5.4)$$

$$\Delta(t) = \det D^0(t) = 1 - \sum_\alpha R_\alpha(t) \beta_\alpha^2, \quad (5.5)$$

$$D_{\alpha\beta}^{0-1}(t) = \delta_{\alpha\beta} - R_\alpha(t) \beta_\alpha \beta_\beta / \Delta(t). \quad (5.6)$$

We presume that $\sum_\alpha R_\alpha(t_0) \beta_\alpha^2 = 1$ and that $f(t_0) = 0$.

From Eqs. (5.3) and (5.6) we find

$$\sum_\beta D_{\alpha\beta}^0(t) D_{\beta\gamma}^{0-1}(0) = \delta_{\alpha\gamma} + \frac{X_\alpha(t) \beta_\gamma}{\Delta(0)}, \quad (5.7)$$

where

$$X_\alpha(t) \beta_\gamma = D_{\alpha\beta}(t) - D_{\alpha\beta}(0). \quad (5.8)$$

Therefore,

$$\delta M_\gamma^2 = \delta \bar{M}_\gamma^2 + X \beta_\gamma / \Delta(0), \quad (5.9)$$

where

$$X = \frac{1}{8\pi^3} C_0 \int_{-\infty}^0 dq^2 \sum_\alpha \frac{1}{\pi} \times \int_{-\infty}^{t_0} \frac{\text{Im} T_\alpha^0(t'; q^2) X_\alpha(t') dt'}{t'}. \quad (5.10)$$

The second term in Eq. (5.9) is the "tadpole" contribution. It is proportional to β_γ , the coupling of the tadpole to the γ th channel, and to $\Delta^{-1}(0)$, where $\Delta^{-1}(t)$ vanishes at $t = t_0$ and represents the tadpole "propagator." The tadpole here, of course, is not a new elementary particle but is simply the effect of the ghost lying on the A_2 trajectory. All results of the tadpole model thus follow directly, even though the tadpole is not a physical particle, does not appear as a

pole in any physically measurable amplitude or form factor, and has a coupling which vanishes at the tadpole "mass" t_0 .

The tadpole model may also be obtained in another way, as follows: We start with the many-channel version of Eq. (2.14), and perform a Sommerfeld-Watson transformation on the right-hand side. The analytic continuations of C_J and $T_\gamma^J(0; q^2)$ into the complex J plane are well defined, and we obtain directly the result

$$\delta M_\gamma^2 = \frac{1}{8\pi^3} \sum_n \frac{C_{\alpha_n}(0)}{\sin \pi \alpha_n(0)} \int_{-\infty}^0 dq^2 \beta_\gamma^n(0; q^2) + \text{B.I.}, \quad (5.11)$$

where $\beta_\gamma^n(0; q^2)$ is the residue associated with the Regge pole $\alpha_n(0)$ in the partial wave amplitude $T_\gamma^J(0; q^2)$, and B.I. denotes the background integral. Because of factorization, we may write

$$\beta_\gamma^n(0; q^2) = \beta^n(0; q^2) \beta_\gamma^n(0),$$

where $\beta_\gamma^n(0)$ is the coupling of the n th trajectory to the γ th channel, and $\beta^n(0; q^2)$ is the coupling of the n th trajectory to the two photons. If a single trajectory (presumably the A_2 trajectory) dominates in Eq. (5.11), then the tadpole contribution to δM_γ^2 , as in Eq. (5.9), is reproduced.

It is well known that the F/D ratio of the A_2 couplings to baryons (namely $F/D \approx -2$), determined from high-energy scattering data,¹⁵ is quite consistent with the value necessary to fit the observed $I=1$ baryon mass splittings.^{6,16} It is also well known that the perturbation theory results for the $I=2$ splittings agree reasonably well with experiment.³ Overall, then, we are justified in saying that there is no obvious disagreement between the experimentally observed electromagnetic mass shifts and the (somewhat crudely estimated) predictions of strong interaction theory. Further predictions, of the ratio of baryon-to-meson mass splittings, can be made when analysis of high-energy data permits the determination of the ratio of the A_2 couplings to baryons and mesons.

VI. CONCLUSION

In concluding this article, let us emphasize again that the approximations involved in obtaining the numerical results of Sec. IV are drastic; we therefore are inclined to view the rather good agreement with experiment as somewhat fortuitous. The qualitative features explained in Sec. III, however, we feel are more reliable, and do permit one to understand in a reasonable way that the neutron is heavier than the proton. We also wish to emphasize that we believe this

¹⁵ V. Barger and M. Olsson, Phys. Rev. Letters **18**, 294 (1967); K. Sarma and G. Renninger, *ibid.* **20**, 399 (1968).

¹⁶ M. Suzuki, Phys. Rev. **171**, 1791 (1968); D. Gross and H. Pagels, *ibid.* **172**, 1381 (1968).

qualitative understanding to be the logical outgrowth of the physical insight developed in Refs. 3 and 4, and that it is indeed the existence of the A_2 , and the dynamics implied by its existence, which are responsible for the major nonperturbative strong interaction effects in the $I=1$ mass differences.

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Inequalities for the K_{l3} Form Factors*

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An inequality for the K_{l3} form factors is derived on the basis of current commutation relations and certain assumptions about the relative importance of the intermediate states. It includes the inequalities due to Quinn and Bjorken and to Suzuki. We find $\xi \leq 0.91/f_+(0) - 1.35$, where $\xi = f_-(0)/f_+(0)$ and $f_+(0) \leq 1$.

ON the basis of current commutation relations, Quinn and Bjorken¹ have argued that the f_+ form factor of K_{l3} decay obeys the inequality

$$f_+(t) \leq 1, \quad t \leq 0. \quad (1)$$

Recently, Suzuki² has shown that, using charge commutators, the following inequality can be derived in second order in symmetry breaking:

$$(4E_\pi E_K)^{-1} [(E_K + E_\pi) f_+ ((E_K - E_\pi)^2) + (E_K - E_\pi) f_- ((E_K - E_\pi)^2)] < 1, \quad (2)$$

where $E_K = (m_K^2 + \mathbf{p}^2)^{1/2}$, and $E_\pi = (m_\pi^2 + \mathbf{p}^2)^{1/2}$. From this he was able to derive an inequality relating $\xi = f_-(0)/f_+(0)$, $f_+(0)$, and λ , where

$$f_+(t) = f_+(0) (1 + \lambda t / m_\pi^2).$$

In the present paper we follow and generalize the method of Quinn and Bjorken.¹ We use the $SU(3)$ vector-current commutation relations and the assumption of the dominance of meson states described by an octet over those that might belong to higher $SU(3)$ representations and have high (Y, I) quantum numbers. We differ from Ref. 1 by using the commutation relations defined below in Eq. (3) between states of arbitrary momentum and not only at $\mathbf{p}^2 \rightarrow \infty$. We arrive at inequalities that depend on two parameters, $\mathbf{p}^2$ and $\mathbf{k}^2$. The limit $\mathbf{p}^2 \rightarrow \infty$ leads to Eq. (1), and the limit $\mathbf{k}^2 = 0$ leads to Eq. (2). However, since two variables

are involved, we get additional restrictions. In particular, we derive a relation between the upper limit on ξ ($\xi_{\max}$) and $f_+(0)$ which is independent of any assumption on the t behavior of $f_+(t)$ and $f_-(t)$.

We start with V -spin currents $j_\mu^i(x)$, obeying the commutation relations

$$\delta(x_0) [j_0^+(x), j_0^-(0)] = 2\delta^4(x) j_0^3(x). \quad (3)$$

We now define the quantity

$$G = \int e^{ik \cdot x} \delta(x_0) \langle p | [j_0^+(x), j_0^-(0)] | p \rangle d^4x, \quad (4)$$

which is equal to the eigenvalue of $2V_z$ of the target. Inserting a sum over intermediate states n , we arrive at

$$G = \sum_n \int d^3p_n [\delta^3(\mathbf{k} + \mathbf{p} - \mathbf{p}_n) |\langle p | j_0^+(0) | n \rangle|^2 - \delta^3(\mathbf{k} + \mathbf{p}_n - \mathbf{p}) |\langle p | j_0^-(0) | n \rangle|^2]. \quad (5)$$

We choose $\mathbf{k} \cdot \mathbf{p} = 0$. We note that if we choose as $|p\rangle$ a π^+ state, then the $\bar{K}^0$ transition is given by the first term of G and therefore will be positive. All n states with $\bar{K}^0$ quantum numbers will give a positive contribution, whereas negative contributions will come only from states with the $K^+\pi^+$ quantum numbers. We assume that the latter transitions are negligible compared to the former. Defining

$$\langle \pi^+, p | j_\mu^+ | \bar{K}^0, q \rangle = (4E_\pi E_K)^{-1/2} [(q_\mu + p_\mu) f_+(t) + (q_\mu - p_\mu) f_-(t)], \quad t = (q - p)^2 \quad (6)$$

we arrive at the inequality

$$(4E_\pi E_K)^{-1/2} [(E_K + E_\pi) f_+(t) + (E_K - E_\pi) f_-(t)] \leq 1, \quad (7)$$

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¹ H. R. Quinn and J. D. Bjorken, Phys. Rev. **171**, 1660 (1968).

² M. Suzuki, Phys. Rev. (to be published).