TABLE II. Noble gas pressures needed to ensure various degrees of mixing. The degree of mixing is defined as the fraction of the high-pressure limit of the D_2/D_1 ratio observed at a given buffer gas pressure, and under conditions of pure D_1 excitation.

Collision partners	Noble gas pressures (in Torr) needed to ensure 65, 80, and 95% mixing 65%	80%	95%
Na-He	7	16	75
Na-Ne	15	32	152
Na-Ar	10	21	99
Na-Kr	10	23	107
Na-Xe	12	26	125
K-He	4	8	32
K-Ne	24	51	215
K-Ar	12	26	108

their particular need for accurate wave functions of the alkali–noble gas quasimolecule.[18]

The present data can be used to predict an upper limit to the degree of mixing between hyperfine structure magnetic sublevels in those optical pumping experiments[8] in which an alkali vapor is exposed to resonance radiation while immersed in a noble gas. We elect to define the "degree of mixing" as the fraction of the high-pressure limit of the D_2/D_1 ratio observed at a given buffer gas pressure, and under conditions of pure D_1 excitation. The high-pressure limit is just twice the Boltzmann factor and appears as $2K$ in Eq. (2). Table II lists the buffer gas pressure required for various degrees of mixing. The fact that relatively high pressures are required to achieve 95% mixing simply reflects the algebraic rather than exponential nature of Eq. (2).

[18] W. R. Thorson (private communication).

ACKNOWLEDGMENTS

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Intensity-Correlation Spectroscopy*

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A survey is given of techniques for spectroscopic analysis using intensity fluctuations. Particular attention is given to counting times, the role of macroscopic sources and detectors, and the electronic constraints placed on the observations.

I. INTRODUCTION

VARIOUS techniques have been suggested in the past few years for applying the study of intensity fluctuations to spectroscopic analysis. An excellent review of these has been given by Wolf and by Glauber.[1] We have recently provided a quantum-mechanical description[2] of intensity correlations in connection with a method for measuring the phase of a scattering amplitude in x-ray scattering. We shall here apply this quantum-mechanical analysis to several of the proposed spectroscopic techniques. We have in mind particularly the observation of the shape and width of a single spectral line. Although the relevant machinery was completely discussed in Ref. 2, we shall utilize some notational simplifications which have been developed in some later work.[3,4]

We shall consider measurements of intensity fluctuations and time correlations in detectors at separate space points. The classical theory of these is described

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[1] E. Wolf, Proceedings of the I.C.O. Conference on Photographic and Spectroscopic Optics, Tokyo, Japan, 1964, J. Appl. Phys. (Japan) (to be published); R. J. Glauber, in *Quantum Optics and Electronics*, edited by C. DeWitt *et al.* (Gordon and Breach Science Publishers, Inc., New York, 1965).

[2] M. L. Goldberger, H. W. Lewis, and K. M. Watson, Phys. Rev. **132**, 2764 (1963). This paper will be referred to as I.

[3] M. L. Goldberger and K. M. Watson, Phys. Rev. **137**, B1396 (1965). This paper will be referred to as II.

[4] M. L. Goldberger and K. M. Watson, Phys. Rev. **140**, B500 (1965). This paper will be referred to as III.

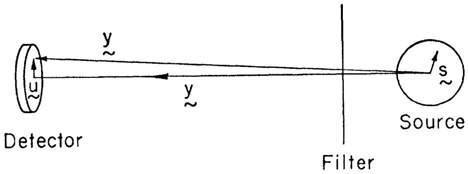


FIG. 1. Schematic illustration of photon counting.

by Born and Wolf.[5] The study of fluctuations in connection with spectroscopy has been reviewed by Mandel.[6] The use of space correlations is essentially the technique of Hanbury-Brown and Twiss.[7] A related method involving interference of Fourier components in a nonlinear device has been suggested by Forrester.[8]

In Sec. II we review the general features of the problem, paying particular attention to the effect of macroscopic sources and detectors and to electronic limitations. The presentation will be reasonably self-contained, but will not include the derivation of some basic formulas which were given in I, II, and III. Specific applications will be discussed in detail in Secs. III and IV. In Sec. V we describe the use of lenses and other optical instruments in such experiments.

II. THE OBSERVATION OF INTENSITY CORRELATIONS

In this section we review those results of II and III of relevance to the present study. Our discussion will hopefully be sufficiently complete that reading papers I, II, and III is not necessary unless missing derivations are desired.

We consider a quasicoherent source,[9] S, of optical radiation, as illustrated in Fig. 1. Light from the source is detected by a photon counter D after passing through a filter which restricts the radiation to an angular frequency interval $\Delta\omega_B$ at a frequency ω_0. We suppose that

$$\Delta\omega_B \ll \omega_0. \tag{2.1}$$

The source-detector separation is described by a vector $\mathbf{Y}$ from a fixed point in the source to a fixed point in the detector. Arbitrary points in source and detector are designated by vectors $\mathbf{s}$ and $\mathbf{u}$, respectively, measured from the fixed reference points (see Fig. 1). The linear dimensions of the source (detector) are characterized by the parameter $L_s(L_d)$ while the corresponding areas are written as Σ_s and Σ_d. We imagine that source and detector have small angular apertures in the sense that

$$L_s/Y \ll 1, \quad L_d/Y \ll 1. \tag{2.2}$$

The photon flux (number of photons/cm^2 sec) at a point $\mathbf{y}=\mathbf{Y}+\mathbf{u}$ at a point in the detector is

$$F(\mathbf{y}) = R_B/4\pi y^2, \tag{2.3a}$$

where R_B is the equivalent isotropic source intensity. The corresponding differential flux at frequency ω, in $d\omega$, is

$$dF = F(\mathbf{y})g(\omega)d\omega, \tag{2.3b}$$

where the spectral function $g(\omega)$ is normalized to unity:

$$\int d\omega\, g(\omega) = 1. \tag{2.4}$$

The spectral width of the source $\Delta\omega_B$ is defined in terms of g by

$$\frac{1}{\Delta\omega_B} = \int d\omega [g(\omega)]^2. \tag{2.5}$$

[The definition of $\Delta\omega_B$ is somewhat arbitrary; for a Lorentz shape

$$g(\omega) = (\Gamma/2\pi)[(\omega-\omega_0)^2+\Gamma^2/4]^{-1}, \quad \Delta\omega_B = \Gamma\pi].$$

Following the notation of our earlier papers, we represent the detector [called detector 1 since we shall shortly introduce a second detector 2] by the counting-rate operator at time T:

$$G_1(T_1) = \int_{-\infty}^{\infty} dt_1\, L_1(T_1-t_1) \int_1 d^3y_1\, \gamma_1(\mathbf{y}_1) \times \sum_{l=1}^{n} e^{iK_l t_1} \delta(\mathbf{y}_1 - \mathbf{x}_l) e^{-iK_l t_1}. \tag{2.6}$$

Here the sum on l runs over the n photons emitted by the source during the time interval T of a given observation. The quantity $\mathbf{x}_l$ is the space coordinate of the lth photon, and K_l is its kinetic-energy operator. The integral on $\mathbf{y}_1$ runs over the volume of detector 1. We shall assume that γ_1, a factor taking into account the efficiency and calibration of the counter, is a constant. Finally, L_1 is the transient response function of the counter, which we write as

$$L_1(\tau) = \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} B_1(\Omega) e^{-i\Omega\tau}, \qquad L_1(\tau)=0, \quad \text{for} \quad \tau<0. \tag{2.7}$$

A characteristic response time $\Delta\tau_r$ for the detector is defined by the expression

$$\frac{1}{\Delta\tau_r} = \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} |B_1(\Omega)|^2. \tag{2.8}$$

[5] M. Born and E. Wolf, *Principles of Optics* (The Macmillan Company, Inc., New York, 1964), 2nd ed.
[6] L. Mandel, in *Symposium on Electromagnetic Theory and Antennas* (Pergamon Press, Inc., Oxford, England, 1963).
[7] R. Hanbury-Brown and R. Q. Twiss, Phil. Mag. **45**, 663 (1954); Proc. Roy. Soc. (London) **243A**, 291 (1957).
[8] A. T. Forrester, J. Opt. Soc. Am. **51**, 253 (1961).
[9] We used the term "incoherent" in II and III to describe what is often called "quasicoherent" radiation in optics. In this paper we revert to the more conventional notation.

[For a simple RC filter, where

$$L(\tau)=(\exp(-\tau/RC))/RC, \quad \Delta\tau_r=2RC.]$$

The wave function at time t for the n-photon system is [See Eq. (2.1) of II]

$$\psi(t)=\mathcal{S}\prod_{i=1}^{n}\Phi_i(\mathbf{x}_i,t), \tag{2.9}$$

where Φ_i is that for the ith photon. The symbol $\mathcal{S}$ means to take the symmetrized product of the Φ's. As in I and II, we are interested in the ensemble average of many observations, each conducted for a time interval T. We suppose that on performing the ensemble average, the Φ_i have random phases and are effectively orthogonal. Mean beam properties such as the photon flux are considered to remain constant during the interval T. There are some delicacies associated with a coordinate space representation of photons which we shall not go into here. They are of no quantitative significance.

The mean rate of counting photons is then

$$\langle G_1\rangle=\langle(\psi(0),G_1(T_1)\psi(0))\rangle, \tag{2.10}$$

where $\langle\cdots\rangle$ denotes the ensemble average. By assumption this rate is independent of T_1 and has the form [see[10] Eq. (2.15), III]

$$\langle G_1\rangle=B_1(0)\Sigma_1\eta_1F(\mathbf{Y}_1). \tag{2.11}$$

Here Σ_1 is the area of the active detector volume and η_1 is the detector efficiency. Actually Eq. (2.11) is just a definition of η_1 since all of the other factors must enter into the counting rate. In our previous papers we assumed either

$$B_1(0)=1, \tag{2.12a}$$

or

$$B_1(0)=0, \tag{2.12b}$$

corresponding to placing a dc blocking filter in the detector output. The latter choice is convenient when discussing fluctuation experiments, so it is worthwhile to define the mean counting rate in the absence of a blocking filter, namely,

$$\langle G_1\rangle_0=\langle G_1\rangle/B_1(0). \tag{2.13}$$

An explicit evaluation of the counting rate, Eq. (2.10), in terms of the wave function of the system, Eq. (2.9), yields

$$\langle G_1\rangle=B_1(0)\Sigma_1w_1\gamma_1\bar{n}\chi(1), \tag{2.14}$$

where

$$\bar{n}=\langle n\rangle, \tag{2.15}$$

w_1 is the detector thickness, and [see Eq. (2.19), III]

$$\begin{aligned}\chi(1)&=\langle(\Phi_i^*(\mathbf{y}_1,t_1)\Phi_i(\mathbf{y}_1,t_1))\rangle\\&=R_B/4\pi c y_1^2,\end{aligned} \tag{2.16}$$

with c the velocity of light. The point $\mathbf{y}_1$ may be taken anywhere in the detector volume because we have assumed that $\chi(1)$ is constant over the detector and also independent of the time t_1 in deducing Eq. (2.14). By comparing our two counting rate expressions, Eqs. (2.11) and (2.14), we complete the definition of the efficiency η_1 or, as we prefer to use it, γ_1:

$$\gamma_1=(c/w_1)\eta_1. \tag{2.17}$$

Although the counter thickness w_1 does not enter into our results in a critical way, it is worthwhile saying a little about it. Since our counting rate operator G_1 defined by Eq. (2.6) does not take into account the stopping of photons in the detector, we interpret w_1 as a measure of the depth of penetration of the photons into the counter, assuming this to be less than the actual counter thickness.

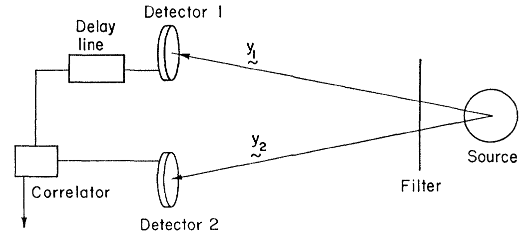


FIG. 2. An intensity correlation experiment.

We turn now to the description of an intensity correlation experiment, schematically illustrated in Fig. 2. Here we have added a second detector, referred to as 2. This will be described by a counting-rate operator, Eq. (2.6), etc., but distinguished by a subscript 2. In a correlation experiment, both detectors are used simultaneously to count photons from the source. We imagine the instantaneous output from detector 1 to be fed into a delay line and then mixed with that from 2 in a correlator which multiplies the two outputs. The correlator output in turn is represented by the operator

$$G_{12}(\tau)=G_{12}(T_2,T_1)=G_2(T_2)G_1(T_1). \tag{2.18}$$

Here $\tau=T_2-T_1$ is the delay deliberately introduced by our delay line. In writing Eq. (2.18) we are tacitly assuming that the counting operators G_1 and G_2 commute. This is not rigorously true, but this particular quantum-mechanical effect does not lead to quantitatively important corrections. A precise formulation of the theory of correlated counting rates is given in an earlier paper.[3]

A special case of the experiment just described is that in which a single detector is used. In this case we imagine that the detectors 1 and 2 referred to in Eq. (2.18) coalesce into one. To do such an experiment, one might split the detector output into two equal signals, pass one through a delay line, and then mix them in a correlator. [A specific example will be discussed in Sec. IV.] Formally we may go from the general two-detector analysis to the single-detector case by equating the subscripts 1 and 2 at an appropriate point.

[10] We use the notation Eq. (2.15), III to indicate Eq. (2.15) of Paper III, etc.

If the correlator in Fig. 2 were a simple square-law device and if the signals were added linearly ahead of it, the relevant quantity for our intensity correlation experiment would become

$$G_{SL}(T_2,T_1)=[G_1(T_1)+G_2(T_2)]^2$$
$$=[G_1(T_1)]^2+[G_2(T_2)]^2+2G_{12}(T_2,T_1). \quad (2.19)$$

Evidently all of the terms in Eq. (2.19) may be obtained from suitable specialization of $G_{12}(T_2,T_1)$, for example by setting 2 equal to 1 and getting G_1^2.

For subsequent order-of-magnitude estimates we shall feel free to set

$$\begin{aligned}\Sigma_1&\approx\Sigma_2\approx\Sigma_d,\\ w_1&\approx w_2\approx w,\\ B_1&\approx B_2\approx B,\\ Y_1&\approx Y_2\approx Y,\end{aligned} \quad (2.20)$$

although in practice this is entirely unnecessary.

The average correlator output during an interval T, as obtained in I and II, in the notation of II,[11] is

$$\langle G_{12}(\tau)=\langle G_1\rangle\langle G_2\rangle+\tfrac{1}{2}\bar{n}^2\int(1)\int(2)|\chi(12)|^2. \quad (2.21)$$

Here we have written

$$(1)=\int_{-\infty}^{\infty}dt_1L_1(T_1-t_1)\int_1 d^3y_1\gamma_1\cdots, \quad (2.22)$$

and similarly for 2, and [compare Eq. (2.16)]

$$\chi(12)=\langle\Phi_i^*(\mathbf{y}_1,t_1)\Phi_i(\mathbf{y}_2,t_2)\rangle$$
$$=\frac{R_B}{4\pi c\bar{n}}\int_S\frac{d^3s}{V_sD_1(\mathbf{s})D_2(\mathbf{s}_2)}\int d\omega\, g(\omega)$$
$$\times\exp\{i\omega[(1/c)(D_2(\mathbf{s})-D_1(\mathbf{s}))-(t_2-t_1)]\}, \quad (2.23)$$

where

$$\begin{aligned}\mathbf{D}_1(\mathbf{s})&=\mathbf{y}_1-\mathbf{s},\\ \mathbf{D}_2(\mathbf{s})&=\mathbf{y}_2-\mathbf{s},\end{aligned} \quad (2.24)$$

and the integral over source points extends over the source volume V_s.

For analytical (and presumably practical) convenience we shall assume that the experimental geometry is so chosen that [here $\lambda=2\pi c/\omega_0$, and strictly speaking $|Y_1-Y_2|$ should be replaced by $\max|y_1-y_2|$]

$$\Sigma_s|Y_1-Y_2|/\lambda Y^2\ll1, \quad (2.25a)$$

and

$$(\Delta\omega_B/c)\theta_d(\Sigma_s)^{1/2}\ll1, \quad (2.25b)$$

where θ_d is the angular spacing of the two detectors as seen from the source (or simply the angular size $(\Sigma_d)^{1/2}/Y$ in the case of a single detector). It is also true that except in oscillating exponentials the replacement

$$D_1\approx Y_1,\quad D_2\approx Y_2, \quad (2.25c)$$

is harmless.

It follows from the conditions (2.25) that the fundamental quantity $\chi(12)$ defined by Eq. (2.23) may be split into a purely geometrical factor and one which depends intrinsically on the beam spectral function $g(\omega)$. [See Eqs. (2.26), III for further discussion.] We find

$$\chi(12)\cong\chi_p(12)Q(12), \quad (2.26a)$$

where

$$\chi_p(12)=\frac{R_B}{4\pi c\bar{n}Y_1Y_2}\int d\omega\, g(\omega)$$
$$\times\exp\left\{i\omega\left[\frac{1}{c}(y_2-y_1)-(t_2-t_1)\right]\right\}, \quad (2.26b)$$

and

$$Q(12)=\int_S\frac{d^3s}{V_s}\exp\left\{i\frac{\omega_0}{c}(\hat{y}_1-\hat{y}_2)\cdot\mathbf{s}\right\}. \quad (2.26c)$$

We may now express the average correlator output $\langle G_{12}(\tau)\rangle$, Eq. (2.21), in the form

$$\langle G_{12}\rangle=\langle G_1\rangle\langle G_2\rangle+\tfrac{1}{2}\bar{n}^2I_s\int(1)\int(2)|\chi_p(12)|^2, \quad (2.27)$$

where [see Eq. (2.31), III]

$$I_s=\int_1\frac{d^2v_1}{\Sigma_1}\int_2\frac{d^2v_2}{\Sigma_2}|Q(12)|^2. \quad (2.28a)$$

Here $\mathbf{v}_1$ is the projection of $\mathbf{u}_1=\mathbf{y}_1-\mathbf{Y}_1$ on a plane perpendicular to $\mathbf{Y}_1$, etc. I_s is a function of the dimensionless quantity $\sigma=Y^2\lambda^2/\Sigma_s\Sigma_d$ (taking $Y_1\approx Y_2\approx Y$ here) and has the limiting values

$$I=1,\qquad \sigma\gg1, \quad (2.28b)$$
$$=Y^2\lambda^2/\Sigma_s\Sigma_d,\quad \sigma\ll1. \quad (2.28c)$$

We shall henceforth assume that $\sigma\ll1$, so that the limit Eq. (2.28c) applies.

It will be convenient to assume in what follows that we put a dc blocking filter in the detector outputs which means

$$B_1(0)=B_2(0)=0, \quad (2.29)$$

so that $\langle G_1\rangle=\langle G_2\rangle=0$ [see Eqs. (2.11) and (2.13)]. Then

$$\langle G_{12}\rangle=\langle\Delta G_{12}\rangle_pI_s, \quad (2.30a)$$

where [as given in Eq. (2.30 III)]

$$\langle G_{12}\rangle_p=\tfrac{1}{2}\langle G_1\rangle_0\langle G_2\rangle_0I_c, \quad (2.30b)$$

$$I_c=\int_1\frac{d^3y_1}{\Sigma_1w_1}\int_2\frac{d^3y_2}{\Sigma_2w_2}\int d\omega\int d\omega'$$
$$\times g(\omega)g(\omega')B_1(\omega'-\omega)B_2(\omega-\omega')$$
$$\times\exp\{i(\omega-\omega')[(1/c)(y_2-y_1)-(T_2-T_1)]\}. \quad (2.31)$$

[11] See Eq. (2.37), II.

It should be possible and it is desirable to design sufficiently thin detectors, well enough aligned, so that we may set $y_2-y_1=Y_2-Y_1$ in the exponential of Eq. (2.31). The precise tolerances involved here clearly depend on both the electronic and spectral bandwidths, but they do not appear too severe. We shall assume in what follows that it is legitimate to write, in place of Eq. (2.31),

$$I_c=\int d\omega\int d\omega' g(\omega)g(\omega')B_1(\omega'-\omega)B_2(\omega-\omega')$$
$$\times\exp\left\{i(\omega-\omega')\left[\frac{1}{c}(Y_2-Y_1)-(T_2-T_1)\right]\right\}. \quad (2.30c)$$

We note in passing that the average value of the square-law correlator output, $G_{\rm SL}$ is obtained from Eqs. (2.30) in the form

$$\langle G_{\rm SL}\rangle=\tfrac{1}{2}I_s\{\langle G_1\rangle_0{}^2I_{c1}+\langle G_2\rangle_0{}^2I_{c2}+2\langle G_1\rangle_0\langle G_2\rangle_0I_c\}, \quad (2.32)$$

where

$$I_{c1}=\int d\omega\int d\omega' g(\omega)g(\omega')|B_1(\omega'-\omega)|^2,$$
$$I_{c2}=\int d\omega\int d\omega' g(\omega)g(\omega')|B_2(\omega'-\omega)|^2. \quad (2.33)$$

The signal-to-noise ratio is of vital importance in analyzing a correlation experiment of the sort under consideration. To discuss this we first define, as in III, the quantity

$$G_{\rm av}(\tau)=\int_0^T dT_1G_{12}(T_1+\tau, T_1). \quad (2.34)$$

Thus,

$$\langle G_{\rm av}(\tau)\rangle=T\langle G_{12}(\tau)\rangle. \quad (2.35)$$

The fluctuations in $G_{\rm av}$ have been computed in III[12] from

$$\langle G_{\rm av}{}^2\rangle=\langle(\psi(0),G_{\rm av}{}^2(\tau)\psi(0))\rangle. \quad (2.36)$$

The result obtained there for the large source case, $\sigma\ll1$, Eq. (2.28c), is

$$\langle G_{\rm av}{}^2\rangle-\langle G_{\rm av}\rangle^2=T\langle G_1\rangle_0\langle G_2\rangle_0M, \quad (2.37)$$

where

$$M=\int\frac{d\Omega}{2\pi}|B_1(\Omega)|^2|B_2(\Omega)|^2\left[\frac{\sin(\Omega w/2c)}{\Omega w/2c}\right]^4, \quad (2.38)$$

and we have set $B_1(0)=B_2(0)=0$ according to Eq. (2.29). Under the conditions that our previous replacement of y_2-y_1 by Y_2-Y_1 [i.e., going from Eq. (2.31) by Eq. (2.30c)] is justified we can take for M,

$$M\sim1/\Delta\tau_r, \quad (2.39)$$

where $\Delta\tau_r$ is the detector response time. Then we find

[12] The quantity $\langle G_{\rm av}{}^2\rangle$ was given in I for the limit of "narrow-band electronics."

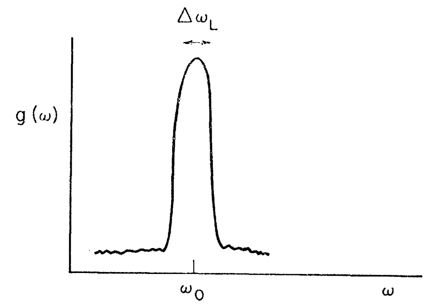


FIG. 3. Spectral function for a single line.

for the signal-to-noise ratio

$$S/N=\langle G_{\rm av}(\tau)\rangle/\{\langle G_{\rm av}{}^2\rangle-\langle G_{\rm av}\rangle^2\}^{1/2}$$
$$=\tfrac{1}{2}I_cI_s\{\Delta\tau_rT\langle G_1\rangle_0\langle G_2\rangle_0\}^{1/2}. \quad (2.40)$$

III. THE BROAD-BAND (bb) LIMIT

Let us suppose that $g(\omega)$ describes a spectral line of width $\Delta\omega_L$ at frequency ω_0 superimposed on a background of low intensity, as is illustrated in Fig. 3. Since we are interested in measuring the line shape we, of course, assume that $\Delta\omega_L$ is less than $\Delta\omega_B$, the frequency band passed by the filter. In this section we are concerned with the limiting case

$$\Delta\tau_r\Delta\omega_L\ll1, \quad (3.1)$$

corresponding to the bandwidth ahead of the correlator being broader than the spectral linewidth. This is the best situation for tracing out the line shape, but one may in practice have to be content with $\Delta\tau_r\Delta\omega_L\sim1$.

The band-pass characteristic $B_1\approx B_2\approx B$ is illustrated schematically in Fig. 4. We have again taken $B(0)=0$. We suppose[13] $B\cong1$ in the interval $\Delta\Omega_l\lesssim\Omega\lesssim(\Delta\tau_r)^{-1}$, where we assume

$$\Delta\Omega_l\ll\Delta\omega_L. \quad (3.2)$$

Let us first consider the case that the filter is so chosen that $\Delta\omega_B\approx\Delta\omega_L$. Then we obtain from Eq. (2.30c)

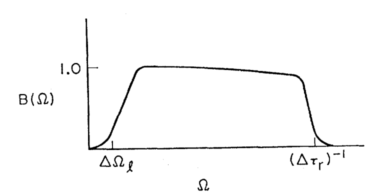
$$I_c=I_{c\rm bb}(P)\equiv\left|\int d\omega\, g(\omega)e^{i\omega P}\right|^2+O(\Delta\Omega_l/\Delta\omega_L), \quad (3.3a)$$

where

$$P\equiv(1/c)(Y_2-Y_1)-(T_2-T_1). \quad (3.4)$$

Because of the condition (3.2) we shall drop the terms.

FIG. 4. Electronic response function.



[13] The actual scale factor by which B should be multiplied is irrelevant.

of order $\Delta\Omega_r/\Delta\omega_L$. We may also write I_c as

$$I_{\text{cbb}}(P)=\left|\int d\omega\, g(\omega)e^{i(\omega-\omega_0)P}\right|^2, \qquad (3.3\text{b})$$

where ω_0, the central line frequency, is defined by

$$\omega_0=\int d\omega\, \omega g(\omega). \qquad (3.5)$$

In the measurement of the autocorrelation function with a single detector [where P reduces to $-(T_2-T_1)$] or the use of two detectors, the measured quantity, in fact, is $I_{\text{cbb}}(P)$. Unfortunately, an observation of $I_{\text{cbb}}(P)$ is not sufficient to determine the spectral function $g(\omega)$ uniquely, since the phase of the integral over $g(\omega)$ is unspecified. This "phase problem" arises in a number of contexts, most notably in x-ray structure analysis. It has been discussed in the present context by Wolf.[14] It was argued in I that the observation of I_{cbb} can be used to deduce a finite set of $g(\omega)$. It is possible that the correct one of these can be found from physical considerations, such as the non-negative character of $g(\omega)$. This seems to be usually the case in x-ray structure analysis. On the other hand, there are a number of features of the line that are independent of the phase question, and are therefore best suited to an initial exploration of intensity interferometry. For example, the second moment of the line is determinable from the dependence of the correlation function on P for small P, as illustrated by

$$\begin{aligned}&(dI_{\text{cbb}}/dP)\,|_{P=0}=0,\\ &(d^2I_{\text{cbb}}/dP^2)\,|_{P=0}=-2\int d\omega(\omega-\omega_0)^2 g(\omega),\end{aligned} \qquad (3.6\text{a})$$

where we recall the previous definition of ω_0, Eq. (3.5).

A probably useful example can be discussed, in which a collision-broadened line is further Doppler broadened in the center. Such a line may be observed in the emissions from a hot plasma; we can simulate its shape (for a narrow line) by

$$g(\omega)=\frac{\Gamma\alpha}{2\pi^{3/2}}\int\frac{e^{-\alpha^2\epsilon^2}}{(\omega-\omega_0-\epsilon)^2+\Gamma^2/4}d\epsilon, \qquad (3.6\text{b})$$

where

$$\alpha^2=Mc^2/2\omega_0^2kT$$

is the Doppler-broadening parameter. For this shape, according to Eq. (3.3b), the correlation function is

$$I_{\text{cbb}}(P)=e^{-\Gamma P-P/^2 2\alpha^2}, \qquad (3.6\text{c})$$

so that both the Lorentzian parameter P and the Doppler parameter α are directly determined by a measurement of the correlation function.

It is sometimes convenient to write $I_{\text{cbb}}(P)$ as a Fourier integral, in which case we have

$$I_{\text{cbb}}(P)=\int_{-\infty}^{\infty} d\omega\, \mathcal{G}(\omega)e^{i\omega P},$$

with

$$\mathcal{G}(\omega)=\int_0^{\infty} d\omega' g(\omega'+\omega)g(\omega').$$

It is easy to see that if $g(\omega)$ is concentrated in a line of width $\Delta\omega_L$, $G(\omega)$ has practically zero amplitude outside the interval $-2\Delta\omega_L<\omega<+2\Delta\omega_L$. It is this feature that makes intensity-correlation experiments less sensitive to the geometrical alignment problems than are classical interferometric techniques.

The all-important signal-to-noise ratio may be obtained from our general expression, Eq. (2.40). We use [from Eqs. (2.11), (2.3a), and (2.8c)]

$$\langle G_1\rangle_0\approx\langle G_2\rangle_0\approx\eta\Sigma_d R_B/4\pi Y^2,$$
$$I_s=Y^2\lambda^2/\Sigma_s\Sigma_0,$$

and also set $I_c\approx 1$. We find

$$S/N\cong(S/N)_{\text{bb}}\equiv\tfrac{1}{2}\eta(T\Delta\tau_r)^{1/2}(\lambda^2R_B/4\pi\Sigma_s). \qquad (3.7\text{a})$$

This expression may appear surprising, since it does not depend on the source-detector distance Y or on the detector area Σ_D. The reason is that we have assumed the limit $\sigma\ll 1$ in Eq. (2.28c). For large enough Y, $I_s\cong 1$ and S/N becomes

$$S/N=\tfrac{1}{2}\eta(T\Delta\tau_r)^{1/2}(R_B\Sigma_D/4\pi Y^2). \qquad (3.7\text{b})$$

It is clear that to maximize the ratio S/N one should choose $\Delta\tau_r$ as large as possible consistent with the restriction $\Delta\tau_r\Delta\omega_L\ll 1$. Had we considered the case $\Delta\tau_r\Delta\omega_L\gg 1$ we should have found that S/N was reduced by a factor $(\Delta\tau_r\Delta\omega_L)^{-1}$, so that the maximum signal-to-noise ratio is obtained for $\Delta\tau_r\Delta\omega_1\approx 1$.

For a source with black-body (BB) intensity on the spectral line of frequency ω_0 and temperature θ we find from (3.7a)

$$\begin{aligned}(S/N)_{\text{BB}}=(\Delta\omega_L/2\pi)\eta(T\Delta\tau_r)^{1/2}&\\ \times\{\exp[\hbar\omega_0/\theta]-1\}^{-1}.&\end{aligned} \qquad (3.8)$$

As another example, let us assume the mercury-arc source of Forrester *et al.*[15] We take $\Delta\tau_r=10^{-10}$ sec, $R_B/4\pi\Sigma=2\times 10^{15}$ photons/cm^2 sec, $\lambda=5.48\times 10^{-5}$ cm, and obtain

$$S/N\approx 50\eta\sqrt{T}, \qquad (3.9)$$

where T is measured in seconds.

Up to this point we have assumed that $\Delta\omega_B\approx\Delta\omega_L$. Another case of interest is that where the electronics is still fast insofar as the line is concerned [i.e., $\Delta\tau_r\Delta\omega_B\ll 1$] but $\Delta\omega_B$ is so broad that

$$\Delta\tau_r\Delta\omega_B\gg 1. \qquad (3.10)$$

[14] E. Wolf, Proc. Phys. Soc. (London) **80**, 1269 (1962).

[15] A. T. Forrester, R. A. Gudmundsen, and P. O. Johnson, Phys. Rev. **99**, 1691 (1955).

We now write

$$g(\omega)=g_L(\omega)+g_c(\omega), \tag{3.11}$$

where g_L represents the line spectrum and g_c the continuous background contribution passed by the filter. The spectral width of g_c is $\Delta\omega_B$. We suppose the line to be much more intense than the background.

Our basic quantity I_c, Eq. (2.30c), involving both the electronics and source characteristics, becomes

$$I_c=I_{cL}+I_{cc}, \tag{3.12}$$

where

$$I_{cL}=\left|\int d\omega\, g_L(\omega)e^{i\omega P}\right|^2, \tag{3.13}$$

and

$$I_{cc}=\int d\omega\int d\omega' g_L(\omega)g_c(\omega')\times B_1(\omega'-\omega)B_2(\omega'\omega')e^{i(\omega-\omega')P}+\text{c.c.} \tag{3.14}$$

Making use of our assumptions about $\Delta\tau_r\Delta\omega_L$ and $\Delta\tau_r\Delta\omega_B$, Eqs. (3.1) and (3.10), we have, approximately,

$$I_{cc}\cong\left[\int d\omega\, g_L(\omega)e^{i\omega P}\right]\int d\omega' g_c(\omega')\times B_1(\omega'-\omega_0)B_2(\omega_0-\omega')e^{-i\omega' P}+\text{c.c.}$$

$$\approx(f/\Delta\tau_r\Delta\omega_B)\left[\int d\omega\, g_L(\omega)e^{i\omega P}\right]+\text{c.c.}, \tag{3.15}$$

where f is the [small] ratio of continuum to line intensity. Thus to the extent that $f/\Delta\tau_r\Delta\omega_B\ll 1$,

$$I_c\cong I_{c1}, \tag{3.16}$$

and the background gives a negligible contribution to the observation.

The condition on the electronic resolving time imposed by the requirement $\Delta\tau_r\Delta\omega_B\ll 1$ is a severe one. If no gain is required between the detectors and the correlator, wave guide or coaxial line couplings might be used to achieve $\Delta\tau_r$ as small as 10^{-11} sec. If gain is required, there are available photodetectors followed by traveling wave amplifiers having bandwidths of about 10^{10} cps.[16] We conclude that with "conventional electronic techniques" the method described in this section is restricted to the analysis of linewidths not much broader than

$$\Delta\omega_L/2\pi\sim 10^{10}\text{ cps}.$$

IV. THE NARROW-BAND LIMIT

Let us suppose that a single photoelectric detector is followed by a tuned circuit and then by a square-law detector, as illustrated in Fig. 5. The two detector situations may be similarly analyzed. The function $B_1(\Omega)=B_2(\Omega)\equiv B(\Omega)$ will peak at the resonance frequency Ω_0 and will be taken to have a bandwidth $\delta\Omega$. We suppose that $\delta\Omega$ is very much less than either $\Delta\omega_L$ or $\Delta\omega_B$. In this case we set $Y_1=Y_2$ and $T_1=T_2$ in the expression for I_c, Eq. (2.30c), which becomes

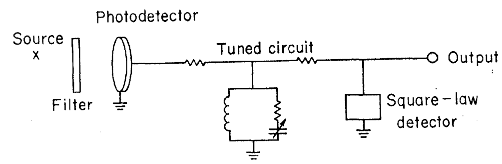


FIG. 5. Use of a tuned circuit in counting photons.

$$I_c=\int d\omega\int d\omega' g(\omega)g(\omega')B(\omega'-\omega)B_2(\omega-\omega')$$

$$=\int d\omega\int d\Omega\, g(\omega+\tfrac{1}{2}\Omega)g(\omega-\tfrac{1}{2}\Omega)|B(\Omega)|^2$$

$$\cong\left[\int d\omega\, g(\omega+\tfrac{1}{2}\Omega_0)g(\omega-\tfrac{1}{2}\Omega_0)\right]\int d\Omega|B(\Omega)|^2$$

$$\equiv\mathcal{G}(\Omega_0)(\Delta\tau_r)^{-1}, \tag{4.1}$$

where we have introduced the previously defined function $\mathcal{G}(\Omega_0)$, and our old definition of the resolving time $(\Delta\tau_r)^{-1}$, Eq. (2.8). We expect that $(\Delta\tau_r)^{-1}\sim\delta\Omega$.

The function $\mathcal{G}(\Omega_0)$ can thus be measured by varying the frequency Ω_0 of the tuned circuit.[17] As we have noted, $\mathcal{G}(\Omega_0)$ is just the Fourier transform of $I_{cbb}(P)$ so that measurement of $\mathcal{G}$ is in principle equivalent to measuring I_{cbb} [see equations following 3.6a)].

The signal-to-noise ratio is again obtained from Eq. (2.40) but now with I_c given by (4.1). For macroscopic sources and detectors [i.e., $I=\sigma\ll 1$] we have

$$S/N=(S/N)_{bb}[\mathcal{G}(\Omega_0)/\Delta\tau_r], \tag{4.2}$$

where $(S/N)_{bb}$ is the broad-band ratio given by Eq. (3.7a).

In conducting the experiment described in this section, one might use a resonant cavity to provide the tuned circuit illustrated in Fig. 5. Both the photo detector and the square-law detector would then be coupled to the cavity. By such means it seems feasible to study linewidths up to 10^{11} cps. The choice of $\delta\Omega$ will depend on the precision with which it is desired to measure $\mathcal{G}(\Omega_0)$ and on the acceptable counting times. Since $\mathcal{G}(\Omega_0)$ has a width of the order $\Delta\omega_B$ and $g(\omega)$ has magnitude $\sim(\Delta\omega_B)^{-1}$, $\mathcal{G}(\Omega_0)\sim(\Delta\omega_B)^{-1}$ [recall that $\mathcal{G}=\int d\omega\, g(\omega+\Omega_0)g(\omega)$] and also $\Delta\tau_r\sim 1/\delta\Omega$, so we may

[16] See, for example, D. E. Caddes and B. J. McMurtry, Electronics **37**, 125 (1964), for a review of wide-bandwidth light demodulators.

[17] The observation of $H(\Omega_0)$ has been suggested by L. Mandel, in *Electromagnetic Theory and Antennas*, edited by E. C. Jordan (The Macmillan Company, Inc., New York, 1963), Part 2, p. 811. A related suggestion has been made by Forrester, Ref. 8.

write (4.2) roughly as

$$S/N \sim (S/N)_{\rm bb}(\delta\Omega/\Delta\omega_B). \tag{4.3}$$

V. USE OF SUPPLEMENTARY OPTICAL INSTRUMENTS

Such optical devices as half-silvered mirrors, lenses, and diffraction gratings may be inserted between source and detectors, as may be convenient, in intensity-correlation experiments. To take account of these we need only replace $\chi(12)$, as defined by Eqs. (2.26), by

$$\chi(12)=\frac{R_B}{4\pi c\bar{n}Y_1Y_2}\int_S\frac{d^3s}{V_s}\int d\omega\, g(\omega) \times\exp\left\{i\omega\left[\frac{1}{c}(V_2-V_1)-(t_2-t_1)\right]\right\}, \tag{5.1}$$

where [here $\mu(\mathbf{y})$ is the refractive index at point $\mathbf{y}$]

$$V_1=\int_s^{\mathbf{y}_1}\mu(\mathbf{x})dx, \tag{5.2}$$

etc., is the optical-path-length integral (eikonal)[18] taken along the ray path leading from point $\mathbf{s}$ in the source to point $\mathbf{y}_1$ in detector 1. The appropriate distances Y_1 and Y_2 in Eq. (5.1) may be deduced from the photon intensity at the detectors, or from an analysis of the geometry used [in principle these are given by the eikonal treatment].

Let us write V_1^0 and V_2j for the respective values of V_1 and V_2 when the point $\mathbf{s}$ is chosen to be $\mathbf{s}=0$, the fixed reference point in the source. Then for a source of small aperture we have

$$\begin{aligned} V_1&\cong -s\cdot\mathbf{y}_1^0+V_1^0,\\ V_2&\cong -s\cdot\mathbf{y}_2^0+V_2^0, \end{aligned} \tag{5.3}$$

where $\mathbf{y}_1^0$ and $\mathbf{y}_2^0$ are the respective directions of those ray paths from $\mathbf{s}=0$ to the points $\mathbf{y}_1$ and $\mathbf{y}_2$. This permits us to write, as in Eqs. (2.26),

$$\chi(12)=\chi_p(12)Q(12), \tag{5.4a}$$

$$\chi_p(12)=\frac{R_B}{4\pi c\bar{n}Y_1Y_2}\int d\omega\, g(\omega) \times\exp\left\{i\omega\left[\frac{1}{c}(V_2^0-V_1^0)-(t_2-t_1)\right]\right\}, \tag{5.4b}$$

$$Q(12)=\int_S\frac{d^3s}{V_s}\exp\left[i\frac{\omega_0}{c}(\hat{y}_1^0-\hat{y}_2^0)\cdot\mathbf{s}\right]. \tag{5.4c}$$

[18] See, for example, Ref. 5, p. 109, or Steven Weinberg, Phys. Rev. **126**, 1899 (1962), for a very general discussion of the eikonal treatment.

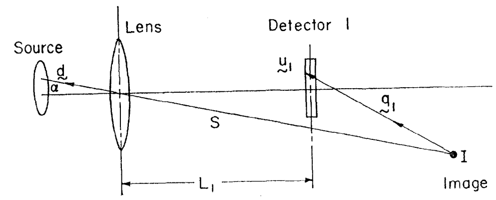


FIG. 6. Illustration of the use of an optical system.

On interpreting Σ_s and Σ_d as "effective areas" defined by the ray paths and on replacing Eq. (3.4) by

$$P\equiv(1/c)(V_2^0-V_1^0)-(T_2-T_1), \tag{5.5}$$

we see that the discussion given in Secs. II, III, and IV is unchanged, except for detail.

We illustrate this with the example shown in Fig. 6. An ideal lens is placed between the source and the two thin detectors, with the source near the focal point of the lens. A point on the source is a distance $\mathbf{d}$ from the center of the lens. A point on detector 1 is at $\mathbf{L}_1+\mathbf{u}_1$, where $\mathbf{L}_1$ is the vector from the center of the lens to a fixed point on detector 1. An image of the source point $\mathbf{d}$ is at I, a distance S from the lens center. The phase of a wave arriving at $\mathbf{u}_1$ from $\mathbf{d}$ is $(\omega/c)V_1$, where

$$V_1=(\mu-1)H\sec\alpha+(d+S)-q_1. \tag{5.6}$$

Here μ is the refractive index and H is the thickness of the lens at its center, α is the angle between $\mathbf{d}$ and the direction of $(-\mathbf{L}_1)$, and q_1 is the distance from the image to u_1. Assuming that S is very large and that the source and detectors are small, we obtain again Eqs. (2.30) for the correlated counting rate, but with Y replaced by the focal length of the lens in Eqs. (2.28).

A different arrangement is to focus the source on a single detector. In this case we obtain, instead of Eq. (5.1),

$$\chi(12)=\frac{R_B}{4\pi c\bar{n}Y_1Y_2}\int_S\frac{d^3s}{V_s}\int d\omega\, g(\omega)e^{i\omega(t_1-t_2)} \times\left[\frac{J_1((\omega D/2c)\sin\alpha_1)}{(\omega D/2c)\sin\alpha_1}\right]\left[\frac{J_1((\omega D/2c)\sin\alpha_2)}{(\omega D/2c)\sin\alpha_2}\right]. \tag{5.7}$$

Here D is the lens diameter and α_1 and α_2 are the angles formed at the lens center between the image of the point $\mathbf{s}$ and the respective points $\mathbf{u}_1$ and $\mathbf{u}_2$ on the detector. In this case it is the area of the lens, rather than that of the detector, which appears in Eqs. (2.28).