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CONSISTENCY QUESTIONS RAISED BY SIMULTANEOUS MANDELSTAM AND ANGULAR-MOMENTUM ANALYTICITY*

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As the result of a study to determine a Regge-pole formula with Mandelstam analyticity for the elastic scattering of two unequal-mass particles,[1] we were led to raise the following question: What are the constraints, if any, that follow from assuming that scattering amplitudes satisfy both the Mandelstam representation and the condition of meromorphism in the right-half angular momentum plane? To put it another way, are Mandelstam and l-plane analyticity necessarily consistent in every case? We find that if there is to be consistency, one can conclude directly that the high-energy limit of the Regge-pole position, $\alpha(\infty)$, is necessarily negative. One also discovers in the unequal-mass problem some surprising asymptotic requirements on the form and size of the Regge "background term," which have apparently been heretofore unnoticed. Whether these latter requirements are consistent or not depends upon the value of α at zero total energy. Considerations of the type we now discuss may possibly be of importance in gaining a deeper understanding of analytic properties in the angular momentum plane or in detecting subtle deviations from the Mandelstam representation.

We sketch here the basic argument and refer the reader to a forthcoming paper for more details.[1] We consider a scattering amplitude $A(s,t)$, with the usual variables, and the corresponding partial-wave amplitude $a(s,l)$. It is assumed for simplicity that $A(s,t)$ has only an s-t double spectral function. The amplitude $a(s,l)$ is assumed to be a meromorphic function of l in a region that includes $\mathrm{Re}\,l > -\frac{1}{2}+\epsilon$ where $0<\epsilon<\frac{1}{2}$. We now explore the consequences of these two assumptions.

Using the Mandelstam version[2] of the Regge-Sommerfeld-Watson representation, we may write

$$A(s,t)=B(s,t)+\sum_i A_R^{\,i}(s,t),$$

$$A_R^{\,i}=\gamma_i(s)\nu^{\alpha_i(\nu)}Q_{-1-\alpha_i(\nu)}(-1-t/2\nu), \qquad (1)$$

where ν is the square of the center-of-mass momentum (and, hence, determined by s) and

$$\gamma_i(s)\nu^{\alpha_i(\nu)} = [2\alpha_i(\nu)+1]\beta_i(\nu)/\cos\pi\alpha_i(\nu),$$

where $\beta_i(\nu)$ is the actual residue of the pole at $l=\alpha_i(\nu)$. If we include in the summation of Eq. (1) all Regge poles that reach the region $\mathrm{Re}\,l > -\frac{1}{2}+\epsilon$ for any s above threshold, s_0, the background term $B(s,t)$ will have the property

$$B(s,t) \leq \text{const}\; t^{-1/2+\epsilon} \quad (t\to\infty) \tag{2}$$

for all $s>s_0$.

Now the heart of our development comes by imposing upon (1) the requirement of Mandelstam analyticity. Neither $B(s,t)$ nor the individual pole terms have the correct analyticity, so we require a cancellation between the background and pole terms to produce the desired result (such a cancellation is known to occur in potential scattering). Our approach, roughly speaking, is the following: We consider a given pole term in (1) and correct its analyticity in order to bring it into conformity with the Mandelstam representation. The correction terms can be evaluated explicitly in terms of α and γ, and the consistency requirement imposed by simultaneous l-plane and Mandelstam analyticity is that the correction terms be bounded by $t^{-1/2+\epsilon}$ as $t\to\infty$ at least for $s>s_0$ [that is, that they be of background size and, hence, cancellable by $B(s,t)$].

The simplest way to correct the analyticity of a Regge-pole term $A_\mathrm{R}(s,t)$, in (1) is to evaluate the absorptive part in the t channel, $D_t(t,s)$, for $\nu<0$ and $t\gg 0$. From Eq. (1) we deduce (suppressing the summation)

$$D_t(t,s) = \mathrm{Im}A_\mathrm{R}(s,t)$$
$$= -\gamma(\nu)(-\nu)^\alpha \sin\pi\alpha Q_{-1-\alpha}(-1-t/2\nu);$$
$$\nu<0, \quad t\gg 0. \tag{3}$$

We see that $D_t(t,s)$ has a cut from $\nu=-t/4$ to $\nu=-\infty$ which is at variance with the Mandelstam representation. We remove this cut and also remove a wedge of D_t for $\nu>0$ in order to instate the correct double-spectral-function boundary. The corrected absorptive part $\tilde{D}_t$ can then be written

$$\tilde{D}_t(s,t) = D_t(s,t) - \frac{1}{2}\int_{-\infty}^{-\frac{1}{4}t}\frac{d\nu'}{\nu'-\nu}\gamma(s')(-\nu')^\alpha \sin\pi\alpha(s')P\left(-1-\frac{t}{2\nu'}\right) - \frac{1}{\pi}\int_0^{d/(t-t_0)}\frac{d\nu'}{\nu'-\nu}\,\mathrm{Im}D_t(t,s), \tag{4}$$

where d is a constant and t_0 is the correct t threshold. Equation (4) explicitly assumes equal masses with $\nu=(s/4)-M^2$. We shall give the corresponding formula for unequal masses in a moment. A Regge formula having Mandelstam analyticity can now be written

$$R(s,t) = \frac{1}{\pi}\int_{t_0}^{\infty}\frac{dt'}{t'-t}D_t(t',s). \tag{5}$$

Our consistency condition now requires that $R(s,t)-A_\mathrm{R}(s,t)$ be bounded by $t^{-1/2+\epsilon}$ for asymptotic t. To check this, it is easy to see that to within terms of order t^{-1}, the difference $R-A_\mathrm{R}$ is asymptotically of the same order as $\tilde{D}_t(t,s)-D_t(t,s)$. From Eq. (4) we conclude by direct calculation

$$\tilde{D}_t(t,s) - D_t(t,s) \sim c_1 t^{\alpha(\infty)} + c_2 t^{-3/2} \quad (t\to\infty), \tag{6}$$

where c_1 and c_2 are independent of t. For consistency, we must have $\alpha(\infty) < -\frac{1}{2}+\epsilon$ and thus necessarily $\alpha(\infty)<0$. This means that for certain trajectories of appropriate signature, such as the Pomeranchuk trajectory, there will be ghost states.

In the case of unequal masses, a correction term in addition to the two given in Eq. (6) is required. The reason is kinematical, since for unequal masses, $\nu=[s-(M-\mu)^2][s-(M+\mu)^2]/4s$. This introduces into $D_t(t,s)$ [Eq. (3)] an additional spurious cut from $s=0$ to $s=r^2/u$ ($r=M^2-\mu^2$) which must be removed. The new correction term has the form

$$\text{const}\int_{r^2/(\Sigma-s-t)}^{0}\frac{ds'}{s'-s}\gamma(s')(-\nu')^\alpha \times \sin\pi\alpha(s')P_\alpha\left(-1-\frac{t}{2\nu'}\right),$$

where $\Sigma=2(M^2+\mu^2)$. As $t\to\infty$ this new term is proportional to $t^{\alpha(0)-1}$. Again in line with our consistency argument, this term, being a correction to the Q term in (3) required to bring about the correct Mandelstam analyticity, must be background size. That is,

$$\alpha(0)-1 < -\tfrac{1}{2}+\epsilon,$$
$$\alpha(0) < \tfrac{1}{2}+\epsilon. \tag{7}$$

To summarize, if the scattering amplitude obeys the Mandelstam representation and is meromorphic in the l plane for $\mathrm{Re}\,l > -\frac{1}{2}+\epsilon$, we deduce

$$\alpha(\infty) < -\tfrac{1}{2}+\epsilon,$$

and, in addition, for the unequal-mass problem

$$\alpha(0) < \tfrac{1}{2}+\epsilon.$$

If either of these constraints is untrue, we have a basic contradiction. It could be that cuts in the angular-momentum plane, which we have ignored, will alter the above conclusions. We note that if the Pomeranchuk pole can be constituted in an unequal-mass scattering channel, we have the disagreeable feature that $\alpha_P(0) < \frac{1}{2}+\epsilon$ or $\alpha_P(0) < 1$ if $\epsilon < \frac{1}{2}$. We have no further insight on these points, but we wish to draw attention to the issues which these seemingly straightforward calculations raise.

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Y^*'s WITH SPIN $\frac{7}{2}$†

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Several years ago, Cook et al. measured the total K^-p cross section from 1 to 4 BeV/c K^- lab momentum and observed a broad, low bump centered at about 1.6 BeV/c (total c.m. energy 2065 MeV).[1] Blanpied et al.,[2] Böck et al.,[3] and Eberhard and Shively[4] have observed bumps in invariant-mass distributions which may indicate existence of Y^*'s with masses above 1900 MeV, but statistical accuracy of and/or agreement among the experiments is poor. Recently, Cool et al. measured very accurately the total K^-p and K^-d cross sections from 1.0 to 2.45 BeV/c (1794 to 2411 MeV) and found evidence for a number of Y^*'s.[5] Among them are an isotopic-spin $I=1$ $Y^*(\approx 2030)$, which we have previously reported,[6] and an $I=0$ $Y^*(\approx 2100)$, for which we here give independent evidence. In addition, we determine the $Y_1^*(2030)$ to have spin and parity $J^P=\frac{7}{2}^+$ and the $Y_0^*(\approx 2100)$ to have $J^P=\frac{7}{2}^-$. Table I gives our results for the $J=\frac{7}{2}$ resonances.

The 72-inch hydrogen bubble chamber was exposed at the Bevatron to a K^- beam with momenta 1.22, 1.42, 1.51, 1.60, and 1.70 BeV/c. We found about 30 000 events consisting of a disappearing beam track and an associated V-like charged decay of a neutral particle. After kinematic analysis and imposition of other selection criteria, 8408 $\bar{K}^0$ events ($K_1^0\to\pi^++\pi^-$) and 14 173 Λ events ($\Lambda\to p+\pi^-$) remained. About half the former are $K^-+p\to\bar{K}^0+n$ (charge-exchange) events; about half the latter are $K^-+p\to\Lambda+\pi^0$ events. Cross sections and production and (for the $\Lambda\pi^0$ events) polarization angular distributions were measured. Table II gives the cross sections. Experimental details and a more complete discussion of results than can be given here will appear in an Article.

The scattering amplitudes for the two reactions in terms of pure isospin amplitudes are

$$A(K^-+p\to\bar{K}^0+n)=(A_{\bar{K}N}{}^1-A_{\bar{K}N}{}^0)/2, \tag{1a}$$

$$A(K^-+p\to\Lambda+\pi^0)=A_{\Lambda\pi}{}^1/\sqrt{2}, \tag{1b}$$

where superscripts give isospin. Differential cross sections were expanded in a Legendre polynomial series

$$\frac{d\sigma}{d\Omega}=c\lambda\!\!\!^{-2}\sum_{n=0}^{n_{\max}}a_nP_n(\cos\theta), \tag{2}$$

where $\lambda\!\!\!^-$ is $\hbar/q$ (q is the K^- c.m. momentum), θ is the c.m. scattering angle between mesons, and c is the square of the numerical factor in Eqs. (1): $\frac{1}{4}$ for $K^-+p\to\bar{K}^0+n$, $\frac{1}{2}$ for $K^-+p\to\Lambda+\pi^0$. The equation $\sigma=4\pi c\lambda\!\!\!^{-2}a_0$ relates σ

Table I. Properties of the $J=\frac{7}{2}$ resonances.

I,J^P	ω_0 (MeV)	Γ (MeV)	$x_{\bar{K}N}$	$x_{\Lambda\pi}$
$1,\frac{7}{2}^+$	2030	170	0.25	0.16
$0,\frac{7}{2}^-$	2120	145	0.25	...