Self-Consistent Determination of Coupling Shifts in Broken $SU(3)$*

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The possibility that certain patterns of $SU(3)$ symmetry breaking are dynamically enhanced in baryon-meson couplings is studied by bootstrap methods. For the strong couplings, a single dominant enhancement is found. It produces very large symmetry-breaking terms, transforming like an octet, as often conjectured. Experimental consequences are listed, such as a reduction of K-baryon couplings relative to π-baryon couplings which is in accord with the experimental weakness of K relative to π production in many circumstances, such as photoproduction and multi-BeV cosmic-ray collisions. For parity-violating nonleptonic couplings, a dominant octet enhancement is again found, as mentioned in a previous paper, which leads to an excellent fit with experiment. For parity-conserving nonleptonic couplings, on the other hand, several different enhancements compete, and the only conclusion we can draw is that terms with the "abnormal" transformation properties brought in by strong symmetry-breaking corrections are present. Our work provides a dynamical derivation of various phenomenological facts associated with $SU(6)$, such as the dominance of the **35** representation in parity-violating nonleptonic decays.

INTRODUCTION

RECENTLY, a "bootstrap" or self-consistent theory of the deviations from $SU(3)$ symmetry has been proposed.[1,2] This theory has previously been successfully applied[3] to the strong and electromagnetic mass splittings within the baryon $\frac{1}{2}^+$ octet and the $\frac{3}{2}^+$ decuplet. We wish to report here on further applications to the strong, electromagnetic, and weak $BB\Pi$ and $B\Delta\Pi$ coupling shifts, stressing those results which are of direct experimental interest.

Linear deviations of masses (δM) and couplings (δg) from their $SU(3)$ symmetric values are in part determined, in a bootstrap theory, by requirements of self-consistency.[1,2] They may also depend on driving terms (such as photon exchange in electromagnetic shifts). One is led then to equations of the form

$$\begin{aligned}\delta M_i/M_i&=\sum_j A_{ij}{}^{MM}\delta M_j/M_j+\sum_j A_{ij}{}^{Mg}\delta g_j+D_i{}^M,\\ \delta g_i&=\sum_j A_{ij}{}^{gM}\delta M_j/M_j+\sum_j A_{ij}{}^{gg}\delta g_j+D_i{}^g.\end{aligned}\tag{1}$$

In these equations, A_{ij} is $SU(3)$-symmetric, the violations of $SU(3)$ being contained in the factors δM, δg, and D. If A has an eigenvalue near unity, then the mass and coupling shifts associated with the eigenvector of this eigenvalue are enhanced. Since A is independent of the perturbation, the enhanced eigenvector can apply universally to strong, electromagnetic, and weak symmetry breaking, and corresponds to symmetry violations with definite $SU(3)$ transformation properties (hopefully octet).

A specific model is required in order to evaluate the A matrix. We employ the same model as in Ref. 3: a Chew-Low type of $SU(3)$-symmetric reciprocal bootstrap model for $B(\frac{1}{2}^+)$ and $\Delta(\frac{3}{2}^+)$ baryon supermultiplets with B and Δ exchange. The input parameters are the experimental average masses of the B and Δ supermultiplets, and the D/F ratio and strong $BB\Pi/\Delta B\Pi$ coupling ratio taken from the model. All input parameters are determined solely by strong-interaction $SU(3)$-symmetric considerations, and are held fixed in all subsequent calculations of symmetry breaking. Arguments are given in Refs. 3 and 4 as to why the truncated A matrix obtained here may provide a reasonable estimate of the eigenvalues of the whole A matrix.

We have calculated with this model the A matrix coefficients of Eq. (1) which couple together the B and Δ mass shifts and the $B\Delta\Pi$ and $BB\Pi$ coupling shifts. The results give the ratios of various coupling shifts, but not their over-all strength, which is determined by the hard-to-treat driving terms. The rather complicated details of calculating the A matrix, for which we used the S-matrix perturbation theory described in Refs. 5 and 6, will be discussed elsewhere.[4]

The A matrix is not symmetric. In particular, a detailed analysis of Eq. (1) shows[4] that the submatrix A^{Mg} is very small compared to the elements of the other three submatrices. As a result, eigenvalues of A^{MM} and A^{gg} are to a very good approximation eigenvalues of the entire A matrix. This circumstance provides a justification for a previous study of the mass splittings in the B and Δ supermultiplets,[3] based on an analysis of the submatrix A^{MM}. The results of that analysis were that A^{MM} had no eigenvalue near one for a **27** violation of

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[1] R. Cutkosky and P. Tarjanne, Phys. Rev. **132**, 1355 (1963).

[2] R. Dashen and S. Frautschi, Phys. Rev. Letters **13**, 497 (1964).

[3] R. Dashen and S. Frautschi, Phys. Rev. **137**, B1331 (1965).

[4] R. Dashen, Y. Dothan, S. Frautschi, and D. Sharp (to be published).

[5] R. Dashen and S. Frautschi, Phys. Rev. **135**, B1190 (1964).

[6] R. Dashen and S. Frautschi, Phys. Rev. **137**, B1318 (1965).

TABLE I. $B\Delta\Pi$ couplings. The experimental g^2 were calculated from the widths Γ, compiled in Ref. 9, and the relation (see Ref. 10) $g^2=\Gamma(\Delta\to B+\pi)\bar{M}_\Delta{}^2/2q^3$.

$B\Delta\Pi$ coupling	Experimental g^2	$SU(3)$-symmetric g^2	$(g+x\delta g)^2$
$N^*N\pi$	10.7 ± 0.5	12.2	16.8
$Y_1{}^*\Lambda\pi$	6.1 ± 0.3	6.1 (input)	6.1 (input)
$Y_1{}^*\Sigma\pi$	1.7 ± 1.7	4.1	1.7
$\Xi^*\Xi\pi$	2.6 ± 0.6	6.1	2.6 (input)

$SU(3)$, while A^{MM} had a single eigenvalue near one for octet violations. The predicted mass ratios were in good agreement with experiment. What we have now done is to extend the calculation so as to find the eigenvalues and eigenvectors of the parts of the A matrix involving the couplings, A^{gg} and A^{gM}.

We consider A^{gg} first. Using the model and input parameters described above yields the following results: (i) The eigenvalue of A^{gg} closest to one is ≈0.93 and corresponds to an octet violation of $SU(3)$. This number is very insensitive to variations of the F/D ratio[7] of the $SU(3)$ symmetric $BB\Pi$ coupling.[8] (ii) There are a number of other eigenvalues fairly close to one. For octet violations of $SU(3)$, the eigenvalues next closest to one are ≈0.66 and 0.49. Furthermore, there is an eigenvalue close to one for **27** violations of $SU(3)$; it is ≈0.88.

The effect of these eigenvalues on the coupling shifts is influenced by the behavior of A^{gM}. To understand this, we consider the 2×2 schematic problem for octet violations of $SU(3)$:

$$\begin{pmatrix}\delta M/M\\ \delta g\end{pmatrix}=\begin{pmatrix}A^{MM} & 0\\ A^{gM} & A^{gg}\end{pmatrix}\begin{pmatrix}\delta M/M\\ \delta g\end{pmatrix}+\begin{pmatrix}D^M\\ D^g\end{pmatrix}. \quad (2)$$

Solution of this problem gives

$$\delta M/M=D^M/(1-A^{MM});$$
$$\delta g=[A^{gM}/(1-A^{gg})]\delta M/M+D^g/(1-A^{gg}). \quad (3)$$

We have found that the element of A^{gM} connecting the leading octet mass eigenvector to the leading octet eigenvector (i.e., the one whose eigenvalue is ≈0.93) is large (≈1.0). Since both A^{MM} and A^{gg} have eigenvalues near one, one sees from the above equations that the coupling shift lying along the leading octet eigenvector A^{gg} will receive a *double enhancement*, leading to large coupling shifts. The same methods applied to the other eigenvalues of A^{gg} near one indicate that they will not benefit from double enhancement. In the case of the octet eigenvectors, the corresponding component of $|A^{gM}|$ is down from ≈1.0 to ≈0.2, while in the **27** violation case there is no mass shift to feed the coupling shift. So the double enhancement appears in this case to favor the leading octet eigenvector uniquely, and we will study the effect of this eigenvector on the pattern of coupling shifts.

STRONG-COUPLING SHIFTS

Our predictions for the medium-strong coupling shifts have the form $g+x\delta\hat{g}$ where g is the $SU(3)$-symmetric coupling, $\delta\hat{g}$ is an arbitrarily normalized octet eigenvector of A^{gg} corresponding to the eigenvalue 0.93, and x is an over-all strength parameter.

The parameter x can be fixed in either of two ways: (a) One can fit to the ratio of any two $\frac{3}{2}^+$ resonance decay widths. Of these, the decay $Y_1{}^*\to\Sigma\pi$ is too poorly known experimentally. The N^* decay width is well-known experimentally, but the static model we are using does not reproduce its shape well and thus does not give an accurate estimate for its width. So we choose the ratio $\Gamma(Y_1{}^*\to\Lambda\pi)/\Gamma(\Xi^*\to\Xi\pi)$. (b) Alternatively, we can use Eq. (3) to estimate the magnitude of the coupling shifts. We suppose that D^g is small compared to the enhanced δM's, for which quantities we take experimental values. For $(1-A^{gg})^{-1}$ and A^{gM} we use our calculated values.

The two estimates of x are in rough agreement. Neither estimate is very good, but (b) is especially uncertain because the quantity $(1-A^{gg})^{-1}$ is so sensitive to the precise magnitude of the eigenvalue of A^{gg} which is near one. For this reason, we used x from the ratio $\Gamma(Y_1{}^*\to\Lambda\pi)/\Gamma(\Xi^*\to\Xi\pi)$ in predicting the other coupling shifts.

Our results for the resonance decays are displayed in Table I, where they are compared with the $SU(3)$ symmetric results and with experiment.[9,10]

Our principal results for the whole set of medium strong $BB\Pi$ and $B\Delta\Pi$ coupling shifts are summarized in Table II. These results support the following main conclusions:

(i) The "medium strong" coupling shifts are *very large*, often near 100% of the $SU(3)$ symmetric values of the couplings. This fact indicates that it is not consistent in an approximate dynamical calculation to include mass shifts while omitting coupling shifts.

(ii) We have found that although there are large changes in the magnitudes of the couplings, the couplings do not change sign.

[7] We have varied F/D between 0.3 and 0.6 which covers the usual range of theoretical and experimental determinations of this quantity. The eigenvalue changed from 0.92 to 0.94.

[8] The results presented here were calculated using linear approximations to the D functions, but we have checked that our results are insensitive to changes in the curvature of the D function within reasonable bounds.

[9] A. Rosenfeld, A. Barbaro-Galtieri, W. Barkas, P. Bastien, J. King, and M. Roos, Rev. Mod. Phys. **36**, 977 (1964).

[10] This relation is obtained by comparing the $\frac{3}{2}^+$ scattering amplitude $T=\bar{M}_\Delta{}^2q^{-3}e^{i\delta}\sin\delta$ (appropriate for the static crossing relations employed in our calculation) with its resonance form $T\approx-g^2(W-M_\Delta-i\Gamma/2)^{-1}$. The theoretical overestimate of the N^* width is not unexpected since the static model always greatly overestimates the width of the high-energy tail of this resonance; such energy-dependent effects may be less for the other, considerably narrower, members of the decimet.

(iii) A very interesting qualitative rule is apparent. The breaking of $SU(3)$ *raises* the coupling strengths for *low-lying channels*, and *decreases* the coupling strengths to *high-mass channels*. This result tends to support the neglect of high-mass channels in approximate dynamical calculations, even though they might appear to enter in an important way from $SU(3)$ symmetry considerations. In this connection, we note that the pattern of coupling shifts is determined by the eigenvalue of A^{qq} near one. The structure of A^{qq} itself is determined by the exchange forces, and does not contain any direct information about the thresholds. Therefore, it is rather remarkable that the coupling shifts nevertheless follow the pattern of the thresholds.

(iv) The strange-particle components of the nucleon wave function are reduced, with various experimental consequences: (a) In high-energy nucleon-nucleon collisions[11] from about 10 BeV to about 10^4 BeV, strange-particle production accounts for only $\approx 15\%$ of the secondaries. It is clear from Table II that in $SU(3)$ strange particles should amount to $\approx 30\%$ of the secondaries produced off nucleons, while our broken $SU(3)$ model would predict $\approx 17\%$. (b) From a comparison of the experimental cross section $d\sigma(\gamma+p\rightarrow K^++\Lambda^0)/d\Omega$ near threshold with a simple pole model for this cross section, it appears that the physical couplings $g_{N\Lambda K}^2$ and $g_{N\Sigma K}^2$ are smaller than their $SU(3)$ values by an order of magnitude.[12] This situation is improved if one includes the symmetry breaking, which materially reduces the values of the squares of the coupling constants.

(v) Note that the η has a very small coupling to all members of the baryon octet and decuplet, which helps explain why it is relatively hard to produce.

(vi) There is some experimental information on the $\Sigma\Sigma\pi$ and $\Sigma\Lambda\pi$ couplings. Their ratio has been deduced from rough dynamical arguments on hypernuclear binding energies and low-energy ΣN scattering.[13,14] It was found that $g_{\Sigma\Lambda\pi}/g_{\Sigma\Sigma\pi}\approx 3$, and this fact was used to deduce[13,14] a value of ≈ 0.2 for the F/D raio—somewhat smaller than that obtained from other considerations. With our corrected values of the couplings, a ratio of $g_{\Sigma\Lambda\pi}/g_{\Sigma\Sigma\pi}\approx 3$ corresponds to an F/D ratio somewhat nearer 0.3, which is slightly more in accord with other determinations.

ELECTROMAGNETIC COUPLING SHIFTS

Since the eigenvalue dominating both strong and electromagnetic shifts is independent of the "direction" taken by the symmetry violation in $SU(3)$ space, we can estimate that the same ratio $\delta g/\delta M$ holds for electromagnetic as for strong shifts. The theoretical value for octet shifts δg_{em} then follows from our experimental knowledge of baryon electromagnetic mass splittings.

There is no firm data on any electromagnetic shifts in $BB\Pi$ or $\Delta B\Pi$ couplings, but the isotopic-spin violations in $NN\pi$ couplings are relevant to possible violations of charge independence in nuclear physics, where one-pion exchange is an important part of the two-nucleon potential, and shifts in $N^*N\pi$ couplings are relevant to the recent experimental search for differences between $N^{*++}\rightarrow p+\pi^+$ and $N^{*-}\rightarrow n+\pi^-$.[15] We find for the $\Delta I=1$ couplings which participate in the octet $SU(3)$-breaking that $\delta g_{NN\pi}$ is extremely small,[16] and $\delta g_{N^*N\pi}$ is quite small—we estimate crudely $\Gamma(N^{*-}\rightarrow n+\pi^-)-\Gamma(N^{*++}\rightarrow p+\pi^+)\approx 1$ MeV.

NONLEPTONIC WEAK INTERACTIONS

In calculating symmetry-breaking, one always hopes that the simplest case will occur—a single eigenvalue dominating all types of symmetry breaking. This hope

TABLE II. $BB\Pi$ and $B\Delta\Pi$ couplings in broken $SU(3)$.[a]

Particle	Component	Threshold (MeV)	$SU(3)$ %	Broken $SU(3)$ %	Normalized total strength in broken $SU(3)$
$N^*(1240)$	$N\pi$	1080	50.0	90.9	1.00
	ΣK	1690	50.0	9.1	
$Y_1^*(1380)$	$\Lambda\pi$	1250	25.0	46.9	0.70
	$\Sigma\pi$	1330	16.7	13.1	
	$N\bar{K}$	1430	16.7	30.3	
	$\Sigma\eta$	1740	25.0	5.3	
	ΞK	1820	16.7	4.4	
$\Xi^*(1530)$	$\Xi\pi$	1460	25.0	30.0	0.47
	$\Lambda\bar{K}$	1610	25.0	46.7	
	$\Sigma\bar{K}$	1690	25.0	15.1	
	$\Xi\eta$	1870	25.0	8.2	
$\Omega^-(1680)$	$\Xi\bar{K}$	1820	100.0	100.0	0.28
$N(940)$	$N\pi$	1080	68.4	82.6	1.00
	$N\eta$	1490	0.1	0.1	
	ΛK	1610	18.7	10.9	
	ΣK	1690	12.8	6.4	
$\Lambda(1115)$	$\Sigma\pi$	1330	46.8	57.4	0.65
	$N\bar{K}$	1430	37.3	33.4	
	$\Lambda\eta$	1660	15.6	8.8	
	ΞK	1810	0.3	0.4	
$\Sigma(1190)$	$\Lambda\pi$	1250	15.6	30.6	0.41
	$\Sigma\pi$	1330	14.7	21.8	
	$N\bar{K}$	1430	8.5	10.5	
	$\Sigma\eta$	1740	15.6	7.5	
	ΞK	1810	45.6	29.6	
$\Xi(1320)$	$\Xi\pi$	1460	12.8	29.9	0.28
	$\Lambda\bar{K}$	1610	0.1	0.4	
	$\Sigma\bar{K}$	1680	68.4	64.3	
	$\Xi\eta$	1870	18.7	12.4	

[a] The entries are defined as follows: We write the $\Delta^\alpha B_j\Pi_k$ coupling as $g_{jk}^\alpha(\alpha=1,\cdots,10;\ j,k=1,\cdots,8)$. The total coupling strength for a given α is defined as $\sum_{jk}(g_{jk}^\alpha)^2$, while the percentage of the jk state is defined $(g_{jk}^\alpha)^2/\sum_{lm}(g_{lm}^\alpha)^2$. The total couplings and percentages for the $BB\Pi$ couplings are defined in an analogous manner. The total couplings are arbitrarily normalized to unity for the (3–3) resonance and the nucleon. The numbers in this Table correspond to an F/D ratio of 0.4.

[11] D. Perkins, in *Proceedings of the International Conference on Theoretical Aspects of Very High Energy Phenomena* (CERN, Geneva, 1961), p. 99.

[12] C. Peck (private communication).

[13] J. J. de Swart and C. Iddings, Phys. Rev. **128**, 2810 (1962); **130**, 319 (1963).

[14] R. H. Dalitz, Phys. Letters **5**, 53 (1963).

[15] G. Gidal, A. Kernan, and S. Kim, Phys. Rev. **141**, 1261 (1966).

[16] For a preliminary estimate of the $\Delta I=2$ corrections to $g_{NN\pi}$, involving a calculation of the driving term as well as the A matrix, see H. Goldberg, Nuovo Cimento **40**, 243 (1965).

was borne out in the study of mass shifts,[3] where a single eigenvalue of A^{MM} controlled both the strong and electromagnetic mass shifts. Furthermore, we have produced evidence in the present paper that a particular eigenvalue of A^{gg} (≈ 0.93) provides a near-instability which controls the pattern of $SU(3)$ symmetry breaking in strong and electromagnetic couplings. For weak interactions, however, we have found that a number of new near-instabilities come into play, compete with the instability that controlled the stronger interactions, and lead to a more complicated pattern of symmetry breaking.

A. Parity-Violating Coupling Shifts

One complication of the weak interactions, of course, is the presence of parity-violating couplings. A previous investigation of these couplings[17] was made by searching for nearly self-consistent parity-violating couplings in the reciprocal bootstrap model of B and Δ. For "normal" charge conjugation properties ($CP=+$, $\mathcal{C}=-$),[18] a clear dominance of a single octet eigenvector was found. Octet dominance ensured the $\Delta I=\frac{1}{2}$ rule and predicted the remaining ratios of decay amplitudes as well. These predictions are compared with experiment[19] in Table III; as one can see, the agreement is quite good.

B. Parity-Conserving Coupling Shifts

A second special feature of the weak interactions is that all the observed decays are strangeness-changing. This is important because to first order, the strangeness-changing weak interactions produce no physical mass shifts. Thus for parity-conserving coupling shifts, although we use the same A matrix as before,[20] the terms A^{MM}, A^{gM}, and A^{Mg} that involve mass now play no role, and the equation for δg simplifies to the form

$$\delta g = A^{gg}\delta g + D^g. \tag{4}$$

A consequence of this is that the leading octet eigenvector no longer receives a double enhancement, and one can expect that the relative influence of the other eigenvalues lying near one will be correspondingly greater. In the absence of information on the driving terms D^g, one can no longer make a unique prediction of δg.

We can, however, do phenomenology, trying linear combinations of the enhanced eigenvectors with various coefficients. Doing so, we find that no single one of the three leading octet eigenvectors (eigenvalues 0.93, 0.66, and 0.49) or the leading **27** eigenvector (eigenvalue 0.88) fits the data well.[21] More surprisingly, an arbitrary linear combination of the three leading octet eigenvectors still does not fit the data well.[22]

To escape this impasse, we recall a third peculiarity of the weak interactions. The octet part of parity-conserving, $CP=+$, weak interactions, for example, can transform either like the sixth component of an octet with $\mathcal{C}=+$ or the seventh component of an octet with $\mathcal{C}=-$.[18] The first case is the normal one derived from the current-current interaction in the absence of strong symmetry breaking, and is the one used in Cabibbo theory.[23] The second, abnormal case becomes possible when strong symmetry-breaking is included. Now there are at least two reasons why excluding the abnormal terms appears unjustified: (i) the strong symmetry breaking is not much weaker than the $SU(3)$ symmetric part of the strong interaction in either the observed pseudoscalar masses or in our theoretical results for $BB\Pi$ and $\Delta B\Pi$ couplings. (ii) In the absence of the abnormal term, $K_1 \to 2\pi$ is forbidden,[23] in flagrant contradiction with experiment. Thus we are motivated to consider possible "abnormal" instabilities by studying the A^{gg} matrix which relates couplings of this type. We find that this matrix has eigenvalues (Table IV) $A_8^{gg}=0.89, 0.82, \cdots, A_{27}^{gg}=0.83, \cdots$, so again several near-instabilities are available.

TABLE III. Comparison of theory (Ref. 17) and a particular fit to experiment (Refs. 9, 19) for nonleptonic, parity-violating hyperon decay amplitudes A, defined as in Stevenson *et al.* (Ref. 19).

Decay	Sign of theoretical amplitude	Theoretical $\lvert A\rvert\times10^{-4}$ $\mathrm{sec}^{-1}\, m_{\pi^-}^{-1}$	Experimental $\lvert A\rvert\times10^{-4}$ $\mathrm{sec}^{-1}\, m_{\pi^-}^{-1}$
Λ_0^0	$-$ (input)	2.1 (input)	2.1 ± 0.2
Λ_-^0	$+$	3.0	3.1 ± 0.1
Σ_+^+	$+$	0.0	0.1 ± 0.3
Σ_0^+	$+$	3.5	3.6 ± 0.35 ($\gamma_\Sigma>0$)
			1.7 ± 0.2 ($\gamma_\Sigma<0$)
Σ_-^-	$+$	4.9	3.9 ± 0.15
Ξ_0^0	$+$	3.1	3.1 ± 0.2
Ξ_-^-	$+$	4.4	4.1 ± 0.2

[17] R. Dashen, S. Frautschi, and D. Sharp, Phys. Rev. Letters **13**, 777 (1964).

[18] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964). For a self-charge-conjugate representation, $\mathcal{C}$ is equal to the charge-conjugation parity of the $I=0$, $Y=0$ member of the representation.

[19] M. Stevenson, J. Berge, J. Hubbard, G. Kalbfleisch, J. Shafer, F. Solmitz, S. Wojcicki, and P. Wholmut, Phys. Letters **9**, 349 (1964); R. Dalitz, 1964 Varenna lectures (to be published).

[20] Here we refer to couplings with "normal" charge-conjugation properties, $\mathcal{C}=+$. For couplings which have no strong-interaction analog, such as $\mathcal{C}=-$ couplings and parity-violating couplings, a new A^{gg} matrix must be calculated.

[21] The signs of the P-wave amplitudes to be compared with the theory were determined from experimental asymmetry parameters and the S-wave predictions of Table III. The only eigenvector that comes at all close to agreeing with these amplitudes is the leading octet eigenvector. It predicts too large a Σ_-^- amplitude, however, and predicts the wrong sign for the P-wave Ξ-decay amplitude relative to the Λ-decay amplitude.

[22] Taking arbitrary amounts of the three octet eigenvectors, one is left with the sum rule

$$\Sigma_+^+ = 1.8\Xi_-^- - 0.8\Sigma_0^+ - 0.4\Lambda_-.$$

Experimentally, the left side is -4.1 ± 0.1 and the right side is ≥ -1.9. Adding in the leading **27** eigenvector does not help since the largest effect of this eigenvector is a $\Delta I=\frac{3}{2}$ component in Λ decay, which is not observed.

[23] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).

The phenomenological situation for parity-conserving decays is now as follows. We consider all eigenvectors, both normal and abnormal, with eigenvalues greater than 0.5, as listed in Table IV. No individual eigenvector fits the data well. The only combinations of two eigenvectors which give a fair fit are the first normal octet eigenvector ($A^{gg}=0.93$) combined with either of the leading abnormal octet eigenvectors (eigenvalues 0.89 and 0.82). A linear combination of these three eigenvectors (about three parts 0.93 and three parts 0.82 to one part 0.89) yields an excellent fit. No other set of three eigenvectors can be made to fit well.

Putting parity-conserving and parity-violating effects together, we can summarize our understanding of the observed nonleptonic decays as follows:

(i) The squares of the effective couplings which enter into nonleptonic decays are enhanced compared to those entering into leptonic decays by a factor $(1-A^{gg})^{-2}$, which is of the order of 10 to 100 for the leading octet eigenvalues. This appears to be the case experimentally.[24]

(ii) As pointed out above, the parity-violating decays are in good shape. It is worth noting that we predict[17] the Lee sum rule[25] for these amplitudes. A further point is that five of the six predictions made for the ratios of these amplitudes hold for either $\mathcal{C}$[17]; thus the good agreement obtained in Table III persists even if strong symmetry breaking mixes in terms of abnormal $\mathcal{C}$.

(iii) A number of competing eigenvectors are found for parity-conserving decays, so that $\Delta I=\frac{1}{2}$ rule and ratios of the amplitudes cannot be predicted in this case. The only phenomenological fit, employing linear combinations of eigenvectors, that succeeds involves comparable amounts of "normal" and "abnormal" $\mathcal{C}$.

(iv) If we take account of the possibility of a sizable violation of CP, our conclusions are essentially unchanged. Five of the six predictions for parity-violating amplitudes are independent of any charge-conjugation consideration.[17] The parity-conserving amplitudes can still be analyzed in terms of $\mathcal{C}=+$ and $\mathcal{C}=-$, and we find once again that both types of $\mathcal{C}$ are needed.

Although we were unable to predict the parity-conserving amplitudes, the results of the phenomenological fits for these amplitudes are at least "reasonable" in the following senses:

(a) The large abnormal-$\mathcal{C}$ term would plausibly be introduced by the large deviations from $SU(3)$ symmetry in strong interactions, observed both in the pseudoscalar meson masses and in our theoretical results (Table II) for baryon couplings.

(b) The only successful fit employed the three octet eigenvalues lying nearest unity.

(c) On the other hand, we do not gain any understanding of why **27** terms are experimentally small; apparently this information is buried in the hard-to-treat driving terms.

It is interesting to consider the relation of our results to $SU(6)$. For the strong mass shifts and for the parity-violating decays, it has been found phenomenologically that symmetry violations transforming like the **35** representation of $SU(6)$ dominate. In each of these cases, our dynamical method produces a single enhanced eigenvector which predicts essentially the same results as **35**, thus providing a dynamical justification for **35** dominance. On the other hand, the assumption that parity-conserving nonleptonic decays transform like the **35** representation with normal $\mathcal{C}$ works less well.[26] We expect that this is due to the presence of several competing eigenvectors.[27]

TABLE IV. Eigenvalues of A^{gg}, for various types of parity-conserving couplings, which are 0.5 or greater.

Transformation property of coupling shift	Eigenvalues
8, $\mathcal{C}=+$	0.93, 0.66, 0.49
27, $\mathcal{C}=+$	0.88, 0.63
8, $\mathcal{C}=-$	0.89, 0.82
27, $\mathcal{C}=-$	0.83

COMPARISON WITH OTHER STUDIES

The most closely related previous treatment of coupling shifts was the study of strong $B\Delta\Pi$ coupling shifts by Wali and Warnock.[28] Essentially they estimated A^{gM} but not A^{gg}. Since the largest term in $A^{gM}dM$ feeds the same eigenvector that is favored by A^{gg}, their results obtained using only A^{gM} are in qualitative agreement with ours. After the present work was completed, an extension of their study to strong $BB\Pi$ coupling shifts[29] came to our attention.

In matters of principle, Ernst, Wali, and Warnock[29] stress two points common to all these studies:

(i) The approximations are not symmetric in the various channels and do not automatically guarantee relations like $g_{\Lambda N\bar{K}}^2=g_{N\Lambda K}^2$.

(ii) The shifts are so large that second-order effects should be included.

We have found that difficulty (i) is not so serious in our self-consistent treatment of symmetry breaking, in the sense that our leading eigenvector satisfies the necessary symmetry relations almost perfectly. We agree that

[24] R. Dashen, S. Frautschi, M. Gell-Mann, and Y. Hara, in *Proceedings of the International Conference on High Energy Physics, Dubna 1964* (Atomizdat, Moscow, 1965).

[25] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964).

[26] See, for example, G. Altarelli, F. Buccella, and R. Gatto, Phys. Letters **14**, 70 (1965).

[27] Another approach we can compare with is tadpole theory [S. Coleman and S. Glashow, Phys. Rev. **134**, B671 (1964)]. To produce the enhancements we have found in $\mathcal{C}=-$ parity-violating decays and in parity-conserving decays of both $\mathcal{C}$, the tadpole theory would require two 0^+ octets of opposite $\mathcal{C}$, and one 0^- octet of opposite $\mathcal{C}$ to the known $\pi K\eta$ octet.

[28] K. Wali and R. Warnock, Phys. Rev. **135**, B1358 (1964).

[29] F. Ernst, K. Wali, and R. Warnock, Phys. Rev. **141**, 1354 (1966).

second-order effects represent an important correction, though we feel that the basic pattern described in the present paper will continue to hold.

In another unpublished report just received, Diu, Rubinstein, and Van Royen[30] have studied the eigenvalues of A^{00} for $BB\Pi$ and $B\Delta\Pi$ coupling shifts. Their results for these eigenvalues are in complete agreement with ours.

[30] B. Diu, H. Rubinstein, and R. Van Royen (to be published).

Note added in proof. The entries in the last two columns of Table II in the unpublished version of this article contained a numerical error.

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S-Wave Baryon-Baryon Scattering in Broken $SU(3)$

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S-wave baryon-baryon scattering is considered in first-order broken $SU(3)$. One obtains nine sum rules for the various hyperon-nucleon amplitudes for scattering in the state $^1S_0$, and six for scattering in the state $^3S_1$. One of the sum rules for the state $^1S_0$ involves reactions with nucleon targets only, and can therefore be used for an experimental check. A few other sum rules involve only one reaction with a non-nucleon target and could be useful in estimating the latter.

I. INTRODUCTION

THE $SU(3)$ symmetry has been very successful in providing a useful classification scheme for hadrons.[1] It has also provided a basis for the correlation of various experimental data on these particles. This basis has either been in terms of the exact symmetry, or in terms of the symmetry broken in the first order in a particular fashion. The well-known example in the latter category is the Gell-Mann–Okubo mass formula. More recently, the baryon-decimet decay into the usual baryon octet and the pseudoscalar-meson octet has also been studied with broken $SU(3)$, and the results have been quite encouraging.[2]

On the other hand, the analysis of scattering processes has so far been mainly in the exact-symmetry limit, and one finds that the results do not generally agree with experiment.[3] This situation is, however, only to be expected, as the comparison with the results of the exact symmetry would be meaningful only if the mass differences between the members of various meson and baryon multiplets were quite small. As these mass differences are by no means small, it would be necessary to analyze scattering processes in the broken $SU(3)$ symmetry, if reasonable agreement with experiment is to be expected. But such analysis is rather involved, as the number of independent amplitudes in terms of which the various reactions (which are to be correlated) can be expressed becomes very large. For instance, consider the case of pseudoscalar-meson–baryon scattering. The number of independent amplitudes one needs in the exact symmetry is only seven. But if the symmetry is broken to first order, this number becomes 31. Consequently, it becomes extremely difficult to obtain sum rules between the various reactions involved.

In the case of baryon-baryon scattering, however, certain simplifications occur, especially if one restricts the discussion only to s waves.[4] One has to note that the two-baryon wave function must be antisymmetric under the combined interchange of space, spin and isotopic-spin coordinates; and in the context of $SU(3)$, the isotopic-spin wave function can be replaced by the

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[1] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964).

[2] V. Gupta and V. Singh, Phys. Rev. **135**, B1442 (1964); C. Becchi, E. Eberle, and G. Morpurgo, Phys. Rev. **136**, B808 (1964); M. Konuma and Y. Tomozawa, Phys. Letters **10**, 347 (1964). See also R. Acharya, Nuovo Cimento **35**, 108 (1965). This problem was discussed earlier by Shumeskevich's method by C. Dullemond, A. J. MacFarlane, and E. C. G. Sudarshan, Phys. Rev. Letters **10**, 423 (1963).

[3] For a review, and for other references, see H. Harari, Proceedings of the 1965 Trieste Summer School (to be published).

A sum rule in broken $SU(3)$ has been obtained and compared with experiment in S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **13**, 213 (1964). This sum rule connects reactions $\pi^-p\to N^{*-}\pi^+$, $\pi^-p\to Y_1^{*-}K^+$, $K^-p\to Y_1^{*-}\pi^+$ and $K^-p\to \Xi^{*-}K^+$. See also M. Konuma and Y. Tomazawa, *ibid.* **12**, 493 (1964). These authors obtain a sum rule connecting reactions initiated by π^-p and K^-p, the final states being the usual baryon–pseudoscalar-meson states and, of course, do not involve any baryon-decuplet resonances.

[4] P. D. DeSouza, G. A. Snow, and S. Meshkov, Phys. Rev. **135**, B565 (1964). The complications that arise if one considers higher angular momentum waves have been discussed in this reference. See also S. Iwao, Nuovo Cimento **33**, 455 (1964).