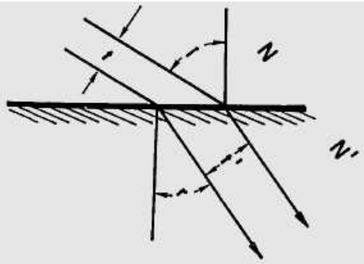

Fig. 1. Magnification by refraction for a plane surface.

cylindrical lenses. In addition, rotation of the prisms leads to linear adjustments in system magnification with no focus errors.

The subject of prismatic magnification is not often discussed in texts[1] on optics. Referring to Fig. 1, a beam of width t is shown as being incident on a prism surface at the angle i. After being refracted at the angle r due to the refractive index change from N to N', the beam has the width t'. The relationship between the beam widths is easily seen to be:

$$t/t' = \cos i/\cos r. \qquad (1)$$

The angular magnification of this surface is the differential ratio dr/di. By differentiating Snell's law and combining the result with Eq. (1), we obtain

$$\mathrm{d}r/\mathrm{d}i = N \cos i/N' \cos r = Nt/N't'. \qquad (2)$$

The magnification M of a series of surfaces is the product of the magnifications of each surface, so

$$M = \prod_{j=1}^{n} \frac{\mathrm{d}r_j}{\mathrm{d}i_j} = \prod_{j=1}^{n} \frac{N_{j-1} \cos i_j}{N_j \cos r_j} = \frac{N_1 t_1}{N_n t_n'}. \qquad (3)$$

If the first and last media are both air, $M = t_1/t_n'$, i.e., the magnification is inversely proportional to the beam diameter.

For the case of a single wedge prism of angle A, Eq. (3) can be written as $M = (\cos i_1 \cos i_2)/(\cos r_1 \cos r_2)$. Using the series approximation $\cos\theta \approx 1 - \theta^2/2$, we find that this relationship can be reduced to

$$M = \left[1 - \frac{i_1^2}{2} - \frac{i_2^2}{2} + \frac{r_1^2}{2} + \frac{r_2^2}{2}\right], \qquad (4)$$

the terms of the third and higher orders being ignored. We can find three relationships that will prove useful with the aid of the first-order form of Snell's law:

$$\begin{aligned} r_1 &= (N_0/N_1)i_1, \\ i_2 &= (N_0/N_1)i_1 - A, \\ r_2 &= (N_0/N_2)i_1 - (N_1/N_2)A. \end{aligned} \qquad (5)$$

Substituting the last three variables into Eq. (4) results in

$$M = 1 - \frac{A^2}{2}\left[\frac{N_1^2}{N_2^2} - 1\right] - N_0 A i_1 \left[\frac{N_1^2 - 1}{N_1 N_2^2}\right] + i_1^2 \left[\frac{N_0^2 - 1}{N_2^2}\right]. \qquad (6)$$

For the most common situation of a prism in air, we have $N_0 = N_2 = 1$, $N_1 = N$, and Eq. (6) reduces to an expression linear in i_1:

$$M = 1 + (A^2/2)(N^2 - 1) - (Ai_1/N)(N^2 - 1). \qquad (7)$$

If we consider the important case of a cemented pair of prisms in air, an equation similar to (4) can be written:

$$M = \left[1 - \frac{i_1^2}{2} - \frac{i_2^2}{2} - \frac{i_3^2}{2} + \frac{r_1^2}{2} + \frac{r_2^2}{2} + \frac{r_3^2}{2}\right]. \qquad (8)$$

As with (5), we can write the appropriate first-order relationships relating the ray angles to the prism angles A and B,

$$\begin{aligned} & r_1 = \frac{i_1}{N_1},\ i_2 = \frac{i_1}{N_1} - A,\ r_2 = \frac{i_1}{N_2} - \frac{N_1}{N_2}A, \\ & i_3 = \frac{i_1}{N_2} - \frac{N_1}{N_2}A - B, r_3 = i_1 - N_1 A - N_2 B; \end{aligned} \qquad (9)$$

and substitute them into (8) to obtain

$$M = 1 - \frac{1}{2}(A^2 + B^2) - \frac{N_1}{N_2}AB + \frac{1}{2}(N_1 A + N_2 B)^2 - i_1\left[\frac{A}{N_1}(N_1^2 - 1) + \frac{B}{N_2}(N_2^2 - 1)\right]. \qquad (10)$$

The prism wedge angle requirement for achromatization is that $B = -KA$. [The condition for simple achromatization is that $A\Delta N_1 + B\Delta N_2 = 0$, or $B = -KA$ with $K = \Delta N_1/\Delta N_2$. A nonachromatized direct vision prism has a similar relationship between its two components, with a constant $K' = (N_1 - 1)/(N_2 - 1)$.] Substitution of this condition into Eq. (10) leads to

$$M = 1 + \frac{A^2}{2}\left[2K\frac{N_1}{N_2} + (N_1 - KN_2)^2 - 1 - K^2\right] - i_1 A\left[\frac{N_1^2 - 1}{N_1} - K\frac{N_2^2 - 1}{N_2}\right]. \qquad (11)$$

Observing that the terms in brackets are functions of only the prism materials we may write Eq. (11) as

$$M = 1 + C_1 A^2 - C_2 A i_1. \qquad (12)$$

The last equation can be solved for either the prism angle A, or the ray angle of incidence angle i_1. Equations (7) and (10) indicate that magnification is a linear function of the prism tip, although as is shown by the last term in Eq. (6), this linearity holds only for thin prisms with the same medium on both sides. The accuracy of (12) should usually be more than sufficient, since magnification is not ordinarily a critical parameter.

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1. James P. C. Southall, *Mirrors, Prisms and Lenses* (Dover Publications, Inc., New York, 1964). Chapter 5 and parts of chapters 14 and 16 comprise one of the more thorough references on prisms, although anamorphic applications are not directly discussed. We generally follow Southall's sign conventions.

A New Reflecting Microscope Objective with Two Concentric Spherical Mirrors

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It is well known that a microscope objective, corrected simultaneously for spherical aberration, coma, and astigmatism, can be

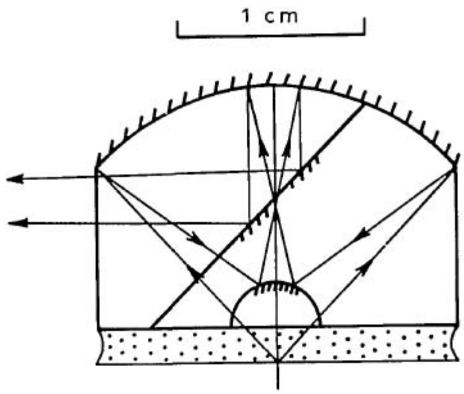

Fig. 1. Scaled drawing of the oil immersion objective described.

constructed from a concentric pair of spherical mirrors—a large concave primary and a small convex secondary. Unfortunately, the secondary mirror in these systems obstructs an objectionably large central part of each incoming pencil of light. Norris, Seeds, and Wilkins[1] showed that the central obstruction can be reduced while spherical aberration remains corrected with a nonconcentric mirror pair, but coma is then no longer corrected. Bouwers[2] retained concentricity while reducing the effective central obstruction with a semireflecting surface. Steel[3,4] has suggested systems of three concentric mirrors which are as well corrected for aberrations as the concentric mirror pair, but have a smaller central obstruction. He appears to have overlooked a convenient special case, to which this letter draws attention, in which the primary and tertiary mirrors combine to form a continuous concave spherical mirror surface, as shown in Fig. 1.

Although the system could have the mirrors facing air, it would generally be easier to build with the mirrors immersed in a built-up block of glass (or fused quartz, for use in uv light) as suggested in the figure. This not only simplifies the support of the reflecting surfaces, but also makes possible an increase in numerical aperture with oil immersion, or, without oil immersion, correction of field curvature with a concave entrance face. In the latter case, both longitudinal and transverse chromatic aberration will be absent if the entrance and exit faces are concentric, respectively, with the axial object and image points. In the case of oil immersion, although longitudinal chromatic aberration will not be present if the exit face is concentric with the axial image point, transverse chromatic aberration cannot be avoided without an increase in complexity of the objective, and is probably better corrected with a compensating eyepiece.

Figure 1 illustrates a particular design for an oil immersion objective, of this all glass type, that has zonal spherical aberration corrected within the Rayleigh limit down to a wavelength of 310 mm. The design has a numerical aperture of 0.66, a magnification of 34.6×, and a focal length of 2.99 mm when the mirrors are in air, while the radius of curvature of the concave primary/tertiary mirror is 12.70 mm, that of the convex secondary mirror is 2.37 mm, and the short and long conjugates, measured from the pole of the primary/tertiary mirror are, respectively, 14.64 mm and 79.38 mm. The obstruction of the incoming axial pencil of light by the secondary mirror is 10% (area). In the all glass design, an exit face will have to be polished on the side of the cylindrical block of glass to the right depth to allow the light to leave the system via the auxiliary plane mirror. This will mean that a small segment of the area of the primary/tertiary mirror will be lost in the particular design illustrated.

An objective of the type described is capable of being designed with a much higher magnification than that for the particular design given here, for sensible use of the large resolving powers obtainable with uv light and oil immersion.

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Experimental Investigation of Some Anomalies in Photographic Plates

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In our holographic experiments with photographic plates, two inconsistencies with theory have repeatedly occurred. Specifically, when simple grating holograms are made by recording the interference pattern between two plane waves, it is often necessary to reorient the hologram from the angle predicted by the Bragg relation in order to obtain the maximum diffraction intensity. In addition, as pointed out by Leith, *et al.*,[1] and verified by additional experiments performed by us, the shape of the orientation sensitivity curves, showing the diffracted intensity as a function of the incidence angle of the reconstructing beam, deviate from theory in that they have sidelobe asymmetry.

The first anomaly, that of the incorrect orientation, is of course expected with thick photographic emulsion and obliquely oriented interference planes. Under normal development procedure, the emulsion shrinks, thereby affecting the orientation of the planes as well as the spacing between them. This, in turn, requires the reconstruction beam to be reoriented from the recording position in order to get maximum diffracted intensity. The second anomaly, that of asymmetric sidelobes, can occur at small offset angles where a transfer of energy between the real and virtual reconstructed images can take place.[2]

In order to eliminate the effect of emulsion shrinkage and energy transfer, holograms were recorded with high offset angles and with interference planes perpendicular to the emulsion surface. Nevertheless, the anomalies persisted. We assumed at the start that these two anomalies stemmed from a common cause and proceeded with an attempt to locate it and find an effective solution.

Simple grating holograms were obtained by recording the interference pattern between two plane waves. The offset angle θ subtended by the two beams was maintained at a constant 60 deg throughout most of these experiments. The recording media was Kodak 10 cm × 12.7 cm 649F spectroscopic plates. We oriented the plates so that their normal bisected the angle θ, thereby achieving fringe surfaces which were perpendicular to the emulsion surface.

In investigating the anomalies, several factors were initially considered as having possible significance. These were development procedure, signal beam to reference beam ratio, real vs virtual image, and exposure time. In order to investigate the effect of the various factors, a series of recordings was made in which one factor was varied while the others were held con-