moves parallel or normal to their common line. However the mutual momentum is always in the plane containing the two charges and the direction of motion.

For more than two charges, the mutual effect may be treated as the sum of the effects for all possible pairs, due to the quadratic property of the momentum and energy. The foregoing results have application in the study of "packing effect" in atomic structure.

FURTHER EXPERIMENTS ON THE MASS OF THE ELECTRIC CARRIER IN METALS

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Introduction.—The production of an electromotive force by the acceleration of a metallic conductor was apparently demonstrated by the work of Tolman and Stewart [*Physic. Rev.*, **8**, 97 (1916); **9**, 164 (1917)], by measuring the pulse of electric current produced by suddenly stopping a coil of wire rotating around its axis. The purpose of the work described in the present article has been twofold. In the first place it seemed desirable to obtain a new demonstration of this production of an electromotive force by the acceleration of a metal, using some method of attack as different as possible from that of Tolman and Stewart, in order to increase our certainty as to the reality of the effect. In the second place it seemed desirable to try to find a method which would eliminate direct electrical connections between moving and stationary parts, and would avoid the sudden stopping of a coil of wire, with the attendant chance of irregular electromotive forces due to buckling or slipping of the wire.

Apparatus.—The apparatus finally used consisted of a copper cylinder $9^1/_8$ inches long, 4 inches outside diameter, and 3 inches inside diameter, oscillating about its axis with a frequency of 18.9 cycles per second. Surrounding this copper cylinder was a coil containing about 60 miles of No. 38 copper wire (diam. 0.1 mm.), which acted as the secondary of a transformer. Connection from this secondary was made through a specially designed three stage amplifier with a vibration galvanometer. The tendency of the electrons in the oscillating copper cylinder to lag behind because of their inertia leads to an electromotive force, the effects of which were finally measured by the deflection of the vibration galvanometer. These galvanometer deflections were then compared with those produced by the known electromotive force accompanying transverse oscillation of the cylinder in such a way as to cut the earth's magnetic field.

The apparatus was mounted on a massive concrete pier in a special location 150 yards from the nearest electrical circuits, was constructed without the use of magnetic materials, and was driven by air pressure to avoid the disturbances which would have been produced by electrical driving. The axis of the oscillating cylinder was made parallel to the earth's magnetic field in order to reduce accidental effects.

Theory of the Experiment.—The experiments consisted in comparing the electromotive force produced in the cylinder by its rotary oscillation with the electromotive force produced by its transverse oscillation in such a way as to cut the earth's magnetic field. The elementary theory of the experiment may be developed as follows.

If a longitudinal acceleration a is applied to a metallic conductor, the electrons within the conductor will tend to move relative to the main body of the metal as though the conductor were stationary and the electrons were acted on by the force.

$$f = ma \tag{1}$$

where m may be called the "effective mass" of the electron. On the other hand if an electromotive force E is applied to a stationary metallic conductor of length l and uniform cross-section, the electrons within the conductor will be acted on by the force

$$f = Ee/l \tag{2}$$

where e is the charge of one electron. Since the "fictitious" force given by equation (1) and the "real" force given by equation (2) both tend to make the electrons move relative to the main body of the metal, it is evident that they may be equated in order to get an expression for the electromotive force produced by the longitudinal acceleration of a metallic conductor. We obtain, for the electromotive force E, produced in a metallic conductor of length l, by an acceleration a the expression

$$E = mla/e \tag{3}$$

Let us now consider the rotary oscillations of the cylinder around its axis. At any radius r we may evidently write for the instantaneous acceleration, the expression

$$a = 4\pi^2\nu^2\theta_e r \sin 2\pi\nu t \tag{4}$$

where ν is the frequency of harmonic oscillation and θ is half the angular amplitude of oscillation. Substituting in equation (3) and taking the length of the conductor at the radius in question as $2\pi r$, we obtain

$$E_e = 8\pi^3\nu^2r^2(m/e)\theta_e \sin 2\pi\nu t \tag{5}$$

as an expression for the electromotive force around a current sheet located in the cylinder at the radius r.

Let us now compare this electromotive force with the electromotive force produced by the transverse oscillation of the cylinder in the earth's field used in calibrating. If θ_c is the half angular amplitude of transverse oscillation, we may write for the maximum flux through a current sheet of radius r, the expression

$$\phi_{\max} = \pi r^2 H \sin \theta_c = \pi r^2 H\theta_c \text{ (for small amplitudes)} \tag{6}$$

where H is total intensity of the earth's field. Hence for harmonic oscillation of frequency ν, we may write for the electromotive force produced in carrying out the calibration the expression

$$E_c = 2\pi^2\nu r^2 H\theta_c \sin 2\pi\nu t. \tag{7}$$

Dividing equation (5) by (7) we obtain for the ratio of the electromotive forces produced by the effect and in calibration the expression

$$E_e/E_c = (4\pi\nu/H) . (m/e) . (\theta_e/\theta_c) \tag{8}$$

or solving for the thing of interest, namely the ratio of the effective mass of the electron to its charge, we obtain,

$$m/e = (H/4\pi\nu) . (E_e/E_c) . (\theta_c/\theta_e) \tag{9}$$

This is the equation which was used in calculating our experimental results. It will be noted that the radius r of the particular current sheet has dropped out so that E_e/E_c may be taken as the ratio of the total electromotive forces produced by the rotary oscillation and transverse oscillation of the effect cylinder.

In carrying out the actual experiments, a comparison was made of the galvanometer deflections produced by the rotary oscillation of the main cylinder and the transverse oscillation of a much thinner walled calibration cylinder, which was driven with the same frequency as the main cylinder and surrounded by a similar secondary coil. This made it possible to eliminate variability in the behavior of the amplifier by obtaining nearly simultaneous readings from the main cylinder and the calibration cylinder. At the close of the experiments a "master calibration" was made comparing the electromotive forces produced by the transverse oscillation in the earth's field of the main cylinder and the calibration cylinders which had been employed. We could than calculate the ratio E_e/E_c which occurs in equation (9) by putting it equal to the ratio of the galvanometer deflections obtained from the main cylinder and from the calibration cylinder multiplied by the ratio determined in the master calibration. The other quantities in equation (9) were determined by direct measurement.

Experimental Results.—In all, eighty-six measurements were made. The average value of m/e in grams per abcoulomb was 5.18×10^{-8}, with an average deviation of 1.33×10^{-8}. The average deviation divided by the square-root of the number of observations was 0.14×10^{-8}.

Conclusion.—It is felt that the work presented above may be regarded as another fairly satisfactory demonstration of the production of electromotive forces by the acceleration of a metallic conductor, and as indicating again that the mass of the carrier in metals is about the same as the mass of an electron in free space. The new work taken by itself alone is perhaps not as convincing as the work of Tolman and Stewart, because of the greater complexity of the apparatus, because of the fact that time did not permit a satisfactory neutralization of the earth's field, and because further developments of the method would be necessary in order to show that the direction of the effect is that predicted on the basis of a mobile *negative* carrier. Our total certainty as to the reality of the effect is, however, greatly increased by the fact that two such widely divergent methods have led to concordant results.

Values of m/e obtained in different ways are given below, in grams per abcoulomb.

m/e in free space	5.66×10^{-8}	(cathode rays)
m/e in copper	6.24×10^{-8}	(Tolman and Stewart)
m/e in silver	6.73×10^{-8}	(Tolman and Stewart)
m/e in aluminum	6.50×10^{-8}	(Tolman and Stewart)
m/e in copper	5.18×10^{-8}	(Tolman, Karrer and Guernsey)

It is evident that our data are not yet accurate enough to determine whether the mass of the electron in a metal is precisely the same as that in free space or not.

A more complete account of the experimental work containing a discussion of the sources of error in the work will be published in the *Physical Review.* The investigation is being continued at the California Institute of Technology.