Are Textures Natural?

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We make the simple observation that, because of global-symmetry-violating higher-dimension operators, as for instance might be expected to be induced by Planck-scale physics, textures are generically much too short lived to be of use for large-scale structure formation.

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The texture scenario for large-scale structure formation is an attractive alternative to inflation or other, defect-mediated, proposals based on new physics at a high energy scale [1]. The model assumes the existence of a non-Abelian global continuous symmetry which, when broken at the grand-unification-theory (GUT) scale, leads to a topological defect known as a texture. With one free parameter, the symmetry-breaking scale v, the model reproduces galaxy-galaxy correlation functions, finds significant galaxy clustering on scales of 20–50h^{-1} Mpc, structure on larger scales, and coherent velocity fields on scales similar to those observed in large-scale surveys. The optimal value of v turns out to be 10^{16} GeV, the GUT scale [2]. Another advantage is that the theory also predicts a distinctive signature in the cosmic microwave background and will therefore be testable in the near future [3]. We feel that given the recent attention devoted to, and the astrophysical promise of, the model, an investigation of the particle physics behind the model is in order.

In this Letter we point out that the possible existence of higher-dimension symmetry-violating operators may invalidate the texture scenario. We find that the couplings of such operators, which, for example, are expected to be induced at the Planck scale by quantum-gravity effects, would have to be exponentially small in order for textures to be effective in generating large-scale structure. In other words, in order for textures to work, the assumed global symmetry must be essentially *exact*. Our argument is simply that explicit symmetry-breaking terms align the field at a unique minimum of the vacuum manifold so that as larger scales come into the horizon, the Higgs field is already correlated and textures do not form.

To illustrate our point, we will first briefly review how textures work. The simplest model assumes a global $G=\mathrm{SU}(2)$ symmetry broken by a complex doublet ϕ^a with potential $\lambda(\phi^2-v^2)^2$. At the earliest times the temperature T is greater than v and the global symmetry is unbroken; as the Universe cools and T falls below v the Higgs field falls to the vacuum manifold, but it falls to different locations on the vacuum manifold in different causally disconnected regions of the Universe. As the Universe expands and these different regions come into causal contact, the field aligns itself quite rapidly within the horizon. Since the homotopy group π_3 of the vacuum manifold is nontrivial, in becoming aligned the Higgs field may become wound around the vacuum manifold and form a texture, an unstable topological defect, which rapidly collapses to a point [4]. When the size of the texture has decreased to roughly v^{-1} the gradient energy pulls the Higgs field out of the vacuum manifold, the texture unwinds, and the energy is radiated away in Nambu-Goldstone bosons which results in the astrophysically relevant density perturbation. As new scales come across the horizon new knots continue to form resulting in perturbations on larger scales. We should remind the reader that if the symmetry G is gauged then the texture field configuration is a gauge transformation of the vacuum having no physical consequences.

We now wish to consider the effect on the evolution of textures of higher-dimension operators that violate the continuous global symmetry G. It is widely believed that Planck-scale physics results in the violation of all global symmetries—both continuous and discrete. We note that superstring theories generically do not enjoy exact global symmetries [5]. In a more general context wormholes provide a specific mechanism for this violation [6–9], but for those uneasy about wormhole arguments [10] there is a very simple physical argument for why one still expects all global symmetries to be violated. It is well known that as a consequence of the black-hole no-hair theorems [11] the global charge of a black hole is not defined; therefore, if in a scattering process a virtual or nonvirtual black hole is formed from an initial state of definite global charge, the black hole will then unprejudicially decay (Hawking evaporate) [12] into final states of differing global charge. In other words, black holes provide a specific physical mechanism for mediating between states of different global charge. At energies small compared to the Planck mass we can represent these symmetry-violating effects by higher-dimension operators in an effective theory of the light modes. On dimensional grounds, the higher-dimension operators are expected to be suppressed by the appropriate power of the Planck mass

$$\Delta L = g M_{\mathrm{Pl}}^{4-n}\phi^n , \qquad (1)$$

where we expect the coupling constant $g\sim 1$. Note that we are considering the situation where, to a first approximation, textures do exist. We therefore assume that for $n \leq 4$ global-symmetry-violating operators are "accidentally" forbidden by, for instance, the existence of other gauge interactions (as discussed below).

It is easy to qualitatively see what the effect of these explicit violations of the global symmetry on texture production is going to be. Instead of an isopotential vacuum manifold (typically of the form of an S^3 in field space), we now have a "tipped" manifold with only a single true vacuum configuration. (In general, a single higher-dimension operator would leave a discrete set of degenerate vacuum states, so one might be concerned about the appearance of domain walls when ϕ condenses. However, when they occur these walls are generally destabilized by the effects of yet higher-dimension operators. We will return to this point below.) As the temperature falls below the G-symmetry-breaking value, correlation-length-sized domains form, in which the order parameter is aligned. If there were no mechanism to rotate the order parameters in different domains into a common direction before they come into the growing horizon then topological defects, specifically textures, would form. However, there *is* a mechanism—the tipping of the potential by the higher-dimension operators.

To be useful for galaxy or large-scale structure formation, the texture scenario must have new textures still entering the horizon at the time t_{struc} when the horizon volume contains a galactic (or large-scale) mass. In other words, we need to ensure that in the interval between the symmetry-breaking phase transition and t_{struc}, the order parameter in the various domains has not finished rotating to a common direction. Since the expansion of the Universe is so slow compared to particle physics time scales, one can anticipate that this leads to a very stringent condition on the coupling constants g.

As discussed above, textures arise in models where a global continuous symmetry G is broken to H such that $\pi_3(G/H)$ is nontrivial. The simplest example of this is the breaking $\text{SU}(2) \to 1$ as discussed originally by Turok [1]. Rather than consider the effects of higher-dimension operators on this model, a moment's thought convinces one that, if one is only interested in order-of-magnitude estimates, it is enough to consider a simpler toy model of $G = \text{U}(1)_{\text{global}}$. Specifically, we take the Lagrangian of the model to be

$$L = |\partial_\mu \phi|^2 - \frac{\lambda}{4}(|\phi|^2 - v^2)^2 + \frac{g}{M_{\text{Pl}}^{2m+n-4}}|\phi|^{2m}\phi^n + \text{H.c.} - c\,, \tag{2}$$

where c is a constant chosen so that the vacuum energy is zero. We have taken the leading higher-dimension operator that violates G to be of dimension $2m+n$ with global charge n. If we write $\phi = \rho e^{i\theta}$ then after symmetry breaking the equation of motion for θ in an expanding Universe is

$$\ddot{\theta} + 3H\dot{\theta} - \nabla^2\theta + gn\frac{M_{\text{Pl}}^4}{v^2}\left(\frac{v}{M_{\text{Pl}}}\right)^{2m+n}\sin n\theta = 0\,, \tag{3}$$

where $H = 1.66 g_*^{1/2} T^2/M_{\text{Pl}} = 1/2t$ is the Hubble parameter in a radiation-dominated Universe and $g_* \sim 100\text{–}1000$ is the effective number of relativistic degrees of freedom. (In general, there may be a term $\Gamma\dot{\theta}$ in the equation of motion from the coupling of θ to other fields. We will say more about this below.) If we define

$$\tau = \frac{1}{g^{1/2}n}\frac{1}{M_{\text{Pl}}}\left(\frac{M_{\text{Pl}}}{v}\right)^{m+n/2-1}, \tag{4}$$

assume $\sin n\theta \simeq n\theta$, and look at the homogeneous component, we find that the equation of motion for θ is

$$\ddot{\theta} + (3/2t)\dot{\theta} + \tau^{-2}\theta = 0\,. \tag{5}$$

At a temperature $T \sim v$ the phase transition occurs and an initial value of θ is selected. As long as $t \lesssim \tau$ the field rolls slowly toward the minimum until $t \sim \tau$ at which point the field begins to undergo oscillations with an amplitude which decreases as $(t/\tau)^{-3/4}$. These coherent scalar-field oscillations describe a condensate of zero-momentum Nambu-Goldstone bosons, and the decaying amplitude describes their dilution in the expanding Universe. The important point is that from Eq. (5) it is clear that the field becomes localized near the minimum of the potential roughly within a few time scales τ.

For the texture model to account for the large-scale structure in the Universe, the time scale τ for rolling must be longer than or comparable to the time t_{struc} at which galactic (or larger) scales enter the horizon. Taking the smallest scale of interest (for the most conservative limit on the coupling g), namely, the galactic scale, that which encloses $10^{12}M_\odot$, we find that $t_{\text{struc}} \sim 4\times10^6$ sec. So, if textures are to be effective in forming even galactic-sized structure,

$$7\times10^{-47}\frac{1}{g^{1/2}n}(10^3)^{m+n/2}\left(\frac{10^{16}\,\text{GeV}}{v}\right)^{m+n/2-1} \gtrsim 4\times10^6\,. \tag{6}$$

There are various ways of expressing the limits that result from this condition. If we demand $v \sim 10^{16}$ GeV, as required for the appropriately sized density perturbations, and suppose that the explicit breaking is due to a dimension-5 operator, then we find that the coupling is constrained to be

$$g \lesssim 10^{-91}\,. \tag{7}$$

This is the central result of our Letter.

If instead we demand that $g \sim 10^{-2}$ with the same value of v then we find that the first operator that explicitly breaks the symmetry and is consistent with texture-seeded galaxies has dimension $2m+n \simeq 35$. Finally, if we assume a symmetry-breaking operator of dimension 5 and again take $g \sim 10^{-2}$ we find that $v \lesssim 10^{-5}$ eV, clearly incompatible with the texture scenario.

One might be concerned that the field θ could get hung up near $n\theta = \pi$, where the potential is a maximum, in certain regions of the Universe leading us to underestimate the rolling time scale τ. However, horizon-sized ($\lambda \sim H^{-1}$) thermal fluctuations of θ at a temperature $T \sim v$ are roughly

$$\langle (\delta\theta)^2_\lambda \rangle_T \sim g_*^{1/2} \left(\frac{v}{M_{\rm Pl}} \right) \sim 10^{-2}, \tag{8}$$

so $\delta\theta \sim 10^{-1}$, and there is no problem with the field getting hung up at its maximum. This should be no surprise since we know that in inflationary models the inflaton fills slowly only if the potential is extremely flat.

While our analysis is quite simple and we ignore such effects as the running of the coupling constants g, it is clear from Eq. (7) that a more refined calculation would not alter our conclusions. We have also considered the effect of a friction term $\Gamma\dot{\theta}$ in the equation of motion that could arise as a result of coupling of θ to other fields in the theory. However, since the mass of the field θ is so small, we expect Γ to be small; furthermore, astrophysical considerations constrain the coupling of θ to other fields in the theory to be small [13]. Still, we have checked that if we take the extreme conservative limit, that if for some unforeseen reason Γ assumes its maximum (in)conceivable value $\Gamma = M_{\rm Pl}$, our results are qualitatively robust (i.e., g is still constrained to be extremely small).

We should also point out that if Γ is very small (as we expect), then one might worry that the energy density in the condensate of θ particles at some point might become greater than that in radiation. Indeed, the simple condition that relic θ particles do not "overclose" the Universe may be used to constrain g; however, the constraint turns out to be weaker than Eq. (7) and the assumption of a radiation-dominated Universe in our analysis remains valid.

While discrete and continuous global symmetries are in general explicitly violated by Planck-scale physics, this is not true of gauge symmetries. As discussed in Ref. [1], we can imagine the necessary continuous global symmetry G arising as an accidental global symmetry of the effective Lagrangian when we impose a set of discrete symmetries. If these discrete symmetries are discrete *gauge* symmetries [14,15] which are protected from violation by Planck-scale physics, then not only the renormalizable part of the effective Lagrangian, but also some higher-dimension operators will be constrained to be invariant under G (especially if these discrete symmetries are non-Abelian [16]). However, it requires some stretch of the imagination to have a sufficiently large discrete-gauge-symmetry group to forbid the existence of all higher-dimension operators up to dimension 35, although it is a possible way of ensuring the integrity of the texture scenario. On the other hand, if observers find the cosmic-microwave-background distortions predicted by textures, we could construe this to be evidence of a very large discrete-gauge-symmetry group (as have been conjectured in string theory [17]). We note that if the breaking of the global symmetry also involves the spontaneous breaking of discrete gauge symmetries then domain walls form that are *not* destabilized by higher-dimension operators, and the evolution of the Universe is radically different (and ruled out).

We can also consider the more recent models of large-scale structure formation involving perturbations through gradient energy without topological defects [18]. In this case arguments analogous to those above show that this scenario is also destablized by Planck-scale effects. In fact, similar arguments may impact many ideas in particle physics and cosmology which rely on exact (or nearly exact) global symmetries such as the Peccei-Quinn mechanism [19], production of global monopoles and cosmic strings, baryogenesis scenarios, some recently proposed inflationary models, late-time phase transitions, various particle dark-matter candidates, particle-physics models with Nambu-Goldstone bosons, or particle-physics models with spontaneously broken discrete global symmetries [20].

In summary, we find that the couplings of higher-dimension global-symmetry-violating terms in the effective Lagrangian must be unnaturally small in order for textures to be responsible for the formation of large-scale structure. Admittedly, little is known about Planck-scale physics; however, the agnostics who question the existence of higher-dimension operators coming from quantum-gravity effects must nevertheless certainly conclude that the texture model is at least unstable to the unknown. Our argument is simply that unless the (approximate) vacuum manifold of G is exactly flat, the field in causally disconnected regions of the Universe will become aligned before coming into causal contact. In a sense the unnatural flatness of the potential necessary for the survival of the texture scenario echoes that required in new inflationary models, namely, that the field rolls only very slowly to its minimum.

We understand that similar conclusions have been reached by Holman, Hsu, Kolb, Watkins, and Widrow [21].

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