Bulletin of the Seismological Society of America. Vol. 66, No. 1, pp. 325–352. February 1976

LETTERS TO THE EDITOR

COMMENTS ON THE PAPER: "PHASE DISTORTION AND HILBERT TRANSFORMATION IN MULTIPLY REFLECTED AND REFRACTED BODY WAVES", BY G. L. CHOY AND P. G. RICHARDS

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In a recent paper Choy and Richards (1975) have demonstrated that phase-shifted arrivals can introduce systematic travel-time errors due to relative changes in the peak and trough positions of the arrival wavelets. The method they suggest to correct the problem is to cross-correlate an estimated zero-phase-shifted wavelet with the seismogram after all the arrivals have been restored to a zero phase shift. The peaks of the cross-correlation are then taken as the reference times for the arrivals. This method is awkward to use in practice because the arrivals will in general be phase-shifted by various angles that are not usually known *a priori*. This necessitates that each arrival be treated separately by searching for the correct restoring phase shift. The purpose of this note is to point out the existence of a much easier method which is simply to compute the envelope of the cross-correlation of the seismogram with an estimated zero-phase-shifted wavelet (Helmberger and Wiggins, 1971).

The envelope of a signal $x(t)$ is the modulus of the analytic signal $\hat{x}(t)$ which is defined to be (Bracewell, 1965)

$$\hat{x}(t) = x(t) - iH(x(t)) \qquad (1)$$

where $H(\cdot)$ denotes the Hilbert transform. The analytic signal is simply computed in the frequency domain

$$\hat{X}(\omega) = X(\omega)\cdot(1+\operatorname{sgn}\omega) \qquad (2)$$

where the capital letters denote the Fourier transform pairs of the quantities in equation (1), and sgn is the usual signum function. The analytic form of a phase-shifted wavelet $x'(t)$, is a rotation of the zero-phase-shifted analytic signal $\hat{x}(t)$

$$\hat{x}'(t) = \exp(i\varepsilon)\hat{x}(t) \qquad (3)$$

where ε is the phase-shift angle. The real part of equation (3) is equivalent to the formula given by Choy and Richards (1975) for phase-shifted wavelets. It is easily proved with the aid of equation (2) that the analytic cross-correlation $\hat{c}'(t)$ of $\hat{x}'(t)$ and $x(t)$ will also have the same phase shift as $\hat{x}'(t)$. That is

$$\hat{c}'(t) = \exp(i\varepsilon)\hat{c}(t) \qquad (4)$$

where $\hat{c}(t)$ is the zero-phase-shift cross-correlation [i.e., the cross-correlation of $\hat{x}(t)$ and $x(t)$].

The envelope of the cross-correlation $|\hat{c}'(t)|$, will be independent of the phase-shift angle ε, making its peaks suitable for timing purposes and the relationship between the cross-correlation and its envelope will indicate the presence of any phase-shifted arrivals. This is useful for detecting arrivals which have touched a caustic or for deciding whether a particular arrival belongs to a forward or receding branch of the travel-time curve (Choy and Richards, 1975). A similar envelope may be defined for the deconvolution of $\hat{x}'(t)$ by $x(t)$.

The envelope is always broader than the corresponding cross-correlation or deconvolution, so it has an apparent reduced time resolution. However, when the phase of the signal is not known *a priori*, the envelope more closely reflects the true time resolution that is achievable with the given data.

To illustrate the use of the envelope, a synthetic seismogram was constructed with arrivals phase-shifted by 0°, 45°, 90°, 135°, and 180° (Figure 1). This seismogram was cross-correlated with and deconvolved by the zero phase-shifted wavelet, and the

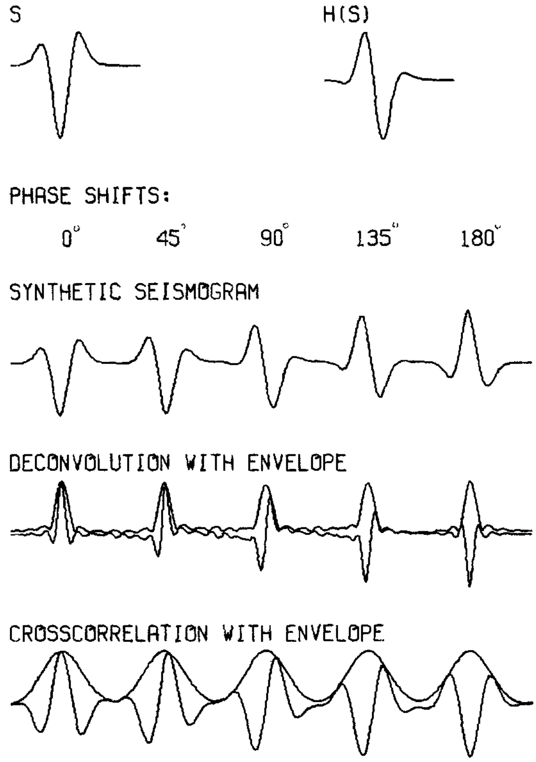

FIG. 1. An example of the use of the envelopes of cross-correlation and deconvolution. The zero phase-shifted wavelet S, and its Hilbert transform $H(S)$, are shown. The phase-shift angles refer to the phase shifts in the three following traces.

envelopes were computed for both cases. A useful way of displaying the envelope is to plot it over the cross-correlation or deconvolution, as was done for this example. This allows the phase-shifted arrivals to be easily detected. For example, a 90° phase shift is characterized by a trough followed by a peak of equal amplitude under the peak of the envelope.

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Manuscript received June 27, 1975