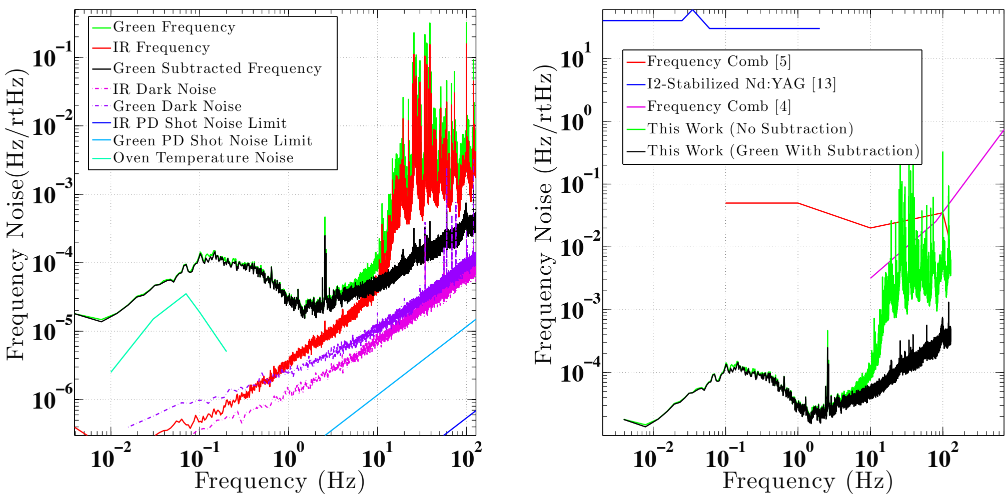

Fig. 3. On the left we see the full noise budget of the experiment: $\theta^A_{1064} - \theta^B_{1064}$ (in red) (which is the in-loop signal of the Mach-Zehnder), $\theta^A_{532} - \theta^B_{532}$ (in green) (which contains excess phase noise), and the subtraction residual (in black). The total RMS frequency fluctuation in the measurement band is 3 mHz. On the right we have a comparison of this work with previous bounds. These bounds show up as frequency noise [13], timing error [5], and phase noise [4].

ignore it. It is highly improbable that the intensity to frequency noise coupling would be above the level shown in Fig. 3. The excess noise below 10 Hz was found to be correlated with air currents on the table, and would be reduced by moving the setup into a vacuum chamber. The total RMS excess frequency noise of the black trace is 3 mHz RMS in the 10 mHz to 128 Hz band.

4. Noise sources

In addition to the usual technical noise sources, it is worth considering whether there is a more fundamental limit to the relative phase between the fundamental and the harmonic. A rough estimate of thermal noise from thermoelastic (Zener) damping was obtained by directly applying [14] the Fluctuation-Dissipation Theorem. We treat the crystal as an 0.8 mm radius cylinder, and follow the calculation done by Heinert et al. [15]. This yields a spectral density of Δk taking into account both thermorefractive and thermoelastic fluctuations. When expressed as frequency fluctuations, it is well approximated by $2.5/(1+500 f^{-7/8})\,\mu\mathrm{Hz}/\sqrt{\mathrm{Hz}}$ above 10 Hz. Below that, it must be flat or continue to decrease, or else the RMS temperature integral would diverge. Practically speaking, in our band of interest, the temperature fluctuations of the ovens is at least 2 orders of magnitude greater than these fundamental thermodynamic temperature fluctuation. In the future, when researchers seek to make frequency comparisons at better than the 10^{-21} level, these thermal noises will have to be calculated with more accuracy.

5. Previous bounds on excess frequency noise

We have examined the results from a number of precision experiments which involve SHG. Although these experiments are likely limited by other noise sources, these data can be used to place upper bounds on the possible noise generated by SHG. In Fig. 3 we compare frequency noise [13], timing errors [5], and phase noise [4]; in Fig. 4 we compare Allan deviations [2, 3, 5, 6, 16–18]. While the comparison in Fig. 3 is straightforward, some caution should be taken interpreting Fig. 4. Our Allan deviation at the 0.1 s time scale is heavily influenced by the high frequency noise in the measurement (10-128 Hz). Since we low pass the signal to acquire data at 256 Hz, we reject noise which would make the Allan deviation increase at all time scales. It should also be emphasized that we only measure relative frequency fluctuations, and that there are some common mode noise sources which the experiment is insensitive to. See [19] for more information on Allan deviations and phase noise.

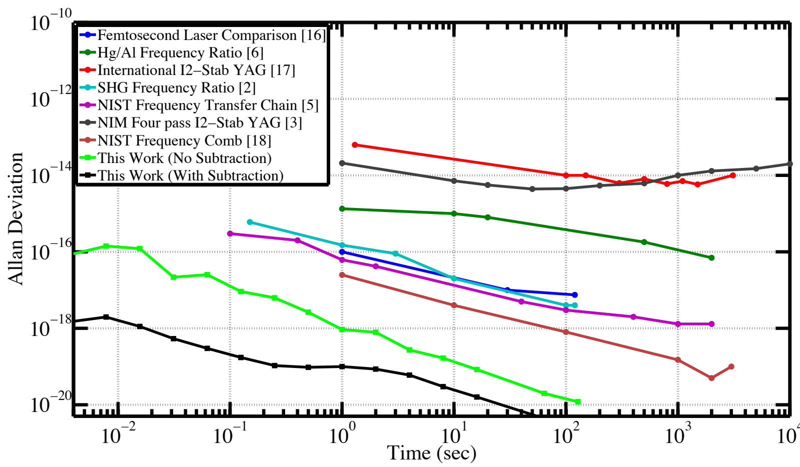

Fig. 4. The Allan deviations from this experiment (green and black) were obtained from the power spectrum as described in [20]. Bounds from previous work are shown for reference.

6. Conclusions

In conclusion, we have demonstrated that the RMS frequency fluctuations added from uncorrelated mechanisms between two SHG crystals is less than 3 mHz at time scales over 10 ms. The obvious correlated mechanisms (temperature and intensity noise coupling) are likely insignificant compared to this as discussed in section 3. This is low enough to not limit the lock acquisition [9, 10] scheme for Advanced LIGO and other gravitational-wave detectors. Additionally, we have shown that there is no excess noise process at a level which is of interest to those doing precision atomic spectroscopy [7], using frequency combs to transfer optical harmonics [4, 5, 21] and other tests of fundamental physics.

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