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A Low-Density Closed Universe

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Matter with an equation of state $p = -\rho/3$ may arise in certain scalar field theories, and the energy density of this matter decreases as a^{-2} with the scale factor a of the Universe. In this case, the Universe could be closed but still have a nonrelativistic-matter density $\Omega_0 < 1$. Furthermore, the cosmic microwave background could come from a causally connected region at the other side of the Universe. This model is currently viable and might be tested by a host of forthcoming observations. [S0031-9007(96)00746-6]

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Of the three possibilities, a closed universe receives far less attention in the current literature than an open or a flat universe. Observations that find a matter density less than critical suggest an open universe. Theoretical arguments, such as the Dicke coincidence and inflation, favor a flat universe. However, there are heuristic arguments for a closed universe that involve, for example, consistency of quantum field theories on a compact space or the idea that it is easier to create a finite universe with zero energy, charge, and angular momentum. Even so, given the observations, it requires some *chutzpah* to suggest that the matter density is greater than critical. For these reasons, models that are closed by virtue of a cosmological constant (Λ) have been recently considered [1]. In this Letter, we consider a variation: a low-density closed universe, which at low redshifts is entirely indistinguishable from a standard open Friedmann-Robertson-Walker (FRW) universe with the same nonrelativistic matter density.

If some form of matter with an equation of state $p = -\rho/3$ exists, then its energy density decreases with the scale factor a of the Universe as a^{-2} and thus mimics a negative-curvature term in the Friedmann equation [2–5]. In this case, the Universe could be closed and still have a nonrelativistic-matter density $\Omega_0 < 1$.

In fact, the energy density contributed by a scalar field with a uniform gradient-energy density would scale as a^{-2}. However, such a scalar-field configuration would collapse within a Hubble time unless it was somehow stabilized. Davis [2] argued that if there was a manifold of degenerate vacua with nontrivial mappings into the three-sphere [which could be accomplished if there was a global symmetry G broken to a subgroup H with $\pi_3(G/H) \neq 1$], then a texture—a topological defect with uniform gradient-energy density—would be stabilized provided that it was wound around a closed universe [2]. Although Davis' configuration is in fact unstable [6], it might be stabilized by higher-derivative terms. Nonintersecting strings would also provide an energy density that scales as a^{-2} [4].

Moreover, if the energy density contributed by the texture is chosen properly, the observed cosmic microwave background (CMB) comes from a causally connected patch at the antipode of the closed universe [7]. (This could similarly be accomplished with $\Lambda \neq 0$, but these models are likely ruled out by lensing statistics [1].) The homogeneity, monopole, and entropy problems are not addressed, and we do not discuss generation of density perturbations. Even so, we find it illustrative and interesting that one can still construct a viable model which looks remarkably like an open universe at low redshifts, even though the largest-scale structure differs dramatically.

The Friedmann equation for a closed universe with nonrelativistic matter and some other form of matter (perhaps a stable texture) with an equation of state $p = -\rho/3$ is

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m + \frac{\gamma - 1}{a^2}$$

$$= H_0^2[\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^2]$$
$$\equiv H_0^2[E(z)]^2, \tag{1}$$

where $H = \dot{a}/a$ is the Hubble parameter (and the dot denotes derivative with respect to time), $z = a_0/a - 1$ is the redshift, G is Newton's gravitational constant, ρ_m is the density of nonrelativistic matter, and γ is a parameter that quantifies the contribution of the energy density of the texture. The second line defines the function $E(z)$. This is exactly the same as the Friedmann equation for an open universe with the same Ω_0, so this closed universe has the same expansion dynamics. At the current epoch (denoted by the subscript "0"),

$$\Omega_0 = 1 + \frac{1-\gamma}{a^2H^2} = 1 - \Omega_t + \frac{1}{a_0^2H_0^2}, \tag{2}$$

where $\Omega_t = \gamma(a_0H_0)^{-2}$ is the contribution of the texture to closure density today. So, $\Omega_0 < 1$ if $\gamma > 1$ even though the Universe is closed, and we require that $\Omega_t + \Omega_0 > 1$.

If the metric of a closed universe is written as

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)], \tag{3}$$

then the polar-coordinate distance between a source at a redshift z_1 and another source along the same line of sight at a redshift z_2 (for $\Omega_0 < 1$) is

$$\chi_2 - \chi_1 = \sqrt{\Omega_0 + \Omega_t - 1}\int_{z_1}^{z_2}\frac{dz}{E(z)}. \tag{4}$$

If Ω_t is chosen such that the polar-coordinate distance of the CMB surface of last scatter is $\chi_{LS} \simeq \pi$, then the CMB we observe comes from a causally connected patch at the antipode of the Universe. Since this universe expands forever, we could also choose $\chi_{LS} \simeq 2\pi$, in which case the CMB photons have traveled precisely once around the Universe. This introduces the intriguing possibility that when we observe the CMB we are looking at the *local* (rather than some distant) region of the Universe as it was at a redshift $z \simeq 1100$. In fact, for $\chi_{LS} \simeq n\pi$ with $n = 1, 2, 3, \ldots$, CMB photons have traveled $n/2$ times around the Universe, and the CMB comes from a causally connected patch on the other side of the Universe (for n odd) or from the local neighborhood (for n even). From Eq. (4), the condition on Ω_t for $\chi_{LS} = n\pi$ is

$$\Omega_t = \left[\frac{n\pi\sqrt{1-\Omega_0}}{\operatorname{arcsinh}(2\sqrt{1-\Omega_0}/\Omega_0)}\right]^2 + 1 - \Omega_0. \tag{5}$$

For $n = 1$ ($n = 2$), Ω_t increases from 1.6 to 2.5 (4 to 10) for Ω_0 between 0.1 and 1.

Is this a realistic possibility? For $n \geq 2$, it requires a radius of curvature for the Universe that is probably too small to be consistent with observations. The $n = 1$ case is still consistent with our current knowledge of the Universe. However, forthcoming observations may be used to distinguish it from a standard open Universe, as we now explain.

Since the expansion dynamics is the same as for an open FRW universe, quantities that depend only on the expansion, such as the deceleration parameter, the age of the Universe, or the distribution of quasar absorption-line redshifts, do not probe Ω_t. Furthermore, the growth of density perturbations is the same as in a standard open universe, so dynamical measurements of Ω_0 (e.g., from peculiar-velocity flows) will also be insensitive to Ω_t. Effects due to geometry arise only at $\mathcal{O}(z^3)$ since $\sin\chi$ and $\sinh\chi$ differ only at $\mathcal{O}(\chi^3)$; therefore, this universe will differ from an open universe only at $z \gtrsim 1$.

Ergo, we now turn to cosmological tests that probe the geometry of the Universe. Underlying these is the angular-diameter distance between a source at a redshift z_2 and a redshift $z_1 < z_2$,

$$d_A(z_1, z_2) = \frac{\sin(\chi_2 - \chi_1)}{(1+z_2)H_0\sqrt{\Omega_0 + \Omega_t - 1}}. \tag{6}$$

The angular size of an object of proper length l at a redshift z is $\theta \simeq l/d_A(0, z)$. Consider first the case where Ω_t is fixed by $n = 2$. Then the antipode $\chi = \pi$ of the Universe must be at some redshift $z_a < 1100$. One finds that $z_a \lesssim 5$ for $\Omega_0 \gtrsim 0.3$, and therefore, the angular sizes of the highest-redshift quasars must be very large. Additional arguments (involving gravitational lenses) against an antipode at $z \lesssim 5$ for this model (and those with a cosmological constant) have been given in Refs. [3,4,8,9]. Therefore, a closed universe with $n \geq 2$ is highly unlikely and we pursue it no further.

In Fig. 1, we plot the angular size as a function of redshift fixing Ω_t so that the CMB comes from the antipode [i.e., Eq. (5) with $n = 1$]. We also plot the re-

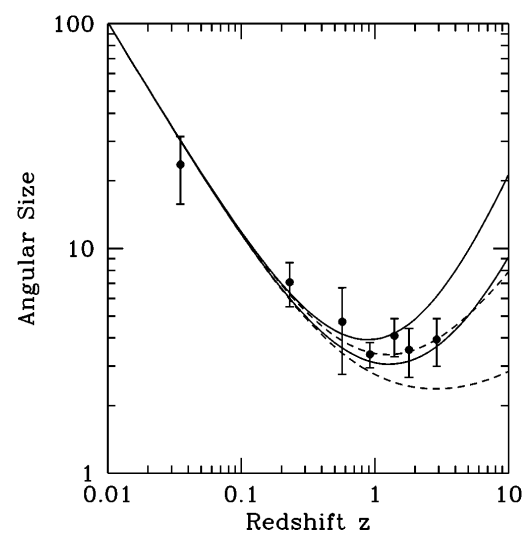

FIG. 1. The angular size of an object of proper length l (in units of lH_0) for the closed universe (solid curves) and for an open and flat FRW universe (dashed curves). In each case, the upper curves are for $\Omega_0 = 1$ and the lower curves are for $\Omega_0 = 0.1$. The points are from Ref. [6]

sults for a FRW universe. The figure shows that the angular sizes in a flat matter-dominated universe can be roughly similar to those in a low-density closed universe. Therefore, an analysis of the angular sizes of some compact radio sources, which shows consistency with a flat universe [10], may also be consistent with a low-density closed universe. Proper-motion distances of superluminal jets in radio sources at large redshift may provide essentially the same probe as do flux-redshift relations. The common caveat is that evolutionary effects must be understood if these are to provide reliable cosmological tests. It has been proposed that these effects may conceivably be understood well enough to discriminate between open and flat Λ models [11]. Figure 1 illustrates, however, that the difference between the angular sizes for the FRW universe and the closed model for the same value of Ω_0 is quite a bit more dramatic than the difference between open FRW and flat Λ models (cf., Fig. 13.5 in Ref. [12]). Therefore, if the angle-redshift relation can distinguish open and flat Λ models, then the distinction between these and the closed model will be even clearer.

Another classical cosmological test is the number-redshift relation. In the low-density closed universe, the differential number of galaxies per steradian per unit redshift is

$$\frac{dN_{\rm gal}}{dz d\Omega} = \frac{n_0 \sin^2[\chi(z)]}{H_0^3(\Omega_0 + \Omega_t - 1)E(z)}, \tag{7}$$

where n_0 is the local number density of galaxies, and the number per comoving volume is assumed to remain constant. In Fig. 2, we plot the number-redshift relation for the low-density closed universe with Ω_t chosen so that the CMB comes from the antipode and for standard open and flat FRW models. The figure shows that an application of this test, which finds values of Ω_0 near unity in a FRW universe [13], can also be consistent with a low-density closed universe. However, galactic evolutionary effects are realistically quite significant, so this remains a controversial test.

A test for Λ discussed by Alcock and Paczyński [14] may also be an especially effective probe of Ω_t. The redshift thickness δz and angular size $\delta\theta$ of a roughly spherical structure that grows with the expansion of the universe will have a ratio

$$\frac{1}{z}\frac{\delta z}{\delta\theta} = \frac{E(z)\sin[\chi(z)]}{z\sqrt{\Omega_0 + \Omega_t - 1}}. \tag{8}$$

As shown in Fig. 3, this function is significantly lower in a low-density closed universe than it is in an open universe (and in a Λ universe; cf., Fig. 13.9 in Ref. [12]). Furthermore, it depends only very weakly on the value of Ω_0 and therefore provides an Ω_0-independent determination of the geometry. A precise measurement may be feasible with forthcoming quasar surveys [15].

We have also checked the probability for gravitational lensing of sources at high redshift. This test provides perhaps the strongest constraint on Λ models [16], and makes it unlikely that the CMB comes from the antipode of a universe that is closed with the addition of a cosmological constant [1]. The probability for lensing of a source at redshift z_s for $\Omega_0 < 1$ and $\Omega_t + \Omega_0 > 1$ relative to the fiducial case of a standard flat universe is

$$P_{\rm lens} = \frac{15}{4}\left[1 - \frac{1}{(1+z_s)^{1/2}}\right]^{-3} \times \int_0^{z_s} \frac{(1+z)^2}{E(z)}\left[\frac{d_A(0,z)d_A(z,z_s)}{d_A(0,z_s)}\right]^2 dz. \tag{9}$$

The current observational constraint is roughly $P_{\rm lens} \lesssim 5$. If Ω_t is chosen so that the CMB comes from the antipode,

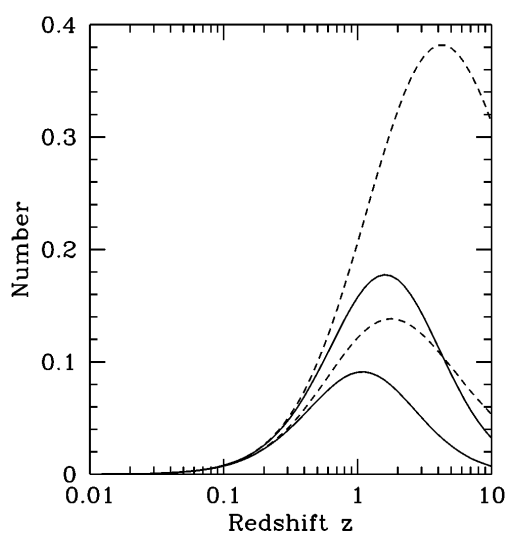

FIG. 2. The differential number of galaxies per unit redshift per steradian in units of $n_0 H_0^{-3}$ for the closed universe (solid curves) and the open FRW universe (dashed curves). The upper curves are for $\Omega_0 = 0.3$ and the lower curves are for $\Omega_0 = 1$.

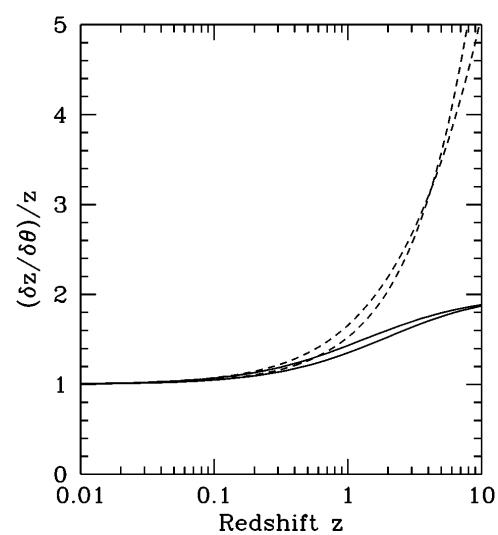

FIG. 3. Plot of $(\delta z/\delta\theta)/z$ for the closed universe (solid curves) and the open FRW universe (dashed curves). Shown are curves for both $\Omega_0 = 0.1$ and $\Omega_0 = 1$.

then $P_{\rm lens} < 2.5$ for $0 < \Omega_0 < 1$. Hence the model is consistent with current data and is likely to remain so.

Finally, if ours is actually a low-density closed universe, it will probably have a dramatic signature in the anisotropy spectrum of the CMB, especially if the CMB comes from the antipode of the universe. Although the detailed shape of the anisotropy spectrum depends on a specific model for structure formation, it quite generically has structure (known as "Doppler peaks") on angular scales smaller than that subtended by the horizon at the surface of last scatter. The angle subtended by the horizon at last scatter depends on the cosmological model; in a standard FRW universe, it is $\theta_{LS} \simeq \Omega^{1/2} 1°$. Therefore, measurement of the location of the first Doppler peak provides a determination of the geometry of the Universe [17], and with forthcoming all-sky CMB maps with subdegree angular resolution, this measurement may be quite precise [18].

The angular scale subtended by the horizon in a low-density closed universe may be approximated by

$$\theta_{LS} \simeq 2° \frac{\sqrt{\Omega_0 + \Omega_t - 1}}{\Omega_0^{1/2} \sin\chi_{LS}}, \tag{10}$$

when θ_{LS} evaluates to small angles; otherwise, $\theta_{LS} = \mathcal{O}(\pi)$. Here,

$$\chi_{LS} = \sqrt{\frac{\Omega_0 + \Omega_t - 1}{1 - \Omega_0}} \operatorname{arcsinh}\left(\frac{2\sqrt{1 - \Omega_0}}{\Omega_0}\right) \tag{11}$$

is the polar-coordinate distance traversed by the CMB photons since last scatter. As expected, this is always larger than θ_{LS} for a flat or open FRW universe. Moreover, if $\chi_{LS} \simeq \pi$, the Doppler-peak structure of the CMB is shifted to the largest angular scales, and the suppression of CMB anisotropies due to Silk damping is also shifted to larger angular scales. The precise shift depends on exactly how close the last-scattering surface is to the antipode. (For example, the anisotropy spectrum might resemble those shown in Fig. 6 of Ref. [1] for the analogous case with a cosmological constant for a flat scale-invariant spectrum of density perturbations. However, the overall tilt of the spectrum depends on the model of primordial perturbations and could therefore be considerably different.) It is almost certain that these signatures will be distinguishable in forthcoming CMB maps if they are indeed there. Additional signatures for a Λ universe have been discussed in Ref. [8].

Although there is no horizon problem in this model, at earlier or later epochs, the CMB is not generally at the antipode. Furthermore, the homogeneity of the Universe is not necessarily explained even if the CMB comes from a causally connected region. Even so, it is worth noting that one can construct a viable model, which is indistinguishable from an open universe at redshifts $z \lesssim 1$, with a closed geometry. Furthermore, the model will be tested by forthcoming observations of the Universe at large redshifts, especially through angular sizes, $\delta z/\delta\theta$, and the CMB.

We have focused in our numerical work on the case where Ω_t is such that the CMB comes precisely from the antipode. However, one could explore other values of Ω_t, perhaps within the context of flat inflationary models.

Finally, what about the homogeneous matter with an energy density which scales as a^{-2}? If this is due to a topologically stabilized scalar-field configuration, as discussed above, then the symmetry-breaking scale must be of the order of the Planck scale if Ω_t is of order unity. Furthermore, the global symmetry must be *exact*. This model would therefore have significant implications for Planck-scale physics if verified [19].

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