

# Supporting Online Material for

## A Photon Turnstile Dynamically Regulated by One Atom

Barak Dayan, A. S. Parkins, Takao Aoki, H. J. Kimble,\* E. P. Ostby, K. J. Vahala

\*To whom correspondence should be addressed. E-mail: hjkimble@caltech.edu

Published 22 February 2008, *Science* **319**, 1062 (2008) DOI: 10.1126/science.1152261

## This PDF file includes:

Materials and Methods Figs. S1 to S4 References

## Supporting Online Information: A Photon Turnstile Dynamically Regulated by One Atom

B. Dayan, A. S. Parkins<sup>a</sup>, Takao Aoki<sup>b</sup>, and H. J. Kimble

Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125, USA

E. Ostby and K. J. Vahala

T. J. Watson Laboratory of Applied Physics, California Institute of Technology, Pasadena, California 91125, USA (Dated: December 18, 2007)

Supporting documentation of the experimental apparatus and theoretical model is provided for our manuscript Ref. [S1].

#### I. EXPERIMENTAL DETAILS

#### A. Setup

An illustration of our apparatus for the measurements in Ref. [S1] is presented in Fig. 1, with further details given in Ref. [S2]. Briefly, a collection of 25 microtoroidal resonators with major diameter  $D \simeq 25 \ \mu m$  and minor diameter  $d \simeq 6 \ \mu m$  is located within a vacuum chamber at  $10^{-9}$  Torr. These resonators are monolithically fabricated from SiO<sub>2</sub> on a Silicon chip by a laser-reflow process [S3]. A probe beam of 5 pW is critically coupled to one particular resonator via a tapered fiber. Critical coupling condition is achieved by adjusting the touching point of the taper on the toroid surface. In the absence of atoms, the forward propagating flux  $\langle a_{out}^{\dagger} a_{out} \rangle$  drops  $100 \times$  for resonant versus nonresonant excitation (i.e.,  $|\alpha_0|^2_{\Delta=0}/|\alpha_0|^2_{\Delta\gg\kappa} \lesssim 10^{-2}$  from Eq. (24) ). Cesium atoms are laser trapped and cooled to  $T \simeq 10\mu$ K at a distance 3 mm above the chip and dropped onto the

Cesium atoms are laser trapped and cooled to  $T \simeq 10\mu$ K at a distance 3 mm above the chip and dropped onto the resonator (Fig. 1A), with some atoms transiting through the evanescent field of the resonator (Fig. 1B). Coupling of individual atoms to the evanescent field modifies the resonance response of the atom-cavity system [S2], resulting in clearly resolved transit events in the forward flux for  $a_{out}$  for individual atoms as recorded by photon counting detectors  $D_1, D_2$  (Fig. 1C). A typical trace for a single drop of the atom cloud is shown in Fig. 1D. For critical coupling with  $\Delta \simeq 0 \simeq \Delta_{AC}$ , these events increase  $\langle a_{out}^{\dagger} a_{out} \rangle$ , with roughly 20 events observed per drop in good



FIG. S1: A – Overall schematic of our experiment in Ref. [S1]. With a coherent input  $a_{in}$ , the interaction of an atom with the cavity fields (a, b) modifies the forward  $a_{out}$  and backward  $b_{out}$  propagating output fields from the cavity. B – A Cesium atom falls through the evanescent field of a microtoroidal resonator. C – The forward propagating field  $a_{out}$  is divided at a 50-50 fiber beamsplitter and directed to two fiber-coupled avalanche photodiodes  $D_1, D_2$  operated in photon counting mode. D – Record of photoelectric detection events  $C(t) \propto \langle a_{out}^{\dagger} a_{out} \rangle$  from the combined outputs of the detectors  $D_1, D_2$ . Evident are peaks above the background that arise from individual transit events as in B [S2].

 $\mathbf{2}$ 

accord with an independent estimate based upon the properties of the atom cloud and structure of the evanescent field of the resonator [S2]. Here,  $\omega_A$  corresponds to the  $6S_{1/2}, F = 4 \rightarrow 6P_{3/2}, F' = 5$  transition in atomic Cesium.  $\omega_p$  is locked to  $\omega_A$  within ±0.5 MHz, while the atom-cavity detuning  $\Delta_{AC}$  is held only to within ±10 MHz, which is however adequate given the broad cavity linewidth (±165 MHz).

#### B. Determination of effective coupling strength

The coherent coupling rate of an atom to the normal modes (A,B) is given by  $g_{A,B} = g_0 f(\rho, z) \{\cos(kx), \sin(kx)\}$ . As described in Ref. [S2], we can experimentally determine  $g_0$  from measurements of transit events as a function of cavity detuning  $\Delta_{AC}$ . Although the transit events are from atoms having different radial distance  $\rho$ , we can define an effective coupling strength  $g_{eff}$  without taking account of  $\rho$  dependence of  $g_{A,B}$ . By following the same procedure as Ref. [S2], we determine  $g_{eff}/2\pi \simeq 70$  MHz. We also obtain the same value from measurements of transit events as a function of intracavity photon number  $\langle n \rangle$ .

#### **II. ANALYSIS RELATED TO FIGURES 3, 4**

In this section, we describe our analysis for extracting transit events for individual atoms from the records  $C_{1,2}(t)$  of photoelectric counts events. For this purpose, we apply a selection criterion to the sum of counts  $\sum_i [C_1(t_i) + C_2(t_i)] \equiv C_{\Sigma}$  over a sliding interval of length  $\Delta t_{atom}$  and require that  $C_{\Sigma} \geq C_{th}$  to accept counts within  $\Delta t_{atom}$  as representing an actual transit event. For  $\Delta t_{atom} = 1.5 \ \mu s$  and  $C_{th} = 4$ , we estimate that the probability for a false positive (i.e., identifying background counts as an actual atom transit) is less than  $10^{-2}$ . Although the results presented here are for  $\Delta t_{atom} = 1.5 \ \mu s$  and  $C_{th} = 4$ , we have verified that they are insensitive to this choice for reasonable values of  $(\Delta t_{atom}, C_{th})$ .

We apply this analysis to  $C_{1,2}(t_i)$  and retain only segments of total duration  $\pm 3 \ \mu$ s around the central window  $\Delta t_{atom}$ , where the time origin t = 0 is determined by the mean of the actual counts in  $\Delta t_{atom}$ . The resulting lists of photocounts are designated as  $A_{1,2}(t_i)$ , from which the probabilities  $p_{1,2}(t_i)$  and  $p_{12}(t_i)$  are determined for single and joint detection events for each atom transit recorded by detectors  $D_{1,2}$ , respectively.

#### III. THEORETICAL MODEL

A schematic of the microtoroidal resonator and fiber taper system investigated in [S1] is shown in Fig. S2. The internal, counter-propagating resonator modes are described in terms of the annihilation operators a and b, while the external input and output fields are described by the operators  $\{a_{in}, a_{out}, b_{in}, b_{out}\}$ . The internal fields suffer an intrinsic loss at the rate  $\kappa_i$  and an extrinsic loss at the rate  $\kappa_{ex}$  due to the fiber coupling. An atom is assumed to couple to the evanescent fields of the intracavity modes with a strength of the form

$$g_{\rm tw} = g_0^{\rm tw} f(\rho, z) \mathrm{e}^{\pm \mathrm{i}kx},\tag{1}$$

where  $\rho$  is the radial distance of the atom from the surface of the toroid, x is the atom's position around the circumference of the toroid, and z is the vertical coordinate. The azimuthal variable  $\phi$  in Ref. [S1] is given by  $\phi = kx$ . The  $\pm$  refers to the clockwise or counterclockwise propagating mode.

#### A. Hamiltonian and master equation

We consider a two-level atom with transition frequency  $\omega_{\rm A}$  and described by the raising and lowering operators  $\sigma^{\pm}$ . The "bare" cavity mode frequency is  $\omega_{\rm C}$ , and the two counterpropagating modes are assumed to be coupled (due, e.g., to scattering off imperfections) with a strength h. A coherent probe field of frequency  $\omega_{\rm p}$  in the input field  $a_{\rm in}$  drives the mode a with strength  $\mathcal{E}_{\rm p}$ . In a frame rotating at the probe frequency, the Hamiltonian for the system can be written in the form [S2, S4]

$$H = \Delta \sigma^{+} \sigma^{-} + \Delta \left( a^{\dagger} a + b^{\dagger} b \right) + h \left( a^{\dagger} b + b^{\dagger} a \right) + \left( \mathcal{E}_{p}^{*} a + \mathcal{E}_{p} a^{\dagger} \right) + \left( g_{tw}^{*} a^{\dagger} \sigma^{-} + g_{tw} \sigma^{+} a \right) + \left( g_{tw} b^{\dagger} \sigma^{-} + g_{tw}^{*} \sigma^{+} b \right),$$

$$(2)$$



FIG. S2: Schematic of microtoroid and fiber coupler.

where, specifically,  $g_{tw} = g_0^{tw} f(r) e^{ikx}$ , and we assume the atom and cavity frequencies to be resonant, so that the detuning  $\Delta = \omega_A - \omega_p = \omega_C - \omega_p$ . Introducing dissipation, the system can be described by the master equation

$$\dot{\rho} = -i [H, \rho] + \kappa \left( 2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a \right) + \kappa \left( 2b\rho b^{\dagger} - b^{\dagger}b\rho - \rho b^{\dagger}b \right) + \frac{\gamma}{2} \left( 2\sigma^{-}\rho\sigma^{+} - \sigma^{+}\sigma^{-}\rho - \rho\sigma^{+}\sigma^{-} \right), \qquad (3)$$

where  $\rho$  is the density operator for the atom-cavity system,  $\kappa = \kappa_i + \kappa_{ex}$  is the field decay rate for the cavity modes, and  $\gamma = 2\gamma_{\perp}$  is the atomic spontaneous emission rate. For a fully quantum mechanical treatment of the system we can compute numerical solutions to the master equation (3) using truncated number state bases for the cavity modes.

#### B. Normal mode representation

The Hamiltonian and master equation for the atom-cavity system can also be usefully expressed in terms of the normal modes of the cavity, which are defined by the operator combinations

$$A = \frac{a+b}{\sqrt{2}}, \qquad B = \frac{a-b}{\sqrt{2}}.$$
(4)

In particular, one can show that

$$\dot{\rho} = -i[H,\rho] + \kappa \left(2A\rho A^{\dagger} - A^{\dagger}A\rho - \rho A^{\dagger}A\right) + \kappa \left(2B\rho B^{\dagger} - B^{\dagger}B\rho - \rho B^{\dagger}B\right) + \frac{\gamma}{2} \left(2\sigma^{-}\rho\sigma^{+} - \sigma^{+}\sigma^{-}\rho - \rho\sigma^{+}\sigma^{-}\right),$$
(5)

with

$$H = \Delta \sigma^{+} \sigma^{-} + (\Delta + h) A^{\dagger} A + (\Delta - h) B^{\dagger} B + \frac{1}{\sqrt{2}} \left( \mathcal{E}_{p}^{*} A + \mathcal{E}_{p} A^{\dagger} \right) + \frac{1}{\sqrt{2}} \left( \mathcal{E}_{p}^{*} B + \mathcal{E}_{p} B^{\dagger} \right) + g_{A} \left( A^{\dagger} \sigma^{-} + \sigma^{+} A \right) - i g_{B} \left( B^{\dagger} \sigma^{-} - \sigma^{+} B \right),$$
(6)

where  $g_A = g_0 f(\rho, z) \cos(kx)$ ,  $g_B = g_0 f(\rho, z) \sin(kx)$ , and  $g_0 = \sqrt{2}g_0^{\text{tw}}$ . So, depending on the position of the atom, coupling may occur predominantly (or even exclusively) to only one of the two normal modes.

In the absence of an atom  $(g_0 = 0)$ , the normal mode steady state amplitudes are readily derived as

$$\alpha_A = \langle A \rangle = -\left(\frac{\mathrm{i}}{\sqrt{2}}\right) \frac{\mathcal{E}_{\mathrm{p}}}{\kappa + \mathrm{i}(\Delta + h)}, \qquad \alpha_B = \langle B \rangle = -\left(\frac{\mathrm{i}}{\sqrt{2}}\right) \frac{\mathcal{E}_{\mathrm{p}}}{\kappa + \mathrm{i}(\Delta - h)}. \tag{7}$$

#### C. Input and output fields

From the theory of inputs and outputs in optical cavities [S5], the output field operators for the two cavity modes are given in terms of the input and intracavity field operators as

$$a_{\rm out}(t) = -a_{\rm in}(t) + \sqrt{2\kappa_{\rm ex}} a(t) = -a_{\rm in}(t) + \sqrt{\kappa_{\rm ex}} \left[A(t) + B(t)\right], \tag{8}$$

$$b_{\rm out}(t) = -b_{\rm in}(t) + \sqrt{2\kappa_{\rm ex}} b(t) = -b_{\rm in}(t) + \sqrt{\kappa_{\rm ex}} \left[A(t) - B(t)\right], \qquad (9)$$

where  $[a_{\rm in}(t), a_{\rm in}^{\dagger}(t')] = [a_{\rm out}(t), a_{\rm out}^{\dagger}(t')] = \delta(t - t')$ , and similarly  $[b_{\rm in}(t), b_{\rm in}^{\dagger}(t')] = [b_{\rm out}(t), b_{\rm out}^{\dagger}(t')] = \delta(t - t')$ . The particular forms of the probe driving terms in the Hamiltonian correspond to coherent amplitudes of the input fields given by

$$\langle a_{\rm in} \rangle = -\frac{\mathrm{i}\mathcal{E}_{\rm p}}{\sqrt{2\kappa_{\rm ex}}}, \qquad \langle b_{\rm in} \rangle = 0.$$
 (10)

and corresponding input photon fluxes of

$$\left|\langle a_{\rm in} \rangle\right|^2 = \frac{\left|\mathcal{E}_{\rm p}\right|^2}{2\kappa_{\rm ex}}, \qquad \left|\langle b_{\rm in} \rangle\right|^2 = 0.$$
(11)

#### IV. ADIABATIC ELIMINATION OF THE CAVITY MODES

For the situation in which the cavity field decay rate,  $\kappa$ , is much larger than the atom-field coupling strength, we may use standard methods (see, e.g., [S6]) to adiabatically eliminate the cavity modes from the system dynamics. The resulting master equation for the reduced density operator of the two-level atom,  $\rho_A$ , takes the form

$$\dot{\rho}_{A} = -i[H_{A},\rho_{A}] + \frac{\Gamma}{2} \left( 2\sigma^{-}\rho_{A}\sigma^{+} - \sigma^{+}\sigma^{-}\rho_{A} - \rho_{A}\sigma^{+}\sigma^{-} \right), \qquad (12)$$

where

$$H_{\rm A} = \Delta_{\rm A} \sigma^+ \sigma^- + \left(\Omega_0 \sigma^+ + \Omega_0^* \sigma^-\right),\tag{13}$$

with  $\Omega_0 = g_A \alpha_A + i g_B \alpha_B$ ,

$$\Delta_{\rm A} = \Delta - \frac{g_A^2 (\Delta + h)}{\kappa^2 + (\Delta + h)^2} - \frac{g_B^2 (\Delta - h)}{\kappa^2 + (\Delta - h)^2} \tag{14}$$

and

$$\Gamma = \gamma + \frac{2g_A^2\kappa}{\kappa^2 + (\Delta+h)^2} + \frac{2g_B^2\kappa}{\kappa^2 + (\Delta-h)^2}.$$
(15)

The effective detuning  $\Delta_A$  is modified from  $\Delta$  by light shifts due to the (off-resonant) coupling with the normal modes at frequencies  $\Delta \pm h$ , while the parameter  $\Gamma$  gives the cavity-enhanced atomic decay rate. For the case  $\Delta = 0$ , it reduces to

$$\Gamma = \gamma + \frac{2(g_A^2 + g_B^2)\kappa}{\kappa^2 + h^2} = \gamma + \frac{2g_0^2 |f(\rho, z)|^2 \kappa}{\kappa^2 + h^2},$$
(16)

which we can write in the form  $\Gamma = \gamma(1 + 2C)$  with the cooperativity parameter

$$C = \frac{g_0^2 |f(\rho, z)|^2 \kappa}{\gamma(\kappa^2 + h^2)}.$$
(17)

## V. CAVITY-MODIFIED OPTICAL BLOCH EQUATIONS

From the master equation (12) we can derive effective optical Bloch equations for the two-level atom. Defining  $\sigma_x = \sigma^+ + \sigma^-$ ,  $\sigma_y = -i(\sigma^+ - \sigma^-)$ , and  $\sigma_z = \sigma^+ \sigma^- - \sigma^- \sigma^+$ , these equations take the form

$$\langle \dot{\sigma_x} \rangle = -\frac{\Gamma}{2} \langle \sigma_x \rangle - \Delta_A \langle \sigma_y \rangle - 2 \operatorname{Im} \{\Omega_0\} \langle \sigma_z \rangle,$$
 (18a)

$$\langle \sigma_y \rangle = -\frac{\Gamma}{2} \langle \sigma_y \rangle + \Delta_A \langle \sigma_x \rangle - 2 \operatorname{Re}\{\Omega_0\} \langle \sigma_z \rangle,$$
 (18b)

$$\dot{\langle \sigma_z \rangle} = -\Gamma \left( 1 + \langle \sigma_z \rangle \right) + 2 \operatorname{Im} \{ \Omega_0 \} \langle \sigma_x \rangle + 2 \operatorname{Re} \{ \Omega_0 \} \langle \sigma_y \rangle,$$
 (18c)

and have the steady state solutions

$$\langle \sigma_x \rangle = -\frac{2\Delta_{\mathcal{A}} \operatorname{Re}\{\Omega_0\} - \Gamma \operatorname{Im}\{\Omega_0\}}{(\Gamma/2)^2 + \Delta_{\mathcal{A}}^2 + 2|\Omega_0|^2},$$
(19a)

$$\langle \sigma_y \rangle = \frac{2\Delta_A \operatorname{Im}\{\Omega_0\} + \Gamma \operatorname{Re}\{\Omega_0\}}{(\Gamma/2)^2 + \Delta_A^2 + 2|\Omega_0|^2},$$
(19b)

$$\langle \sigma_z \rangle = -\frac{(\Gamma/2)^2 + \Delta_A^2}{(\Gamma/2)^2 + \Delta_A^2 + 2|\Omega_0|^2},$$
 (19c)

### VI. OUTPUT FIELDS

To study the properties of the output fields, we need to compute moments of the output field operators, as defined in (8,9). We can rewrite the expressions (8,9) in the forms

$$a_{\rm out}(t) = -\langle a_{\rm in} \rangle - a'_{\rm in}(t) + \sqrt{\kappa_{\rm ex}} \left[ A(t) + B(t) \right] = \frac{i\mathcal{E}_p}{\sqrt{2\kappa_{\rm ex}}} - a'_{\rm in}(t) + \sqrt{\kappa_{\rm ex}} \left[ A(t) + B(t) \right], \tag{20}$$

$$b_{\rm out}(t) = -\langle b_{\rm in} \rangle - b'_{\rm in}(t) + \sqrt{\kappa_{\rm ex}} \left[ A(t) - B(t) \right] = -b'_{\rm in}(t) + \sqrt{\kappa_{\rm ex}} \left[ A(t) - B(t) \right], \tag{21}$$

where  $a'_{in}(t)$  and  $b'_{in}(t)$  are vacuum noise input operators. Quantum Langevin equations for the normal mode operators A, B can be derived from the Hamiltonian (6), together with the input-output theory of [S5]. From these equations, in the adiabatic regime of large  $\kappa$ , we can write

$$A(t) \simeq \alpha_A - \frac{\mathrm{i}g_A}{\kappa + \mathrm{i}(\Delta + h)} \,\sigma^-(t) + \frac{\sqrt{2\kappa}}{\kappa + \mathrm{i}(\Delta + h)} \,A_{\mathrm{in}}(t), \tag{22}$$

$$B(t) \simeq \alpha_B - \frac{g_B}{\kappa + i(\Delta - h)} \sigma^-(t) + \frac{\sqrt{2\kappa}}{\kappa + i(\Delta - h)} B_{in}(t), \qquad (23)$$

where  $A_{in}(t)$  and  $B_{in}(t)$  are independent, vacuum noise input operators. As far as the computation of normallyordered moments of the output fields (e.g.,  $\langle a_{out}^{\dagger}a_{out}\rangle$  and  $\langle a_{out}^{\dagger}a_{out}^{\dagger}a_{out}\rangle$ ) are concerned, we can therefore make the following substitutions in the correlation functions:

$$a_{\rm out} \to \alpha_0 + \alpha_- \sigma^-, \qquad b_{\rm out} \to \beta_0 + \beta_- \sigma^-$$
 (24)

corresponding to Eq. (1) of [S1], with

$$\alpha_0 = \frac{\mathrm{i}\mathcal{E}_p}{\sqrt{2\kappa_{\mathrm{ex}}}} + \sqrt{\kappa_{\mathrm{ex}}} \left(\alpha_A + \alpha_B\right), \qquad (25)$$

$$\alpha_{-} = \sqrt{\kappa_{\text{ex}}} \left[ -\frac{\mathrm{i}g_A}{\kappa + \mathrm{i}(\Delta + h)} - \frac{g_B}{\kappa + \mathrm{i}(\Delta - h)} \right],\tag{26}$$

$$\beta_0 = \sqrt{\kappa_{\rm ex}} \left( \alpha_A - \alpha_B \right), \tag{27}$$

$$\beta_{-} = \sqrt{\kappa_{\text{ex}}} \left[ -\frac{\mathrm{i}g_A}{\kappa + \mathrm{i}(\Delta + h)} + \frac{g_B}{\kappa + \mathrm{i}(\Delta - h)} \right].$$
(28)

So, in particular, for the output photon fluxes we have

$$\langle a_{\text{out}}^{\dagger} a_{\text{out}} \rangle = |\alpha_0|^2 + \alpha_0^* \alpha_- \langle \sigma^- \rangle + \alpha_0 \alpha_-^* \langle \sigma^+ \rangle + |\alpha_-|^2 \langle \sigma^+ \sigma^- \rangle$$

$$= |\alpha_0|^2 + \operatorname{Re}\{\alpha_0^* \alpha_-\} \langle \sigma_x \rangle + \operatorname{Im}\{\alpha_0^* \alpha_-\} \langle \sigma_y \rangle + \frac{1}{2} |\alpha_-|^2 \left(1 + \langle \sigma_z \rangle\right),$$

$$\langle b_{\text{out}}^{\dagger} b_{\text{out}} \rangle = |\beta_0|^2 + \beta_0^* \beta_- \langle \sigma^- \rangle + \beta_0 \beta_-^* \langle \sigma^+ \rangle + |\beta_-|^2 \langle \sigma^+ \sigma^- \rangle$$

$$(29)$$

$$|\beta_0|^2 + \beta_0 \beta_- \langle \sigma^- \rangle + \beta_0 \beta_- \langle \sigma^+ \rangle + |\beta_-|^2 \langle \sigma^+ \sigma^- \rangle$$

$$= |\beta_0|^2 + \operatorname{Re}\{\beta_0^* \beta_-\} \langle \sigma_x \rangle + \operatorname{Im}\{\beta_0^* \beta_-\} \langle \sigma_y \rangle + \frac{1}{2} |\beta_-|^2 (1 + \langle \sigma_z \rangle),$$

$$(30)$$

while for the second-order correlation functions we have

$$\langle a_{\text{out}}^{\dagger} a_{\text{out}}^{\dagger} a_{\text{out}} a_{\text{out}} \rangle = |\alpha_0|^2 \left( |\alpha_0|^2 + 2\alpha_0^* \alpha_- \langle \sigma^- \rangle + 2\alpha_0 \alpha_-^* \langle \sigma^+ \rangle + 4|\alpha_-|^2 \langle \sigma^+ \sigma^- \rangle \right)$$

$$= |\alpha_0|^2 \left[ |\alpha_0|^2 + 2\text{Re}\{\alpha_0^* \alpha_-\} \langle \sigma_x \rangle + 2\text{Im}\{\alpha_0^* \alpha_-\} \langle \sigma_y \rangle + 2|\alpha_-|^2 \left(1 + \langle \sigma_z \rangle \right) \right], \qquad (31)$$

$$\langle b_{\text{out}}^{\dagger} b_{\text{out}}^{\dagger} b_{\text{out}} b_{\text{out}} \rangle = |\beta_0|^2 \left( |\beta_0|^2 + 2\beta_0^* \beta_- \langle \sigma^- \rangle + 2\beta_0 \beta_-^* \langle \sigma^+ \rangle + 4|\beta_-|^2 \langle \sigma^+ \sigma^- \rangle \right)$$

$$= |\beta_0|^2 \left[ |\beta_0|^2 + 2\operatorname{Re}\{\beta_0^*\beta_-\}\langle\sigma_x\rangle + 2\operatorname{Im}\{\beta_0^*\beta_-\}\langle\sigma_y\rangle + 2|\beta_-|^2 \left(1 + \langle\sigma_z\rangle\right) \right].$$
(32)

#### VII. TRANSMISSION SPECTRA

The normalized output photon fluxes in the forwards and backwards directions are defined by

$$T_{\rm F} = \frac{\langle a_{\rm out}^{\dagger} a_{\rm out} \rangle}{\langle a_{\rm out}^{\dagger} a_{\rm out} \rangle_{\Delta \gg \kappa}} = \frac{\langle a_{\rm out}^{\dagger} a_{\rm out} \rangle}{|\mathcal{E}_{\rm p}|^2 / (2\kappa_{\rm ex})}, \qquad T_{\rm B} = \frac{\langle b_{\rm out}^{\dagger} b_{\rm out} \rangle}{\langle a_{\rm out}^{\dagger} a_{\rm out} \rangle_{\Delta \gg \kappa}} = \frac{\langle b_{\rm out}^{\dagger} b_{\rm out} \rangle}{|\mathcal{E}_{\rm p}|^2 / (2\kappa_{\rm ex})}.$$
(33)

For detunings  $|\Delta| \ll \kappa, h$  (where  $\alpha_0 \simeq 0$ ) and weak driving (i.e.,  $|\Omega_0|$  small), we can write

$$\langle a_{\text{out}}^{\dagger} a_{\text{out}} \rangle \simeq \frac{1}{2} |\alpha_{-}|^{2} \left(1 + \langle \sigma_{z} \rangle\right) \simeq \frac{|\alpha_{-}|^{2} |\Omega_{0}|^{2}}{(\Gamma/2)^{2} + \Delta_{A}^{2}} \,. \tag{34}$$

For  $kx = \pi/2$  as in Fig. 1 of [S1] we have  $|\alpha_-|^2 \simeq \kappa_{\text{ex}} g_B^2/(\kappa^2 + h^2)$ ,  $|\Omega_0|^2 \simeq g_B^2 |\mathcal{E}_p|^2/2(\kappa^2 + h^2)$ ,  $\Delta_A \simeq \Delta + g_B^2 h/(\kappa^2 + h^2)$ , and  $\Gamma \simeq \gamma + 2g_B^2 \kappa/(\kappa^2 + h^2)$ , so (34) describes a Lorentzian in  $\Delta$  of full width  $\Gamma$  centered at  $\Delta = -g_B^2 h/(\kappa^2 + h^2)$ . For  $2g_B^2 \kappa/(\kappa^2 + h^2) \gg \gamma$  (i.e., large single-atom cooperativity C), the normalized maximum of this Lorentzian is approximated by  $(\kappa_{\text{ex}}/\kappa)^2$ , which can approach unity at critical coupling for  $h \gg \kappa_i$ , as illustrated in Fig. 1B of [S1].

#### VIII. PHOTON STATISTICS

For resonant driving  $(\Delta = 0)$ , where  $\alpha_0 = 0$ , it follows immediately from (29) and (31) that

$$g_{\rm F}^{(2)}(\tau=0) = \frac{\langle a_{\rm out}^{\dagger} a_{\rm out}^{\dagger} a_{\rm out} a_{\rm out} \rangle}{\langle a_{\rm out}^{\dagger} a_{\rm out} \rangle^2} = 0,$$

i.e., the light transmitted into  $a_{out}$  is sub-Poissonian as a consequence of the photon blockade mechanism described in [S1]. This result is illustrated numerically in Fig. 1D of [S1] (reproduced in Fig. S3 below) and demonstrated experimentally in Fig. 3B of [S1] for  $\Delta \simeq 0$ .

In contrast, the light transmitted into  $b_{out}$  in this regime is strongly bunched, as illustrated in Fig. S3 below. With the assumption of weak driving, and for  $|\Delta| \ll \kappa, h$ , one may show from the expressions given above for  $\langle b_{out}^{\dagger} b_{out}^{\dagger} b_{out} b_{out} \rangle$  and  $\langle b_{out}^{\dagger} b_{out} \rangle$  that, at a detuning  $\Delta \simeq -g_B^2 h/(\kappa^2 + h^2)$  (where the central resonances in  $T_{\rm F}$  and  $T_{\rm B}$  are located),

$$g_{\rm B}^{(2)}(\tau=0) = \frac{\langle b_{\rm out}^{\dagger} b_{\rm out}^{\dagger} b_{\rm out} \rangle}{\langle b_{\rm out}^{\dagger} b_{\rm out} \rangle^2} \simeq \frac{16h^2 \kappa^2 \left(\kappa^4 + h^4 - h^2 \kappa^2\right)}{\left(\kappa^4 - h^4\right)^2} \gg 1 \quad \text{for} \ h \simeq \kappa.$$
(35)

These contrasting behaviors of  $g_{\rm F}^{(2)}(\tau=0)$  and  $g_{\rm B}^{(2)}(\tau=0)$  at  $\Delta \simeq 0$  form the basis of the photon turnstile. For  $\kappa \simeq \kappa_{\rm ex}$  (i.e., low intrinsic losses in the resonator), such that  $T_{\rm F}(\Delta \simeq 0) \simeq 1$  and  $T_{\rm B}(\Delta \simeq 0) \simeq 0$  (Figs. 1B,C of [S1]), this turnstile can efficiently regulate the transmission of photons into the forward and backward propagating fields of the fiber coupler.

Returning briefly to  $g_{\rm F}^{(2)}(\tau=0)$ , we note from Fig. S3 that the sub-Poissonian nature of the forward light persists over a reasonably wide range of detunings (about  $\Delta = 0$ ), and survives averaging over the azimuthal coordinate  $\phi = kx$ , as demonstrated in Fig. 1G of [S1]. For a certain range of detunings it is also apparent that strong bunching can occur; the center of this range coincides with a distinct minimum in the transmitted forward flux (in the presence of an atom) where, in particular,  $T_F(\Delta) \simeq 0$  (see Fig. 1B in [S1]). The extreme bunching is associated with a large relative increase in the forward flux given the detection of a first photon in  $a_{\rm out}$  [S8].

While the curves in Fig. S3 are for the case of small intrinsic losses, i.e.,  $\kappa_i \ll \kappa \simeq \kappa_{ex} \simeq h$ , in Fig. S4 we plot the corresponding results for the parameters actually realized in the experiment reported in [S1]. Note that the range of  $\Delta$  considered is reduced, given that in this case  $\kappa/2\pi = 165$  MHz (*cf.* 255 MHz in Fig. S3). The large scale structure of the curves differs somewhat from Fig. S3, but the key features of  $g_F^{(2)}(\tau = 0)$  and  $g_B^{(2)}(\tau = 0)$  at  $\Delta \simeq 0$  relating to the photon turnstile effect are still clearly evident. Note that in contrast to the sub-Poissonian nature of the resonant, forward light, the backward light exhibits slight bunching on resonance.



FIG. S3: Intensity correlation functions  $g_{\rm F}^{(2)}(\tau=0)$  (left) and  $g_{\rm B}^{(2)}(\tau=0)$  (right) as a function of probe detuning, for parameters as in Figs. 1B-D of [S1], i.e., for  $(g_0, \kappa_{\rm i}, h, \gamma)/2\pi = (70, 5, 250, 1)$  MHz and  $\kappa_{\rm ex} = \kappa_{\rm ex}^{\rm cr}$ , with  $f(\rho, z) = 1$  and azimuthal position  $\phi = kx = \pi/2$ .



FIG. S4: Intensity correlation functions  $g_{\rm F}^{(2)}(\tau=0)$  (left) and  $g_{\rm B}^{(2)}(\tau=0)$  (right) as a function of probe detuning, for parameters as realized in the experiment reported in [S1], i.e.,  $(g_0, \kappa_{\rm i}, h, \gamma)/2\pi = (70, 75, 50, 5.2)$  MHz and  $\kappa_{\rm ex}/2\pi = \kappa_{\rm ex}^{\rm cr}/2\pi = 90$  MHz, with  $f(\rho, z) = 1$  and azimuthal position  $\phi = kx = \pi/2$ .

For  $\Delta = 0$  the time-dependent correlation function,  $g_{\rm F}^{(2)}(\tau)$ , can be computed straightforwardly from solutions to the modified optical Bloch equations; in particular, one can show that [S7]

$$g_{\rm F}^{(2)}(\tau) = \frac{1 + \langle \sigma_z(\tau) \rangle_{\rho_{\rm A}(0) = |0\rangle\langle 0|}}{2\langle \sigma^+ \sigma^- \rangle},$$

which, for weak driving (i.e.,  $|\Omega_0| \ll \Gamma/2$ ), reduces to

$$g_{\rm F}^{(2)}(\tau) \simeq \left(1 - {\rm e}^{-\Gamma t/2}\right)^2.$$

This closely approximates the theoretical curve shown in Fig. 4 of [S1] and reaches a value of 0.5 at the time

$$\tau_B = -\frac{2}{\Gamma} \ln\left(1 - \frac{1}{\sqrt{2}}\right) \simeq \frac{2.5}{\Gamma}$$

which we can use to characterize the time over which the photon blockade mechanism operates.

- [S1] B. Dayan, A. S. Parkins, Takao Aoki, H. J. Kimble, E. Ostby, and K. J. Vahala, manuscript submitted to Science, October 2007.
- [S2] See Supplementary Information to T. Aoki, B. Dayan, E. Wilcut, W. P. Bowen, A. S. Parkins, H. J. Kimble, T. J. Kippenberg, and K. J. Vahala, Nature 443, 671 (2006).
- [S3] D. K. Armani et al., Nature 421, 925 (2003).
- [S4] K. Srinivasan and O. Painter, Phys. Rev. A 75, 023814 (2007).
- [S5] M. J. Collett and C. W. Gardiner, Phys. Rev. A 30, 1386 (1984); C. W. Gardiner and M. J. Collett, *ibid.* 31, 3761 (1985).
- [S6] C. W. Gardiner and P. Zoller, Quantum Noise, 2nd Ed. (Springer-Verlag, Berlin, 1999).
- [S7] H. J. Carmichael, Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations (Springer-Verlag, Berlin, 1999).
- [S8] H. J. Carmichael, Phys. Rev. Lett. 70, 2273 (1993)
  - <sup>a</sup> TA PRESTO, Japan Science and Technology Agency, Saitama, Japan
  - <sup>b</sup> ASP Department of Physics, University of Auckland, Auckland, New Zealand