Cosmic 21 cm Delensing of Microwave Background Polarization and the Minimum Detectable Energy Scale of Inflation

Kris Sigurdson[1,*] and Asantha Cooray[2,†]

[1]California Institute of Technology, Mail Code 130-33, Pasadena, California 91125, USA

[2]Department of Physics and Astronomy, 4186 Frederick Reines Hall, University of California, Irvine, California 92697, USA

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We propose a new method for removing gravitational lensing from maps of cosmic microwave background (CMB) polarization anisotropies. Using observations of anisotropies or structures in the cosmic 21 cm radiation, emitted or absorbed by neutral hydrogen atoms at redshifts 10 to 200, the CMB can be delensed. We find this method could allow CMB experiments to have increased sensitivity to a background of inflationary gravitational waves (IGWs) compared to methods relying on the CMB alone and may constrain models of inflation which were heretofore considered to have undetectable IGW amplitudes.

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Introduction.—Cosmological inflation [1] provides an explanation for both the predominant isotropy and homogeneity of the Universe and also the tiny, a few parts in 10^5, anisotropies and inhomogeneities observed in the cosmic microwave background (CMB) radiation. These tiny perturbations grew via gravitational instability to form the large-scale structure and galaxies we see today. While the details of a theory of inflation remain a mystery, a generic outcome of inflationary models is the generation of a long-wavelength background of gravitational waves [2]. Since the amplitude of inflationary gravitational waves (IGWs) is proportional to $\mathcal{V}$, the value of the inflaton potential $V(\varphi)$ during inflation, a reliable detection of IGWs could establish the energy scale of inflation $\mathcal{V}^{1/4}$ [3] and help discriminate between physical models of the early Universe.

A promising technique to detect IGWs is to observe and study the polarization statistics of the CMB in detail. Since gravitational waves have a handedness, they lead to curl-like patterns of polarization (the B modes) in the two-dimensional CMB polarization field [4]. Density perturbations (which are scalar rather than tensor fluctuations and possess no handedness) cannot produce curl-like patterns to leading order and instead produce gradientlike patterns (the E modes). The main confusion to the detection of the IGW polarization signal is the transfer of power in the E modes to the B modes via gravitational lensing of CMB photons by foreground density fluctuations between our detectors and the CMB surface of last scattering [5].

In this Letter we propose a new method for discriminating between the lensing-induced contamination and the B modes from the IGW polarization signal. This method, described in detail below, relies on observations of the cosmic 21 cm radiation emitted or absorbed by neutral hydrogen atoms at redshifts 10 to 200. We begin by summarizing how the lensing confusion arises.

Gravitational lensing.—Lensing induces a remapping of the CMB polarization field at the last-scattering surface ${}_{\pm}X(\hat{\mathbf{n}})$ such that ${}_{\pm}\tilde{X}(\hat{\mathbf{n}}) = {}_{\pm}X[\hat{\mathbf{n}} + \nabla\phi(\hat{\mathbf{n}})]$ is the observed polarization field, where ${}_{\pm}X = Q \pm iU$ are linear combinations of the Stokes parameters Q and U and $\alpha(\hat{\mathbf{n}}) = \nabla\phi(\hat{\mathbf{n}})$ is the lensing deflection angle. Here,

$$\phi(\hat{\mathbf{n}}; z_s) = -2\int_0^{r(z_s)} dr' \frac{r - r'}{r' r} \Phi(\hat{\mathbf{n}}, r') \tag{1}$$

is the deflection potential, a line-of-sight projection of the gravitational potential Φ to redshift z_s. The total lensing potential $\phi(\hat{\mathbf{n}}) \equiv \phi(\hat{\mathbf{n}}; z_{\rm CMB})$ is this quantity evaluated at $z_s \to z_{\rm CMB} \approx 1100$.

Using the flat-sky approximation and the E-mode or B-mode decomposition [4], the lensed B-mode polarization power spectrum, in the relevant limit $C_l^{BB} \ll C_l^{EE}$, is

$$\tilde{C}_l^{BB} = C_l^{BB} + \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} [\boldsymbol{l}'' \cdot \boldsymbol{l}']^2 \sin^2(2\theta_l') C_{l''}^{\phi\phi} C_{l'}^{EE}, \tag{2}$$

where $\boldsymbol{l}'' = \boldsymbol{l} - \boldsymbol{l}'$. The second term in this expression is the lensing confusion in the B-mode map which must be separated from C_l^{BB} (the IGW signal). Here, $C_l^{\phi\phi}$ is the angular power spectrum of the total deflection potential and is simply related to a weighted projection of the matter power spectrum [5]; $C_l^{\phi\phi}(z_s)$ is the incomplete power spectrum out to source redshift $z_s < z_{\rm CMB}$.

Unlike the B modes generated by IGWs, C_l^{EE} is dominated by larger amplitude scalar perturbations. The expected few-percent conversion of E modes creates a large signal in the B-mode power spectrum [5]. We can relate the amplitude of fluctuations, in terms of the tensor-to-scalar ratio $\mathcal{T}/S$, to the inflationary energy scale via [6]

$$\mathcal{V}^{1/4} = 3.0 \times 10^{-3} (\mathcal{T}/S)^{1/4} m_{Pl}. \tag{3}$$

For a tensor-to-scalar ratio below 2.6×10^{-4} or $\mathcal{V}^{1/4}$ below 4.6×10^{15} GeV, the IGW signal is completely confused by the lensing contaminant [7]. To bypass this limit one must separate the lensing-induced B modes from those due to IGWs. Clearly, the lensing confusion could be exactly removed if one knew the three-dimensional distribution of mass out to the CMB last-scattering surface. However, as our goal is a measurement of C_l^{BB}, knowledge of the projected quantity $\phi(\hat{\mathbf{n}})$ is sufficient. One way to estimate $\phi(\hat{\mathbf{n}}; z_s)$ is by using quadratic estimators or maximum likelihood methods to statistically infer the deflection-angle field given some lensed random field $\tilde{\chi}(\hat{\mathbf{n}})$ at source redshift z_s. For instance, arcminute resolution CMB temperature and polarization maps could be used to make such an estimate [8–10]. Another way to estimate $\phi(\hat{\mathbf{n}}; z_s)$ is by observing the weak-lensing distortions of the shapes of objects of a known average shape at source redshift z_s. However, observations of the weak lensing of galaxies, which have $z_s \sim 1$–2, cannot be used to delense CMB maps because an order unity fraction of the lensing contamination arises from the structure at $z > 3$. Higher source redshifts are needed for effective delensing.

Cosmic 21 cm radiation.—Neutral atoms kinetically decouple from the thermal bath of CMB photons at $z \sim 200$ and cool adiabatically as $T_g \propto (1+z)^2$ [11]. Since the spin temperature of hydrogen atoms remains collisionally coupled to T_g these atoms resonantly absorb CMB photons at $\lambda_{21} = 21.1$ cm (the hyperfine transition of the ground state of hydrogen). The cosmic 21 cm radiation is thus first observable in absorption by low-frequency radio telescopes which could detect brightness-temperature fluctuations at wavelength $\lambda = \lambda_{21}(1+z)$ [12–14]. During reionization, the neutral gas distribution is likely to be complex due to the first luminous sources [15] and cosmic 21 cm signatures shift to emission [16]. Yet, even before reionization, it is possible the 21 cm sky is brightened by emission from neutral hydrogen gas contained in minihalos with masses $\sim 10^3$–10^7 $M_\odot$ [17].

Like the CMB, 21 cm fluctuations are gravitationally lensed by the foreground large-scale mass distribution. Since 21 cm radiation probes an epoch very close to CMB decoupling, CMB and 21 cm radiation are affected, essentially, by the same foreground mass distribution.

Quadratic estimators.—Quadratic estimators can be used to extract lensing information from the gravitationally lensed field $\tilde{\chi}(\hat{\mathbf{n}})$ of some intrinsic field $\chi(\hat{\mathbf{n}})$ at redshift z_s. The quadratic form $\nabla \cdot [\tilde{\chi}(\hat{\mathbf{n}}) \nabla \tilde{\chi}(\hat{\mathbf{n}})]$ provides an estimate of the deflection angle at position $\hat{\mathbf{n}}$ on the sky given the $\tilde{\chi}$ anisotropy map. For the CMB, the quantity $\tilde{\chi}$ could be the temperature anisotropies [9], the polarization anisotropies, or some combination of both [10]. The brightness-temperature fluctuations in the 21 cm transition of neutral hydrogen at redshifts 10 to 200 could similarly be used [18].

In Fourier space, the quadratic estimator for the deflection potential is

$$\hat{\phi}(\boldsymbol{l}; z_s) = \mathcal{Q}_l(z_s) \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} (\boldsymbol{l} \cdot \boldsymbol{l}' C_{l'}^{\chi\chi} + \boldsymbol{l} \cdot \boldsymbol{l}'' C_{l''}^{\chi\chi}) \frac{\chi(\boldsymbol{l}')\chi(\boldsymbol{l}'')}{2T_{l'}^{\chi\chi} T_{l''}^{\chi\chi}}, \tag{4}$$

where $C_l^{\chi\chi}$ is the unlensed power spectrum and $T_l^{\chi\chi} = \tilde{C}_l^{\chi\chi} + N_l^{\chi\chi}$ is to total power spectrum, including lensing corrections and a noise power spectrum $N_l^{\chi\chi}$. The expectation value of the deflection-potential estimator $\langle \hat{\phi}(\boldsymbol{l}; z_s) \rangle$ (the ensemble average over realizations of the random field χ) is just $\phi(\boldsymbol{l}; z_s)$. Here,

$$[\mathcal{Q}_l(z_s)]^{-1} = \int \frac{d^2 \boldsymbol{l}'}{(2\pi)^2} \frac{(\boldsymbol{l} \cdot \boldsymbol{l}' C_{l'}^{\chi\chi} + \boldsymbol{l} \cdot \boldsymbol{l}'' C_{l''}^{\chi\chi})^2}{2T_{l'}^{\chi\chi} T_{l''}^{\chi\chi}} \tag{5}$$

is the noise power spectrum associated with a quadratic reconstruction of $C_l^{\phi\phi}(z_s)$ using the field $\tilde{\chi}$ [9].

Partial delensing bias.—An estimate of $\phi(\hat{\mathbf{n}})$ can be used to delense the CMB B-mode polarization map. In the limit $z_s \to z_{\rm CMB}$ (conventional CMB delensing) the extraction of C_l^{BB} from the delensed map is limited by the noise introduced during delensing. The residual contamination of the B modes is given by the second term of Eq. (2) with the replacement $C_l^{\phi\phi} \to \mathcal{Q}_l$. However if $z_s < z_{\rm CMB}$ this noise is not necessarily the factor limiting a measurement of the IGW signal. Using $\hat{\phi}(\hat{\mathbf{n}}; z_s)$ as a proxy for $\phi(\hat{\mathbf{n}})$ to delense the map leaves a residual lensing contamination not due to noise. Accounting for this partial delensing bias $\mathcal{B}_l(z_s) \equiv C_l^{\phi\phi} - C_l^{\phi\phi}(z_s)$ (due to the difference in source redshift between the lensed field $\tilde{\chi}(\hat{\mathbf{n}})$ and the CMB) the residual contamination of the B-mode power spectrum is instead the second term of Eq. (2) with $C_l^{\phi\phi} \to \mathcal{B}_l(z_s) + \mathcal{N}_l(z_s)$. This is true whether $\hat{\phi}(\hat{\mathbf{n}}; z_s)$ is estimated using quadratic estimators or by some other method. Here, $\mathcal{N}_l(z_s)$ is the residual noise power spectrum of the deflection potential due to noise associated with the delensing process—for quadratic reconstruction $\mathcal{N}_l(z_s) = \mathcal{Q}_l(z_s)$. If the deflection potential is reconstructed from a line, as is the case for cosmic 21 cm radiation, the source redshift is exactly know and $\mathcal{B}_l(z_s)$ can be reliably estimated.

Quadratic reconstruction.—Unlike the CMB anisotropies, which lack power on angular scales below a few arcminutes due to Silk damping, the cosmic 21 cm anisotropies extend to much higher values of l (limited by the Jeans wavelength of the gas) and peak in amplitude at higher values of l [13,14]. Additionally, measurements of cosmic 21 cm anisotropies in different frequency bins provide several estimates of essentially the same deflection field.

As shown in Fig. 1, we estimate a 21 cm experiment centered around $z_s \sim 30$ with a 20 MHz coverage in frequency space capable of observing anisotropies out to $l \sim 5000$ would have an $\mathcal{N}_l(z_s)$ higher than the planned CMBpol mission. In this case residual confusion arises from noise rather than bias. Following the approach used in Ref. [19], using the residual noise level for lensing contamination after a quadratic reconstruction of the deflection field, a CMB polarization experiment could detect $\mathcal{T}/S \gtrsim 2.5 \times 10^{-5}$ or $\mathcal{V}^{1/4} > 2.6 \times 10^{15}$ GeV. (Limits

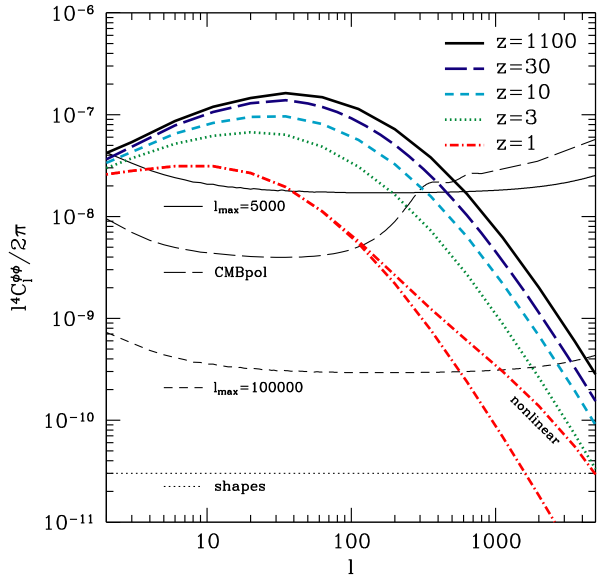

FIG. 1 (color online). Shown (thick solid line) is the angular power spectrum of the deflection potential as a function of source redshift z_s. Curves labeled by $l_{\rm max}$ (thin solid and short-dashed line) are the estimated noise levels for quadratic reconstruction using 21 cm anisotropies in 40 0.5 MHz bins centered around $z_s \approx 30$. We assume a noise power spectrum with $T_{\rm sys} = 3000$ K at 46 MHz, that $l_{\rm max} f_{\rm cov} \approx 15$, and a year of integration [16]. The curve labeled shapes (thin dotted line) shows the residual noise curve if shear is directly measured using resolved minihalos in a 1 MHz bandwidth about $z_s \approx 30$. Noise levels for a CMB-only reconstruction of deflections with CMBpol are shown (thin long-dashed line) assuming a 3 arcminute beam, a noise level of 1 μK $\sqrt{\rm sec}$, and a year of integration.

here and throughout are quoted at the one-sigma level.) Given the statistical nature of quadratic reconstruction, the best noise levels are obtained with all-sky observations of 21 cm anisotropies and we have assumed 21 cm observations over a quarter sky. A next-generation 21 cm experiment capable of observing anisotropies out to $l \sim 10^5$ with the same central redshift and bandwidth would have a $\mathcal{N}_l(z_s)$ an order of magnitude below CMBpol and now be limited largely by the bias $\mathcal{B}_l(z_s)$. Paired with CMB polarization observations, this type of measurement could detect $\mathcal{T}/S \gtrsim 1.0 \times 10^{-6}$ or $\mathcal{V}^{1/4} > 1.1 \times 10^{15}$ GeV—comparable to the ultimate limit from high sensitivity and resolution CMB observations alone [20]. However, in the new scenario, as CMB data will not be used for a lensing extraction, a much lower-resolution CMB experiment would suffice. Furthermore, if lensing information need not be extracted from the CMB observations, the optimal observing strategy is to integrate over a few square degrees patch of the sky as proposed in Ref. [21], though 21 cm observations must be over a wider area due to the statistical nature of the lensing reconstruction.

Other methods.—If minihalos bright in 21 cm emission exist in the early Universe, just as galaxy shapes are sheared by weak gravitational lensing, so will be the shapes of these minihalos. Ellipticity information obtained from such 21 cm minihalos could be used to reconstruct the projected potential out to high z [22]. Based on the dark-matter halo mass function, we expect roughly a surface density of 10^{11}/sr of such minihalos at $z \sim 30$ for a bandwidth of 1 MHz with masses between 10^5 and 10^7 $M_\odot$. A typical halo of mass 10^6 $M_\odot$ has a characteristic projected angular size of ~60 milliarcseconds. If resolved, then techniques currently applied to measure shear in background galaxies in the low-z Universe could be adapted for this application. In the case of minihalos, one can target a smaller area with 21 cm observations than with quadratic reconstruction to determine the projected mass distribution because ellipticities provide a direct measure of the local projected mass distribution (which can be used to delense CMB maps within the same field).

Bias-limited delensing.—To understand to what extent bias-limited reconstructions, where the residual lensing contamination is dominated by $\mathcal{B}_l(z_s)$, would result in the removal of lensing confusion, we have calculated the residual B-mode power spectrum after correcting for the modified lensing kernel when $z_s < z_{\rm CMB}$. We have adapted the formalism of Ref. [19] to estimate the smallest detectable background of IGWs, while the residual noise levels are summarized in Fig. 2. While knowing the projected mass distribution out to $z_s = 1$ does not allow the confusion to be reduced significantly, if it is known to $z_s = 10$ the confusion is reduced by an order of magnitude and the minimum detectable energy scale of inflation is reduced below the limit derived using quadratic CMB statistics [19]. A lensing-source redshift $z_s \gtrsim 30$ would be required to improve beyond the practical 1.1×10^{15} GeV limit of the more sophisticated maximum likelihood method with high-resolution and sensitivity CMB polarization observations [20]. However, we emphasize here that in the case where a bias-limited reconstruction of the deflection field exists for $z_s \sim 30$, a very high-resolution CMB polarization experiment is not necessary and a lower-resolution (but high sensitivity) CMB polarization experiment could do the same job if paired with high-resolution observations of the cosmic 21 cm radiation.

For a bias-limited reconstruction out to a $z_s \sim 100$, the limit on the tensor-to-scalar ratio is 7.0×10^{-8} or $\mathcal{V}^{1/4} > 6.0 \times 10^{14}$ GeV. For $z_s \sim 200$, the maximum redshift where 21 cm fluctuations are expected to be nonzero, one could probe down to $\mathcal{V}^{1/4} > 3 \times 10^{14}$ GeV.

Discussion.—While obtaining bias-limited measurements out to $z_s \approx 100$ is a daunting task, with many experimental and theoretical obstacles to overcome, there is great interest in detecting the fluctuations in the cosmic 21 cm radiation at $z \sim 10$–200 for their own sake and for a variety of other reasons (e.g., [13,22–25]). The observational study of 21 cm fluctuations, especially during and prior to the era of reionization, is now being pursued by a

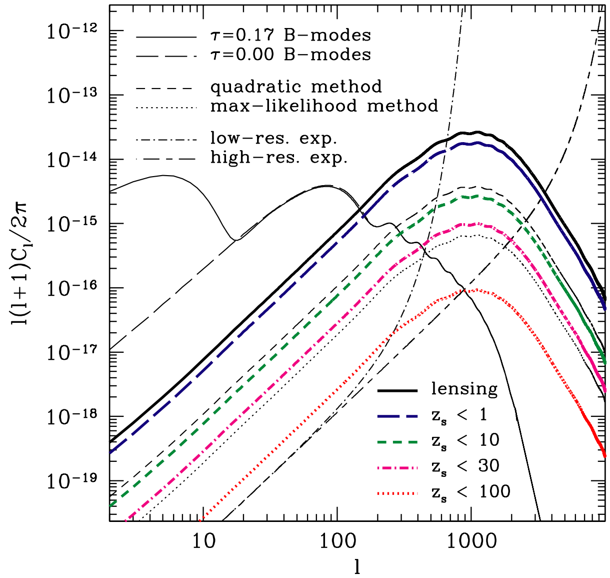

FIG. 2 (color online). Shown (thick line) are the residual contamination of the CMB *B*-mode polarization induced by gravitational lensing for bias-limited delensing with source redshift z_s. Previous estimates of residual confusion arising in CMB-only delensing are shown for quadratic (thin short-dashed line) and maximum likelihood (thin dotted line) methods. The IGW induced *B*-mode signal is shown with (thin solid line) and without (thin long-dashed line) a low-l early reionization bump assuming $\mathcal{T}/S = 1.0 \times 10^{-1}$ or, from Eq. (3), $\mathcal{V}^{1/4} = 2.0 \times 10^{16}$ GeV. Limits quoted in the text can be roughly read from this figure by requiring that the lensing confusion be less than peak of the ($\tau = 0.17$) IGW signal—a lower τ would increase the minimum $\mathcal{V}^{1/4}$ by a factor of $\lesssim 2$–3. For reference, the noise curves of a high-resolution CMB polarization experiment with 2 arcminute beams and a pixel noise of 0.25 μK arcminute (thin long-short-dashed line) and a lower-resolution experiment with 30 arcminute beams (thin dot-dashed line) and similar pixel noise are shown. This latter low-resolution experiment could nevertheless detect IGWs when paired with a cosmic 21 cm lensing reconstruction. For very efficient delensing other foregrounds, such as patchy reionization [27], might dominate confusion.

variety of low-frequency radio interferometers. While certain models of inflation may be related to grand unified theories, some supersymmetric theories of inflation have energy scales of several times 10^{14} GeV [26]. New methods, such as the idea of using observations of the cosmic 21 cm radiation to delense the CMB *B*-mode polarization suggested here, are needed to push the minimum detectable energy scale of inflation below 10^{15} GeV and discriminate between physical theories at these high energy scales.

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*Electronic address: ksigurds@tapir.caltech.edu
†Electronic address: acooray@uci.edu

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