

Letters to the Editor

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Thermal Resistivity of Mercury in the Intermediate State

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THE thermal resistivity along the axis of a 5.2-mm diameter cylinder of approximately 99.99 percent pure mercury was measured as a function of transverse magnetic field at various temperatures between 4.2° and 1.3°K. At temperatures above about 2.1°K, the resistivity in the intermediate state ($0.5 < \eta < 1$, where η is the reduced field H/H_c) was quite well represented by

$$w = 2(w_n - w_s)\eta + (2w_s - w_n), \quad (1)$$

where w_s and w_n are the resistivities in the pure superconducting and pure normal states, respectively. This result might have been anticipated for a specimen composed of alternate superconducting and normal laminae perpendicular to the axis of the cylinder, which is probably a good first approximation to the structure in the intermediate state.¹

Below 2.1°K, however, departures from Eq. (1) were observed corresponding to an additional component of resistivity $w_a(\eta)$ on the right-hand side, similar in character to that observed in pure lead by Webber and Spohr² and in pure tin and indium by Detweiler and Fairbank.³ For mercury, w_a vanished at $\eta = 0.5$ and 1.0 and passed through a maximum at an intermediate value of η which seemed to decrease somewhat with decreasing temperature, but which always lay between 0.74 and 0.70. As shown by Fig. 1, the maximum value of w_a was in good agreement with the empirical formula

$$(w_a)_{\max} = 1.3 \times 10^{-3} t^{-5} \text{ watt}^{-1} \text{ cm deg},$$

where t is the reduced temperature T/T_c . The results of Detweiler and Fairbank³ for two tin specimens of nearly the same diameter and impurity content also yield $(w_a)_{\max}$ values proportional to

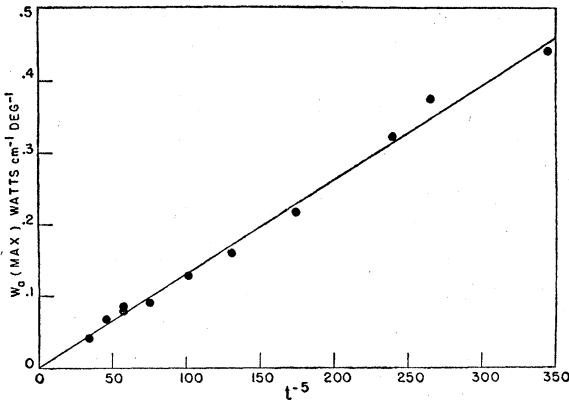


FIG. 1. Maximum additional thermal resistivity versus reduced temperature to the power -5 , mercury.

t^{-5} with a coefficient $7.9 \times 10^{-4} \text{ watt}^{-1} \text{ cm deg}$, which suggests that this type of behavior may be fairly general.

It is worth noting that for the mercury specimen in the normal state the mean free path of electrons l_n at 2°K was about $4 \times 10^{-4} \text{ cm}$. While this is small compared to the probable thickness of the normal laminae¹ for $\eta = 0.72$, roughly 10^{-2} cm , it is greater than the estimated width of the superconducting-normal boundary layer,¹ about $5 \times 10^{-5} \text{ cm}$. On the two-fluid model these figures seem to imply that as one crosses the boundary from a normal to a superconducting region, the equilibrium population of normal electrons decreases by a factor t^4 in a distance small compared to l_n . If it is assumed that a similar population decrease occurs for normal electrons actually crossing the boundary, a fraction $(1-t^4)$ of these electrons being in some way prevented from taking part in the heat transfer process, then a higher thermal resistivity $w_n t^{-4}$ should occur in a normal layer of approximate thickness l_n in contact with the boundary. Although the reason for the existence of such a layer is not clear, it is interesting that for the specimen as a whole this assumption gives rise to an additional resistivity of the form

$$w_a \sim (w_n d_{nc}/Z) t^{-5}, \quad (3)$$

where w_{nc} and l_{nc} are the values of w_n and l_n for $t=1$, and Z is the combined thickness of one superconducting and one normal lamina. Taking Z as about 10^{-2} cm , one obtains $w_a \sim 5 \times 10^{-3} t^{-5} \text{ watt}^{-1} \text{ cm deg}$, which is comparable with the observed $(w_a)_{\max}$.

I am grateful to Mr. R. J. Sladek for his cooperation in this experiment.

¹ D. Shoenberg, *Superconductivity* (Cambridge University Press, Cambridge, 1952).

² R. T. Webber and D. A. Spohr, *Phys. Rev.* **84**, 384 (1951).

³ D. P. Detweiler and H. A. Fairbank, *Phys. Rev.* **88**, 1049 (1952).

The λ -Transition in Liquid Helium

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WE show why the interatomic potential does not alter the existence of an Einstein-Bose condensation¹ in He⁴.

The partition function $Q = \sum_n \exp(-\beta E_n)$, with $\beta = 1/kT$, is the trace of the operator $\exp(-\beta H)$. A coordinate representation of $\exp(-i\hbar H/\hbar)$ may be expressed in terms of an integral over trajectories.² An analogous situation applies to $\exp(-\beta H)$. For a system of N atoms of mass m interacting in pairs with a mutual potential $V(\mathbf{R})$ the partition function becomes

$$Q_B = (N!)^{-1} \int d^N \mathbf{z}_i \int_{tr} \exp \left[- \int_0^\beta \left\{ \frac{m}{2\hbar^2} \sum_i \left(\frac{d\mathbf{x}_i}{du} \right)^2 + \sum_{i,j} V(\mathbf{x}_i - \mathbf{x}_j) \right\} du \right] \mathcal{D}^N \mathbf{x}_i(u). \quad (1)$$

The integral \int_{tr} must be taken over all trajectories $\mathbf{x}_i(u)$, for $i=1$ to N , of the atoms such that initially they are in the same configuration \mathbf{z}_i as finally, i.e., $\mathbf{x}_i(0) = \mathbf{z}_i$ and $\mathbf{x}_i(\beta) = \mathbf{z}_i$ [for we want the diagonal element of $\exp(-\beta H)$]. Also an integral is taken over all such configurations \mathbf{z}_i (to obtain the trace). This Q_B is for atoms which obey Boltzmann statistics, and the $(N!)^{-1}$ is added, as is conventional. Actually He⁴ obeys Bose statistics, the sum on states must only be over symmetrical states. This has the effect that the true Q for He⁴ is

$$Q = (N!)^{-1} \sum_P \int d^N \mathbf{z}_i \int_{tr_P} \exp \left[- \int_0^\beta \left\{ \frac{m}{2\hbar^2} \sum_i \left(\frac{d\mathbf{x}_i}{du} \right)^2 + \sum_{i,j} V(\mathbf{x}_i - \mathbf{x}_j) \right\} du \right] \mathcal{D}^N \mathbf{x}_i(u), \quad (2)$$

where in this case the integral \int_{tr_P} is taken over all trajectories for which $\mathbf{x}_i(0) = \mathbf{z}_i$ and $\mathbf{x}_i(\beta) = P\mathbf{z}_i$.

That is, the final configuration may find the atoms permuted from the initial positions (signified by Pz_i), and we are to sum over all such permutations P .

The main effect of the potential is to limit the important trajectories and configurations z_i to those in which the atoms never overlap (radius of repulsion 2.6A).

In this note we will go into no further detail than is necessary to determine the reason for and character (order) of the transition. These features are not affected by smooth functions of the temperature. We assume Q_B would show no transition. Then we can write $Q = qQ_B$ and argue that the transition is contained in the factor q by which Q differs from Q_B .

Since for a free particle,²

$$\int \exp\left(-\int_0^\beta \frac{1}{2} m \dot{x}^2 du\right) \mathcal{D}x(u) = (2\pi\hbar^2\beta/m)^{-1/2} \exp[-\frac{1}{2}m(x(0) - x(\beta))^2/\beta\hbar^2]$$

holds, a displacement by distance $x(0) - x(\beta) = d$ results in a factor

$$y = \exp(-\frac{1}{2}md^2/\beta\hbar^2) \quad (3)$$

relative to the case of a return to the original position [$x(\beta) = x(0)$].

For He at the temperature of the transition (2.2°K), this factor is $1/e$ for $d = 3.4A$. Hence, near the transition, displacements beyond nearest neighbors do not contribute effectively to Q . Only those permutations are important in which the atoms are either left in their original positions or moved to a neighboring atom. A permutation in which i goes to j 's site must move j to, say, k 's site, etc., until the r th atom in the cycle is adjacent to i 's original position and moves over to occupy it. Such a cycle, or ring, of r adjacent atoms each moved to the position of the next contributes a factor y^r to q , if we use in (3) some mean atomic spacing for d .

The atoms are not free particles. But the potentials will not drastically alter the possibility of moving over to a nearest neighbor. At any rate we can expect such a factor y increasing with decreasing temperature, even though the exact temperature dependence may not be given correctly by³ (3). The important configurations z_i in (2) (those with nonoverlapping atoms) are very much like the configurations for a classical fluid. Each permutation cycle corresponds to a ring or, if we consider lines joining atomic centers, to a polygon on this configuration. Therefore, roughly,

$$q = \sum_L g(L) y^L, \quad (4)$$

where $g(L)$ is the total number of ways that polygons can be drawn on the configuration so that the total number of sides of all the polygons together is L . The polygons must not cross.

A single large polygon of r sides contributes a very small amount y^r ($y < 1$). But a large polygon can be drawn in more ways than a small one. Increasing the length r by one increases the number of polygons available by a factor, say s (perhaps 3 or 4), although the contribution of each is multiplied by y . Thus if $sy < 1$ (high T) large polygons are unimportant. As T falls, suddenly when $sy = 1$ the contributions from very large polygons (limited by the size of the container) begin to be important. This produces a transition in the behavior of q .

The theoretical determination of the order of the λ -transition in helium therefore can be reduced to a definite mathematical problem; namely, the behavior of the sum (4) as a function of y for a cubic lattice. For the character of the transition probably does not depend upon the long-range disorder of the actual configuration.

This idealized mathematical problem has not been solved. The difficulty is that the polygons must not intersect. A rough way of taking this into account is to assume that, when drawing polygons, if K sides are already drawn, the next atom site has a chance of $(N-K)/N$ of being unoccupied. This yields a third-order transition like the ideal Bose gas (for which, indeed, the above assumption is valid).

A solution of the idealized problem which more rigorously describes the geometrical correlations might well give a transition

of a different order. (The experimental transition appears to be second order).

This work resulted from conversations with M. Kac.

¹ F. London, Phys. Rev. **54**, 947 (1938).

² R. P. Feynman, Revs. Modern Phys. **20**, 367 (1948).

³ Actually (3) is essentially correct. Even for large displacements of a given atom i , the atoms which might hinder the motion can temporarily readjust their positions $x(u)$ to permit i to pass more easily. The effect is just to change m to a somewhat higher effective mass. A more detailed account is being prepared.

Heat Pulses in Helium II between 0.1°K and 1.0°K

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THE propagation of heat pulses in helium II has been investigated at temperatures between 0.1°K and 1.0°K by the standard method of Pellam¹ and Atkins and Osborne.² The lengths of the cavities were 3 cm and 6 cm, and their diameter was 0.4 cm. The pulse duration was usually 20 microseconds. Special care was taken to ensure a good thermal contact between cooling agent and liquid. The warming-up times were of the order of one hour.

At the lowest temperatures the sharp start of the heat pulse was found to propagate with a velocity of 230 m/sec. In the 6-cm cavity this velocity decreased, the start becoming unsharp, from about 0.2°K. In the 3-cm cavity that did not occur until about 0.35°K. Between those temperatures and about 0.8°K the apparent velocity of the pulse decreased rapidly, being smaller in the 6-cm than in the 3-cm cavity.

The usual second sound was found above about 0.8°K. The velocities were the same for the two cavities and agreed with those reported by De Klerk, Pellam, and Hudson³ and by Herlin.⁴ The velocities of the start increase somewhat with rising heat input.²

The results suggest that the mean free path of the phonons traveling at the speed of normal sound is several centimeters at the lowest temperatures used and decreases to a few millimeters at most at the highest temperatures.^{5,6} At temperatures of about 0.5°K the behavior of the pulse is rather similar to that of a heat pulse in a normal medium.⁵

Details will be published in *Physica* and the *Communications from the Kamerlingh Onnes Laboratory*.

¹ J. R. Pellam, Phys. Rev. **75**, 1183 (1949).

² D. V. Osborne, Nature **162**, 213 (1948); Proc. Phys. Soc. (London) **A64**, 114 (1951); K. R. Atkins and D. V. Osborne, Phil. Mag. **41**, 1048 (1950).

³ de Klerk, Hudson, and Pellam, Phys. Rev. **89**, 326 (1953).

⁴ R. D. Maurer and M. A. Herlin, Phys. Rev. **76**, 948 (1949).

⁵ C. J. Gorter, Phys. Rev. **88**, 681 (1952).

⁶ L. D. Landau and I. M. Khalatnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) **19**, 637, 709 (1949).

Fountain Effect in Helium II below 1°K

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THE fountain effect in helium II was investigated below 1°K. A long vertical glass capillary, kept at a constant temperature of about 1.1°K, was connected by a tube filled with very fine Fe₂O₃ powder to a thermally insulated vessel which could be cooled down by adiabatic demagnetization. The vessel and the capillary were partly filled with helium II, and in the capillary fountain effects between 20 cm and 40 cm could be observed.

Below 0.8°K the observed fountain effect was smaller than is predicted by London's formula,¹

$$d\phi/dT = \rho S, \quad (1)$$

where ρS is the entropy per unit volume.