Final-state interactions and CP violation in $K_L \to \pi^+\pi^- e^+e^-$

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Using chiral perturbation theory we calculate the imaginary parts of the $K_L \to \pi^+\pi^- e^+e^-$ form factors that arise from $\pi\pi \to \pi^+\pi^-$ and $\pi\pi \to \pi^+\pi^-\gamma^*$ rescattering. We discuss their influence on CP-violating variables in $K_L \to \pi^+\pi^- e^+e^-$.

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The E832 fixed-target experiment at Fermilab, whose primary goal is to look for a nonzero value of ϵ'/ϵ, will reconstruct on the order of 1000 events in the rare decay mode $K_L \to \pi^+\pi^- e^+e^-$ [1]. At present, approximately ten such events have been observed by the E731 fixed target experiment [2], the precursor to E832. Long-distance physics dominates this decay mode, with the leading contribution coming from $K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+e^-$, where a single virtual photon creates the e^+e^- pair. This one photon contribution to the decay amplitude has the form

$$M^{(1\gamma)} = \frac{s_1 G_F \alpha}{4\pi f q^2}[iG\epsilon^{\mu\lambda\rho\sigma} p_{+\lambda} p_{-\rho} q_\sigma + F_+ p_+^\mu + F_- p_-^\mu] \bar{u}(k_-)\gamma_\mu v(k_+), \tag{1}$$

where G_F is Fermi's constant, α is the electromagnetic fine structure constant, $s_1 \simeq 0.22$ is the sine of the Cabibbo angle, and $f \simeq 132$ MeV is the pion decay constant. The π^+ and π^- four-momenta are denoted by p_+ and p_- while the e^+ and e^- four-momenta are denoted by k_+ and k_-. The sum of electron and positron four-momenta is $q = k_+ + k_-$. The Lorentz scalar form factors $G, F_\pm$ depend on scalar products of the four-momenta q, p_+, and p_-. Theoretical predictions for $G, F_\pm$ were first made in Ref. [3].

Chiral perturbation theory allows a systematic expansion of an observable in powers of p^2, where p is a typical momentum involved in the process of interest. Such an expansion was performed for the form factors $F_\pm$ and G defined above in the analysis of Ref. [4]:

$$F_\pm = F_\pm^{(1)} + F_\pm^{(2)} + \cdots,$$
$$G = G^{(1)} + G^{(2)} + \cdots. \tag{2}$$

The superscripts denote the order of chiral perturbation theory at which each term arises [i.e., $F_\pm^{(m)}, G^{(m)}$ give a contribution of order p^{2m-1} to the square brackets of Eq. (1)].

The K_L state has both CP even and CP odd components:

$$|K_L\rangle \simeq |K_2\rangle + \epsilon|K_1\rangle, \tag{3}$$

where $|K_2\rangle$ is the CP odd state $|K_2\rangle = (|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2}$ and $|K_1\rangle$ is the CP even state $|K_1\rangle = (|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2}$. The parameter $\epsilon \simeq 0.0023 e^{i44^0}$ [in a phase convention where the $K^0 \to \pi\pi(I=0)$ amplitude is real] characterizes CP nonconservation in $K^0\bar{K}^0$ mixing. We neglect other (i.e., direct) sources of CP nonconservation in the one-photon part of the $K_L \to \pi^+\pi^- e^+e^-$ decay amplitude. Contributions to the form factors $F_\pm$ from the $|K_2\rangle$ and $|K_1\rangle$ parts of the K_L state have different symmetry properties. Under interchange of the pion four-momenta, $p_+ \to p_-$ and $p_- \to p_+$, the CP-conserving parts of the form factors arising from the $|K_2\rangle$ component transform as

$$F_+ \to F_- \quad \text{and} \quad F_- \to F_+, \tag{4}$$

while the CP-violating parts of the form factors arising from the $|K_1\rangle$ component transform as

$$F_+ \to -F_- \quad \text{and} \quad F_- \to -F_+. \tag{5}$$

At leading order in chiral perturbation theory [i.e., order p in the square brackets of Eq. (1)],

$$G^{(1)} = 0, \tag{6a}$$

$$F_+^{(1)} = -\frac{32 g_8 f^2 (m_K^2 - m_\pi^2)\pi^2 \epsilon}{q^2 + 2q\cdot p_+}, \tag{6b}$$

$$F_-^{(1)} = \frac{32 g_8 f^2 (m_K^2 - m_\pi^2)\pi^2 \epsilon}{q^2 + 2q\cdot p_-}, \tag{6c}$$

$G^{(1)}$ is zero [it enters in the square brackets of Eq. (1) multiplied by three-momentum factors, and is therefore at most an order p^3 effect] and contributions to $F_\pm$ not proportional to ϵ do not occur until higher order in chiral perturbation theory. In Eq. (6), g_8 is the coefficient of the leading two-derivative part of the chiral Lagrangian for $\Delta S = 1$ weak nonleptonic kaon decay [5]. It is real and the measured $K^0 \to \pi\pi(I=0)$ decay amplitude gives $|g_8| \simeq 5.1$.

Since the CP violating contribution to the $K_L \to \pi^+\pi^- e^+e^-$ decay amplitude occurs at a lower order of chiral perturbation theory than the CP conserving contribution, the effects of indirect CP nonconservation are enhanced in this decay. It is convenient for the discussion of CP violation in $K_L \to \pi^+\pi^- e^+e^-$ to use the four-body phase-space variables used by Pais and Trieman for semileptonic K_{l4} decay [6]. They are $q^2 = (k_+ + k_-)^2$; $s = (p_+ + p_-)^2$; θ_π, the angle

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between the π^+ three-momentum and the K_L three-momentum in the $\pi^+\pi^-$ rest frame; θ_e, the angle between the e^- three-momentum and the K_L three-momentum in the e^+e^- rest frame; and ϕ, the angle between the normals to the planes defined (in the K_L rest frame) by the $\pi^+\pi^-$ pair and the e^+e^- pair. Using these kinematic variables the CP violating observable

$$B_{CP}=\langle \mathrm{sgn}(\sin\phi\,\cos\phi)\rangle \tag{7}$$

gets a large contribution from indirect CP nonconservation. Neglecting other sources of CP violation, one has, after integrating over $\cos\theta_e$ and ϕ,

$$B_{CP}=\frac{G_F^2 s_1^2\alpha^2}{3\times 2^7(2\pi)^8 f^2 m_K^3\Gamma_{K_L}}\times\int d\,\cos\theta_\pi\,ds\,dq^2\,\sin^2\theta_\pi\beta^3X^2\times\left(\frac{s}{q^2}\right)\mathrm{Im}[G(F_+^*-F_-^*)]. \tag{8}$$

where

$$\beta=[1-4m_\pi^2/s]^{1/2}, \tag{9a}$$

$$X=\left[\left(\frac{m_K^2-s-q^2}{2}\right)^2-sq^2\right]^{1/2}. \tag{9b}$$

If the variables s and q^2 are not integrated over the entire phase space, then the same is to be done to the $K_L\to\pi^+\pi^-e^+e^-$ width, Γ_{K_L}, in the denominator of Eq. (8).

The form factor G first arises at second order in chiral perturbation theory. Because tree diagrams involving vertices from the Wess-Zumino term do not contribute [7], it is dominated by local order p^4 terms in the chiral Lagrangian [8] which give a real contribution to $G^{(2)}$. The measured $K_L\to\pi^+\pi^-\gamma$ decay rate [9] implies that

$$|G^{(2)}|\simeq 40. \tag{10}$$

In obtaining this result from the data, we have neglected the experimental momentum dependence of G. Higher order terms in the chiral expansion endow G with momentum dependence. At leading order in chiral perturbation theory

$$\mathrm{Im}[G(F_+^*-F_-^*)]\to\mathrm{Im}[G^{(2)}(F_+^{(1)*}-F_-^{(1)*})], \tag{11}$$

in Eq. (8) and the imaginary part comes solely from the phase of ϵ appearing in $F_\pm$. In Ref. [3] the form factors $F_\pm$ and G were estimated by extrapolating from the measured $K_L\to\pi^+\pi^-\gamma$ amplitude. They noted that B_{CP} was large and furthermore showed that final state $\pi\pi$ interactions give an important enhancement of B_{CP}. In this Brief Report we calculate the absorptive parts of G and (F_+-F_-) using chiral perturbation theory and consider their influence on B_{CP}. Our approach includes both $\pi\pi\to\pi^+\pi^-$ and $\pi\pi\to\pi^+\pi^-\gamma^*$ rescattering. Previous estimates of the effect of final-state interactions used the measured pion phase shifts and neglected $\pi\pi\to\pi^+\pi^-\gamma^*$.

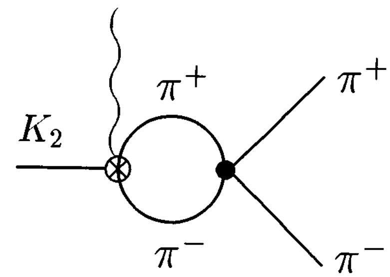

FIG. 1. Feynman diagram contributing to Abs$G^{(3)}$ at leading order. In this figure and those that follow, a solid circle denotes a vertex arising from the leading-order strong and electromagnetic chiral Lagrangian. The other vertex in this figure arises from an $O(p^4)$ counterterm in the chiral Lagrangian.

Dividing the third-order contribution to G into its dispersive and absorptive pieces, $G^{(3)}=\mathrm{Disp}G^{(3)}+i\,\mathrm{Abs}G^{(3)}$, we find that the Feynman graph shown in Fig. 1 gives

$$\mathrm{Abs}G^{(3)}=\frac{G^{(2)}}{48\pi}\left(\frac{s}{f^2}\right)\left(1-\frac{4m_\pi^2}{s}\right)^{3/2}. \tag{12}$$

Unfortunately, the dispersive part of $G^{(3)}$ is not calculable as it receives a contribution not only from the loop graph in Fig. 1, but also from loop graphs involving the Wess-Zumino term and from new order p^6 local operators in the chiral Lagrangian for weak radiative kaon decay.

The absorptive parts of $F_\pm$ first arise at second order in chiral perturbation theory from the Feynman diagrams in Fig. 2 which give

$$\mathrm{Abs}F_+^{(2)}=-g_8(m_K^2-m_\pi^2)\pi\epsilon\left\{\frac{(4m_K^2-2m_\pi^2)}{q^2+2q\cdot p_+}\sqrt{1-4m_\pi^2/m_K^2}-4\left[\int_0^{\xi_-}y_+\,dy-\int_0^{\xi_+}y_-\,dx\right]-\frac{8q\cdot(p_+-p_-)}{s}\left[\int_0^{\xi_+}\frac{xy_-}{(y_+-y_-)}dx+\int_0^{\xi_-}\frac{xy_+}{(y_+-y_-)}dx\right]\right\}. \tag{13}$$

Abs$F_-^{(2)}$ is obtained from Eq. (13) by interchanging p_+ with p_- using the symmetry property in Eq. (5). The limits of integration in Eq. (13) are given by

$$\xi_\pm=\frac{1\pm\sqrt{1-4m_\pi^2/m_K^2}}{2}, \tag{14}$$

and the variables $y_\pm$ are defined by

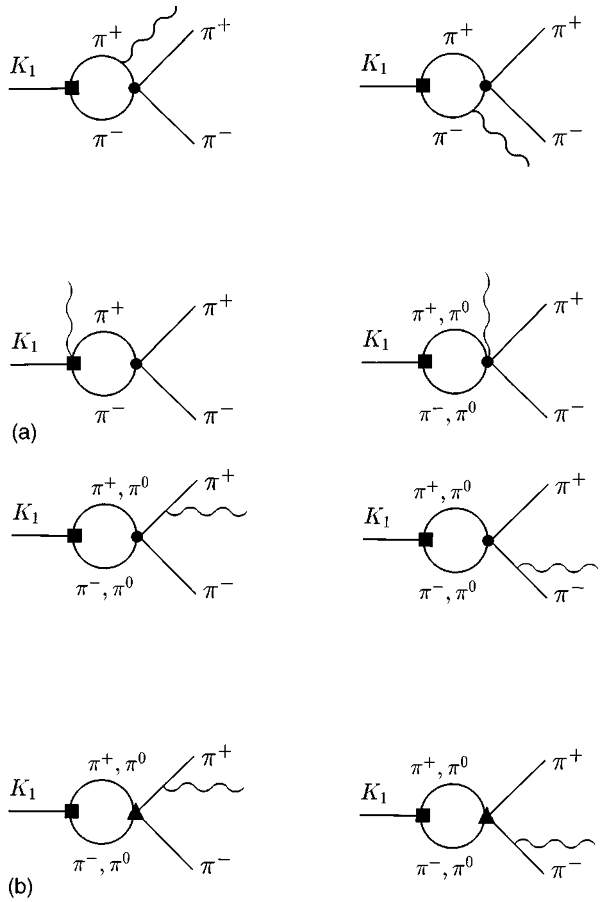

FIG. 2. Feynman diagrams contributing to Abs$F_{\pm}^{2}$ at leading order. A solid square denotes a vertex arising from the $\Delta S=1$ part of the leading-order-gauged weak chiral Lagrangian. A solid triangle vertex arises from the piece of the leading-order strong chiral Lagrangian proportional to the quark masses

$$y_{\pm}=\frac{(1-x)s+x(m_K^2-q^2)\pm\sqrt{[(1-x)s+x(m_K^2-q^2)]^2-4s[m_\pi^2-q^2x(1-x)]}}{2s}. \tag{15}$$

We include the influence of final-state interactions on B_{CP} by setting

$$\begin{aligned}\mathrm{Im}[G(F_+-F_-)^*]\rightarrow\ &\mathrm{Im}[G^{(2)}(F_+^{(1)}-F_-^{(1)})^*]\\ &+\mathrm{Re}[\mathrm{Abs}G^{(3)}(F_+^{(1)}-F_-^{(1)})^*]\\ &-\mathrm{Re}[G^{(2)}(\mathrm{Abs}F_+^{(2)}-\mathrm{Abs}F_-^{(2)})^*],\end{aligned} \tag{16}$$

in Eq. (8). The first of the three terms on the right-hand side of Eq. (16) was calculated in Ref. [4] and the last two represent the effects of final-state interactions.

We find that final-state interactions increase B_{CP} by about 45% over what we presented in Ref. [4]. The first term in Eq. (13), and consequently the third term in Eq. (16), is the dominant contribution from final-state interactions and it enhances B_{CP} by the factor

$$\frac{(4m_K^2-2m_\pi^2)}{32\pi f^2}\sqrt{1-4m_\pi^2/_K^2}\simeq 0.45 \tag{17}$$

over the leading order result obtained in Ref. [4]. The trend that final-state interactions increase B_{CP} is in agreement with Ref. [3]. The rate Γ_{K_L} in the denominator of Eq. (8) depends on the collection of counterterms defined as w_L in Ref. [4]. Setting w_L to zero, we find that $|B_{CP}|\simeq 14\%$ with the cut $q^2>(10\ \mathrm{MeV})^2$ imposed and $|B_{CP}|\simeq 4\%$ with the cut $q^2>(80\ \mathrm{MeV})^2$ imposed. With $w_L=2$, the asymmetry is even larger. We find in this case that $|B_{CP}|\simeq 18\%$ for each of the cuts listed above. Table I gives the predicted values for the magnitude of B_{CP} times the branching ratio for $K_L\rightarrow\pi^+\pi^-e^+e^-$ (in units of 10^{-18}) for various cuts on the minimum lepton pair invariant mass squared, $q_{\min}^2$. In this table, w_L has been set to zero.

We have calculated the leading absorptive parts of the form factors G and $F_{\pm}$ using chiral perturbation theory and included, using Eq. (16), their influence on B_{CP}. However, this is not a completely systematic approach because $\mathrm{Im}[\mathrm{Disp}G^{(3)}(F_+^{(1)}-F_+^{(1)})^*]$ and $\mathrm{Im}[G^{(2)}(\mathrm{Disp}F_+^{(2)}-\mathrm{Disp}F_-^{(2)})^*]$ in Eq. (16) were neglected, despite being the same order in the momentum expansion as the terms that were retained. Nonetheless, including only the absorptive

TABLE I. The CP violating observable $|B_{CP}|\times\mathscr{B}$ (10^{-8}) for a range of values of $q^2_{\rm min}$.

Lower cut $q_2^{\rm min}$	$\lvert B_{CP}(\%)\rvert\times\mathscr{B}(10^{-8})$
$(10\ \mathrm{MeV})^2$	208
$(20\ \mathrm{MeV})^2$	122
$(30\ \mathrm{MeV})^2$	76
$(40\ \mathrm{MeV})^2$	50
$(60\ \mathrm{MeV})^2$	22
$(80\ \mathrm{MeV})^2$	9.7
$(100\ \mathrm{MeV})^2$	3.9
$(120\ \mathrm{MeV})^2$	1.4
$(180\ \mathrm{MeV})^2$	0.013

parts may be a good approximation as they are enhanced by a factor of π.

Finally we note that the absorptive parts of the form factors calculated here are also important for direct CP nonconservation in $K_L\to\pi^+\pi^-e^+e^-$. For example, the variable

$$D_{CP}=\langle \mathrm{sgn}(\cos\theta_e)\rangle, \tag{18}$$

is a CP violating observable that arises from interference of the one-photon amplitude in Eq.(1) with the short-distance contribution to the $K_L\to\pi^+\pi^-e^+e^-$ decay amplitude,

$$M^{(\mathrm{SD})}=\frac{G_F s_1\alpha}{f}(\xi p_-^\mu+\xi^* p_+^\mu)\bar{u}(k_-)\gamma_\mu\gamma_5 v(k_+). \tag{19}$$

In the kaon rest frame, the electron-positron energy difference is proportional to $\cos\theta_e$; D_{CP} is therefore a measure of this e^+e^- energy asymmetry.

The W-box and Z-penguin Feynman diagrams are responsible for producing the short distance amplitude, $M^{(\mathrm{SD})}$. The quantity ξ depends on the charm and top quark masses and on Cabibbo-Kobayashi-Maskawa matrix elements. It has been calculated in the next to leading logarithmic approximation [10]. After integrating over ϕ and $\cos\theta_e$ we find that

$$D_{CP}=\frac{s_1^2G_F^2\alpha^2}{2^7(2\pi)^6m_K^3f^2\Gamma_{K_L}}\int d\ \cos\theta_\pi\ ds\ dq^2\ \beta^3X^2 \times\sin^2\theta_\pi s\ \mathrm{Im}G\ \mathrm{Im}\xi. \tag{20}$$

At leading order in chiral perturbation theory $\mathrm{Im}G=\mathrm{Abs}G^{(3)}$. Unfortunately, we find that D_{CP} is around 10^{-7}, and is therefore too small to be measured in the next generation of kaon decay experiments. We do not provide more detailed data on D_{CP} since the CP violating variable A_{CP} discussed in [4] is also a measure of direct CP violation, and has a much larger magnitude of $\sim 10^{-4}$.

In this work, we have estimated the final-state interactions at lowest order in the chiral expansion for strong interactions. Higher-order contributions which we have not computed may modify our results, particularly in the $I=J=1$ channel where the ρ plays an important role [11].

In summary, we have determined the leading effect of $\pi\pi\to\pi\pi$ and $\pi\pi\to\pi\pi\gamma^*$ final-state interactions on the CP violating asymmetry $B_{CP}=\langle \mathrm{sgn}(\sin\phi\cos\phi)\rangle$. We find that these interactions enhance B_{CP} by about 45% over the estimates given in [4]. We have also shown that the CP violating e^+e^- energy asymmetry D_{CP} arises from the interference of the short-distance amplitude with the absorptive part of the form factor G, but found that D_{CP} is unlikely to be observed in the near future.

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