

GRAVITATIONAL-WAVE EMISSION FROM COMPACT GALACTIC BINARIES

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ABSTRACT

Compact Galactic binaries where at least one member is a white dwarf (WD) or neutron star constitute the majority of individually detectable sources for future low-frequency space-based gravitational-wave (GW) observatories; they also form an unresolved continuum, the dominant Galactic foreground at frequencies below a few mHz. Due to the paucity of electromagnetic observations, the majority of studies of Galactic-binary populations so far have been based on population-synthesis simulations. However, recent surveys have reported several new detections of WD binaries, providing new constraints for population estimates. In this article, we evaluate the impact of revised local densities of interacting WD binaries on future GW observations. Specifically, we consider five scenarios that explain these densities with different assumptions on the formation of interacting systems; we simulate corresponding populations of detached and interacting WD binaries; we estimate the number of individually detectable GW sources and the magnitude of the confusion-noise foreground, as observed by space-based detectors with 5 and 1 Mkm arms. We confirm earlier estimates of thousands of detached-binary detections, but project only a few ten to a few hundred detections of interacting systems. This reduction is partly due to our assessment of detection prospects, based on the iterative identification and subtraction of bright sources with respect to both instrument and confusion noise. We also confirm earlier estimates for the confusion-noise foreground, except in one scenario that explains smaller local densities of interacting systems with smaller numbers of progenitor detached systems.

Key words: binaries: close – Galaxy: general – gravitational waves – methods: data analysis – white dwarfs

Online-only material: color figures

1. INTRODUCTION

Compact Galactic binaries with periods shorter than approximately five hours are of considerable interest as gravitational wave (GW) sources (Nelemans 2009; Marsh 2011a). Such binaries include detached double white dwarf (DDWD) systems, short-period cataclysmic variables (CVs), and AM CVn systems (so named after their prototype, AM Canum Venaticorum; see Solheim 2010). These last are short-period binaries in which a white dwarf (WD) is accreting He-rich material from an H-deficient companion. AM CVn systems are defined spectroscopically by the presence of strong absorption or emission He lines and the absence of H lines, which instead characterize “standard” CVs. In this paper, we focus on those binaries that are evolving toward, or have evolved from, extremely short orbital periods, on the order of minutes. These include DDWD and AM CVn systems, sometimes referred to as “ultracompact” binary systems. Such binaries have received considerable attention, among other reasons, as possible progenitors for Type Ia supernovae, R Coronae Borealis stars (Webbink 1984; Iben & Tutukov 1984), and the so-called Ia explosions (Bildsten et al. 2007). Our understanding of their formation and evolution remains poor.

Together with ultracompact X-ray binaries, DDWD and AM CVn systems will be the most numerous and individually brightest GW sources in the Galaxy for low-frequency (10^{-4} –0.1 Hz) space-based GW detectors. Within this frequency band, observations suggest 0.5 – 2.5×10^7 DDWD systems and theoretical estimates predict 10^5 – 10^7 AM CVn systems in our Galaxy (Maxted & Marsh 1999; Nelemans et al. 2001a, 2001c, 2004).

The aim of this paper is to characterize the detectable GW emission from compact binaries in light of recent

population estimates based on new observations. The first estimates were based solely on population-synthesis simulations; see Appendix A for a detailed discussion and review of the literature. Recent observations of AM CVn systems have provided new information that constrains earlier estimates. For instance, the identification of AM CVn systems among candidates selected from the Sloan Digital Sky Survey (SDSS) spectroscopic catalog allowed Roelofs et al. (2007b) to estimate the density of AM CVn systems by modeling the completeness of the catalog. A follow-up spectroscopic survey of a color-selected subset of the SDSS photometric database seems to find even fewer AM CVn systems than expected (Roelofs et al. 2007b; Rau et al. 2010), leading to local density estimates smaller by 10–20 than suggested by population-synthesis studies.

To date, 27 AM CVn systems have been presented in the literature; the most recent detections have been achieved in synoptic surveys, including a short-period system from Kepler (Fontaine et al. 2011) and a medium-period, outbursting system from the Palomar Transient Factory (Levitan et al. 2011).

Similarly, surveys of low-mass WD candidates selected from SDSS have led to the discovery of several short-period DDWDs that could be progenitors of AM CVn systems (Badenes et al. 2009; Mullally et al. 2009; Kulkarni & van Kerkwijk 2010; Marsh et al. 2011b). For example, the Extremely Low Mass (ELM) survey (Brown et al. 2010; Kilic et al. 2011a; Brown et al. 2012) searches for companions around extremely low mass WDs ($\sim 0.2 M_{\odot}$). The ELM survey has approximately quintupled (with 19 new systems) the known sample of DDWDs that will merge within a Hubble time (Kilic et al. 2011b; Brown et al. 2012), providing evidence for at least one plausible AM CVn formation channel (see Section 2.1).

In this paper, we consider five scenarios for AM CVn formation that span plausible populations, and we study their consequences for space-based GW detectors by modeling the detection and subtraction of sources in simulated data streams. We consider two representative detectors: the standard *LISA* developed as a joint NASA–ESA project until 2011 (Prince et al. 2009; Jennrich 2009), and a descoped configuration with shorter armlengths of 1 Mkm, which is similar, but not equivalent, to the NGO/eLISA design studied by ESA in 2011 (Amaro-Seoane et al. 2012).

We find, first, that the number of detectable AM CVn systems decreases from $\sim 10,000$, as reported in previous assessments of *LISA* science, to ~ 100 – 400 (for one year of observation with a single interferometer—see Appendix B). Precise numbers depend on the assumptions used to explain how a postulated number of DDWD progenitors can result in the number of AM CVn systems seen in surveys. For the shorter detector, these numbers are reduced by a factor of a few. Second, we confirm that mostly DDWD systems determine the residual confusion foreground left after GW signals from detectable binaries are subtracted. The foreground does not change significantly from previous estimates, except in one scenario in which we explain the fewer observed AM CVn systems by reducing the space density of DDWD progenitors.

The rest of this paper is structured as follows. In Section 2, we discuss the current understanding of AM CVn formation. In Section 3, we outline the observations of AM CVn and DDWD systems in the Galaxy, and the resulting calibration of local space densities. In Section 4, we present our analysis: we introduce our explanatory formation scenarios in Section 4.1, discuss the space-based observation of GWs from Galactic binaries in Section 4.2, describe our simulations of GW source detection and subtraction in Section 4.3, and detail our results in Section 5. In Section 6, we recapitulate our conclusions and propose possible future work. In the Appendices, we provide more detailed discussions of specific topics: in Appendix A, we summarize previous analytical and numerical studies of the Galactic foreground for *LISA*, and give a general semi-analytical derivation of its magnitude and shape; in Appendix B, we discuss GW detection and define our reference detector configurations; in Appendix C, we provide more details about the fits of the GW confusion foreground obtained from our simulations. In the rest of the paper, we use the standard notation where G is the gravitational constant and c is the speed of light.

2. THE FORMATION AND EVOLUTION OF COMPACT-BINARY SYSTEMS IN THE GALAXY

In the standard picture of binary-star evolution, compact binaries form following two consecutive mass-transfer episodes; the loss of sufficient angular momentum by friction in the second common-envelope phase ensures sufficiently tight orbits for GW emission to influence dynamics. Here, we concentrate on the processes leading to mass transfer and to the formation of AM CVn systems. Due to space limitations, we do not attempt here to provide an extensive literature review, and refer the reader to Solheim (2010) and references therein for a detailed review on the current understanding of the formation and evolution of DDWD and AM CVn systems.

The subsequent orbital evolution of compact binaries is driven by gravitational radiation, by mass transfer, by a dissipative coupling that can feed the angular momentum added through accretion back into the orbit, and by other poorly modeled processes such as wind loss, etc. Therefore, the rate of change

\dot{J}_{orb} of the orbital angular momentum comprises three or more competing components,

$$\dot{J}_{\text{orb}} = \dot{J}_{\text{GR}} + \dot{J}_{\text{M}} + \dot{J}_{\tau_s}. \quad (1)$$

Here, \dot{J}_{GR} represents the loss of angular momentum to gravitational radiation,

$$\dot{J}_{\text{GR}} = -\frac{32}{5} \frac{G^3}{c^5} \frac{M_1 M_2 M}{a^4} J_{\text{orb}} \quad (2)$$

(for circular orbits) where M_1 and M_2 are the two binary masses (M_2 is the donor’s mass for mass-transferring systems), a is their separation, and $M = M_1 + M_2$. Next, $\dot{J}_{\text{M}} = \sqrt{G M_1 R_h M_2}$, where \dot{M}_2 indicates the donor’s mass-loss rate due to Roche-lobe overflow and R_h defines the orbital radius around the accretor with the same specific angular momentum as the transferred mass (Verbunt & Rappaport 1988).

Mass transfer can proceed either via an accretion disk or via “direct impact” (see, e.g., Marsh & Steeghs 2002; Ramsay et al. 2002; Marsh et al. 2004). “Direct impact” occurs when the donor’s mass stream hits the accreting star directly. The particular route depends on the initial separation in the binary following the ejection of the second envelope, and on the donor’s entropy and equation of state (see, e.g., Gokhale et al. 2007; Dan et al. 2011, 2012). Typically, mass-transferring binaries with initial orbital periods greater than 10 minutes form an accretion disk. As discussed below, the presence or absence of an accretion disk is critical to the stability of the system. Whether a disk is present or not, \dot{J}_{τ_s} denotes the change in the torque due to dissipative coupling, tidal or magnetic, between the accretor’s spin and the orbital angular momentum during the accretor’s synchronization timescale τ_s . We assume that the donor and orbit’s spins are synchronized. In this work, we do not consider the change in angular momentum driven by other physical processes, such as losses due to winds or magnetic braking, because of current limitations in our understanding and modeling of such phenomena.

As the term “detached” in their very name implies, DDWD dynamics are determined solely by gravitational radiation and tides (see, e.g., Piro 2011). By contrast, the formation, stability, and outcome of AM CVn systems depend on all three effects in Equation (1). Three proposed channels exist for the formation of AM CVn systems, and each can vary in its efficiency. We describe them below. Following Nelemans et al. (2001a, 2001b, 2004), we assume that the first two channels listed below can operate with either “optimistic” or “pessimistic” efficiencies, resulting in different AM CVn numbers and period distributions.

2.1. The DDWD Channel

A subclass of DDWDs with periods of several minutes becomes semi-detached when angular-momentum loss due to gravitational radiation drives the two bodies sufficiently close that the lower-mass WD fills its Roche lobe and mass transfer begins, either via direct impact or via an accretion disk. The stability of mass transfer depends critically on the mass ratio of the two initial WDs (requiring typically $q = M_2/M_1 \lesssim 2/3$) and on whether spin–orbit tidal coupling feeds angular momentum accreted by the primary star back to the orbit; the strength of this coupling is characterized by the synchronization timescale τ_s (Marsh et al. 2004).

Under optimistic assumptions, strong tidal coupling stabilizes mass transfer, favoring the formation of stable AM CVn systems

and resulting in fewer mergers. By contrast, under pessimistic assumptions, there is no tidal coupling and many more DDWD systems merge. In the presence of a disk, the accreted material's angular momentum is mainly transferred back into the orbit by tidal forces at the outer edge of the disk. However, the disk's material at the inner edge still has sufficient angular momentum to spin up the accretor. Effects not considered here, such as magnetic braking (Farmer & Roelofs 2010), may also impact the formation of AM CVn systems and favor the merger of the WDs.

If stable mass transfer proceeds, then the donor WD loses mass and expands in size, while the system evolves to longer orbital periods. The minimum orbital period at which mass transfer can begin is ~ 2 minutes for a $0.45 M_{\odot}$ and a $0.25 M_{\odot}$ binary. Maximum orbital periods for evolved systems in the Galaxy are estimated to be about 90 minutes; these correspond to very old systems whose orbits have slowly evolved over a Hubble time.

Surveys specifically targeted at low-mass WD (such as the ELM survey of WDs; Brown et al. 2010; Kilic et al. 2011a; Brown et al. 2011) have resulted in the discovery of numerous DDWD systems with $q < 2/3$, which are expected to get into contact within a Hubble time. Several of these newly discovered low-mass binaries will form stable mass-transferring AM CVn systems, even in the most pessimistic case of no tidal coupling (Brown et al. 2011).

2.2. The Helium Star–White Dwarf Channel

In this channel, a WD accretes from an initially nondegenerate He star with mass $0.4\text{--}0.6 M_{\odot}$ before the He has been exhausted in the He-star core. He stars are produced from stars with masses greater than $2 M_{\odot}$ that lose their envelopes on the red giant branch, and typically have lifetimes comparable to their main-sequence progenitors, allowing for mass transfer before evolution to the WD stage. Mass transfer is stable when the mass ratio of the donor He star to the accretor WD is less than approximately 1.2 (Tutukov & Fedorova 1989; Ergma & Fedorova 1990; see also Nelemans et al. 2001a). Due to mass loss, He burning in the donor's core quenches and at masses $\sim 0.2 M_{\odot}$ the star becomes semi-degenerate. The minimum orbital period reached by these systems is ~ 10 minutes.

It remains in question whether $0.1 M_{\odot}$ or $0.3 M_{\odot}$ of He accreted from an He star will destroy a CO WD by edge-lit detonations (henceforth denoted as ELDs; see Shen et al. 2010 and references therein). Nelemans et al. (2001a) distinguish between optimistic and pessimistic assumptions for this channel depending on whether double ELDs remove CO WD systems.

2.3. The Evolved Cataclysmic Variable Channel

In analogy to traditional CVs, a so-called evolved CV comprises a main-sequence star which transfers mass to a WD or neutron star (Podsiadlowski et al. 2001). The difference is that mass transfer occurs as the donor star evolves off the hydrogen main sequence. However, it is thought that the evolved-CV channel contributes only marginally ($< 2\%$) to the AM CVn population (Nelemans et al. 2004; Roelofs et al. 2007b) for systems with periods less than 1500 s. Therefore, we do not consider this channel in this work, which focuses on the detectability of AM CVn systems by GW interferometers.

3. OBSERVATIONS OF AM CVn AND DDWD SYSTEMS IN THE GALAXY

We now turn our attention to recent population estimates of AM CVn systems based on observations. As mentioned, Roelofs

et al. (2007b) estimate the total space density of AM CVn systems using observations of a source population identified by emission lines in SDSS. Characterizing selection effects for the six observed systems in SDSS is challenging. The orbital periods of these systems are generally shorter than predicted for the majority of AM CVn systems by population estimates, thereby requiring a significant extrapolation to estimate the space density of the entire population.

Table 1 in Roelofs et al. (2007b) presents the modeled and inferred AM CVn local space densities for different population-synthesis models. The inferred densities range from 1.2×10^{-6} to $3.4 \times 10^{-6} \text{ pc}^{-3}$, assuming a thin-disk population with a scale height of 300 pc for most systems. A comparison between the inferred and modeled densities immediately disfavors formation models such as the optimistic DDWD channel, and requires an overall reduction in the estimated number of Galactic AM CVn systems by a factor larger than 10. Roelofs et al. (2007b) hypothesize that lower AM CVn densities arise predominantly due to the lower efficiency of the major formation channels. Later in this paper, we consider an alternative possibility, namely, that systems observed by Roelofs et al. (2007b) may come from a distribution with scale height larger than conventional thin disks, with AM CVn progenitors born early in the history of the Galaxy, before the thin disk was fully developed.

Although the number of known DDWD systems that will merge within a Hubble time has increased to 24, an approximate factor of 5, in the last few years (4 eclipsing systems are now known; the ELM survey has uncovered the eclipsing DDWD binary with the shortest known period of 12.7 minutes), a detailed population analysis of such systems has yet to be undertaken. More recently, on the basis of the assumed Galactic-disk model, the ELM survey predicts 40 kpc^{-3} of DDWDs where at least one WD has a mass $< 0.25 M_{\odot}$, with a factor of two uncertainty in correction factors (Brown et al. 2011). This is much lower than population-synthesis results (Nelemans et al. 2001c). On the other hand, population synthesis results show good agreement with observations made by the Supernova type Ia Progenitor (SPY) survey. The SPY radial-velocity survey estimated the local space density of WDs to be $4.8 \times 10^{-3} \text{ pc}^{-3}$ (Holberg et al. 2008), with ~ 100 new DDWDs found (with orbital periods less than 10 days) from a sample of 1000. At present, estimating space densities from the ELM and SPY surveys is very challenging, requiring proper understanding of selection biases. Detailed population comparisons between the two surveys are not currently available.

4. OUR ANALYSIS

We now describe our analysis: in Section 4.1, we explain our choices for WD binary populations; in Section 4.2, we discuss GW emission from binaries; and in Section 4.3, we describe our simulations of binary detection and subtraction in detector data. Section 5 below presents the resulting estimates of the number of detectable systems and of the residual confusion-noise foreground.

4.1. WD Binary Catalogs

In our analysis, we considered several different assumptions on AM CVn formation, generated simulated binary catalogs and detector data sets for each, and estimated the resulting binary detection and subtraction prospects. Among the large number of cases that we investigated, here for conciseness we report on five scenarios that we believe are representative of the range

of possibilities given the constraints of current observations. Two of the scenarios represent an upper bound (with maximally efficient formation channels) and conversely, a bound derived assuming inefficient formation channels on predictions from population-synthesis models (cases 1 and 2); two others attempt to reproduce SDSS-estimated AM CVn densities (cases 3 and 4 below); a fifth (case 5) explores a Galactic population with a larger scale height at times earlier than 7 Gyr ago. As discussed below, the modified Galactic-disk scenario (case 5) should be consistent with SDSS observations; however, this agreement is difficult to quantify as the SDSS-estimated densities given in Roelofs et al. (2007b) are derived assuming a standard disk model.

We generate DDWD and AM CVn catalogs with the SeBa population-synthesis code (Portegies Zwart & Verbunt 1996; Nelemans et al. 2001b, 2004; Toonen et al. 2011), with the following assumptions and parameters: (1) the time- and position-dependent star formation rate (SFR) based on Boissier & Prantzos 1999 (see Nelemans et al. 2004), (2) initial primary masses distributed with power-law index -2.5 (see Kroupa et al. 1993), (3) a flat initial mass-ratio distribution, (4) a flat distribution for the logarithm of the semimajor axis, up to a radius of $10^6 R_\odot$, (5) eccentricities distributed as $P(e) \propto e^2$, and (6) a 50% binary fraction in the initial population of main-sequence stars. We expect a factor of two difference in population-synthesis results obtained using different assumptions and initial parameters than those stated above (Portegies Zwart & Verbunt 1996; Nelemans et al. 2001b, 2004). Each system is described by seven parameters: the binary masses M_1 and M_2 , the orbital period P_{orb} , the mass-transfer rate \dot{M} , the Galactic latitude l and longitude b , and the luminosity distance d to the solar system.

Following Nelemans et al. (2004), we distribute single and binary stars according to a Galactic model that comprises both disk and bulge components.⁶ The density of systems in the disk is

$$\rho_{\text{disk}}(R, Z) = \rho_{\text{BP}}(R) \text{sech}^2\left(\frac{Z}{Z_{\text{disk}}}\right) \text{pc}^{-3}, \quad (3)$$

where R is the cylindrical radius from the Galactic center, $\rho_{\text{BP}}(R)$ results from integrating the time-dependent plane-projected SFR of Boissier & Prantzos (1999; see Equations (1) and (2) of Nelemans et al. 2004), Z is the perpendicular distance from the Galactic plane, and Z_{disk} is the disk’s scale height. The density of systems in the spherical bulge is given by

$$\rho_{\text{bulge}}(r) \propto e^{-(r/r_{\text{bulge}})^2} \text{pc}^{-3}, \quad (4)$$

where r is the spherical distance from the Galactic center, r_{bulge} is the characteristic radius of the bulge, and Equation (4) is normalized so that there are as many bulge systems as disk systems in the inner 3 kpc of the Galaxy, with a current total mass of $2.6 \times 10^6 M_\odot$.

For the first four scenarios, we set $Z_{\text{disk}} = 300 \text{ pc}$ and $r_{\text{bulge}} = 0.5 \text{ kpc}$ (Jurić et al. 2008; Nelemans et al. 2004; Binney & Tremaine 1987; Sofue et al. 2009); $\rho_{\text{BP}}(R)$ is fit well by a density $\propto \exp(-R/R_{\text{disk}})$ with $R_{\text{disk}} \approx 2.5 \text{ kpc}$ for DDWD systems and $R_{\text{disk}} \approx 2.2\text{--}2.4 \text{ kpc}$ for AM CVn systems. In the

last scenario, we explore a modified model of the Galactic disk with a larger scale height for binaries that are produced at earlier times ($>7 \text{ Gyr}$ ago).

For all but one scenarios, we use the DDWD catalog analyzed in Nelemans et al. (2004), with local density $1.2 \times 10^{-5} \text{ pc}^{-3}$, and orbital periods as large as 5.5 hr ($f_{\text{GW}} = 10^{-4} \text{ Hz}$). Binaries at longer periods are not typically detectable by GW instruments because detector noise is higher at low frequencies; they are therefore not considered further in this analysis. All the catalogs of AM CVn systems, by contrast, show the effects of our different assumptions on AM CVn formation:

Case 1: upper bound (strong tidal coupling and inefficient ELD events). This catalog includes AM CVn systems that have formed via the DDWD channel operating with strong tidal coupling and the He-star–WD channel with few ELD events (see Nelemans et al. 2001b, 2004). Although the resulting AM CVn local density is higher ($2.8 \times 10^{-5} \text{ pc}^{-3}$) than SDSS-estimated values, past investigations of Galactic WD binaries as *LISA* sources have all used variants of this scenario (see, e.g., Edlund et al. 2005; Timpano et al. 2006). Thus, this case provides a meaningful comparison between our study and previously published results, which may vary depending on their idealizations of “detection,” “resolution,” etc. (see Section 4.3).

Case 2: inefficient formation channels (weak tidal coupling and efficient ELD events). This catalog includes AM CVn systems that have formed via the DDWD channel operating with minimal tidal coupling and the He-star–WD channel with many ELD events (see Nelemans et al. 2001b, 2004). The AM CVn local density is $6.1 \times 10^{-6} \text{ pc}^{-3}$, a factor of two greater than SDSS-estimated values (Roelofs et al. 2007a), and falls within the factor of two uncertainty of population-synthesis results.

Case 3: mild tidal coupling and no ELD events. This catalog includes AM CVn systems that have formed via only the DDWD channel, where mild tidal coupling ($\tau_s = 2 \text{ years}$) between the WDs allows for stable mass transfer in up to 10% of systems compared to the optimistic channel of case 1. We assume that the He-star–WD channel is suppressed. (However, observations discussed in Roelofs et al. 2007a suggest the presence of hot donors, which imply either that the He-star–WD channel is active with core He burning having been quenched only recently, or that following the second common-envelope ejection, the initial DDWD orbital separation is so small that the donor WD cannot cool within the gravitational-radiation timescale.) The AM CVn local density is $1.1 \times 10^{-6} \text{ pc}^{-3}$.

Case 4: fewer progenitor DDWDs. In this scenario, we explain the SDSS-estimated AM CVn densities by postulating a 1/5 reduction in the number of their DDWD progenitors. This reduction in DDWD systems is consistent with existing observations and with the factor of two uncertainty introduced in population-synthesis models by our limited understanding of binary evolution and Galactic structure. This catalog also includes AM CVn systems that have formed both via the DDWD channel with mild tidal coupling ($\tau_s = 1 \text{ year}$) and via the pessimistic He-star–WD channel (see Nelemans et al. 2001b, 2004); their densities are also reduced by 1/5 (across all periods). The DDWD and AM CVn local densities are 2.4×10^{-6} and $1.5 \times 10^{-6} \text{ pc}^{-3}$.

⁶ The bulge is not included in the analysis of Roelofs et al. (2007b). Furthermore, both Ruiter et al. (2009) and Yu & Jeffery (2010) also include a halo component; however, the impact of halo DDWD systems seems negligible for GW interferometers.

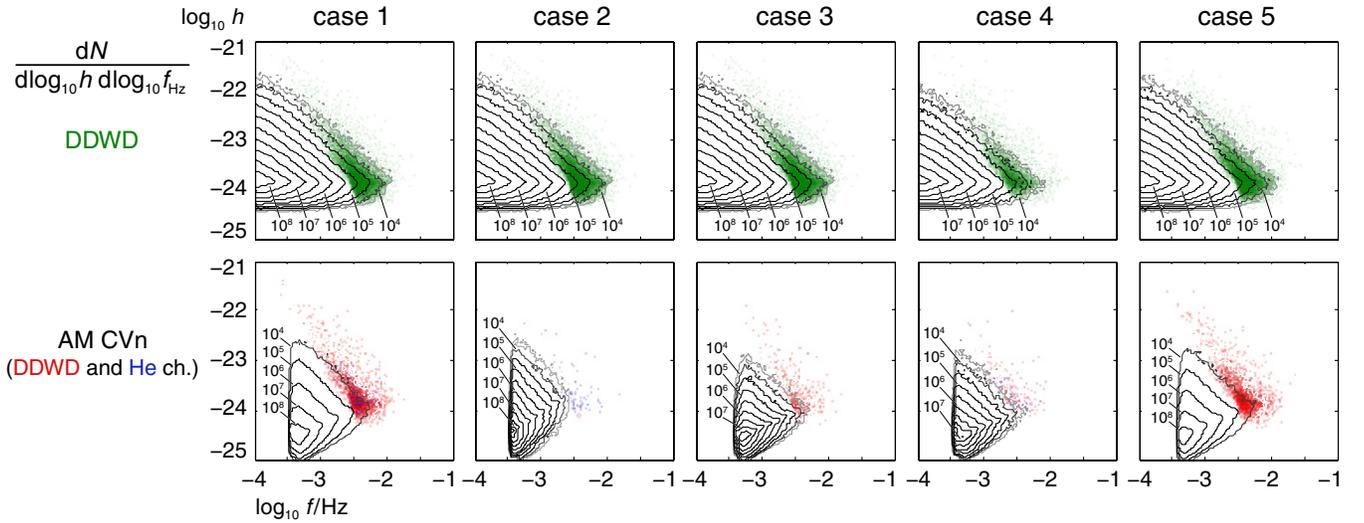


Figure 1. Number density as a function of GW-strain amplitude h and GW frequency f for DDWD (top row) and AM CVn (bottom row) systems in our five scenarios. Green dots denote individually detected DDWD systems (for one year of observation with the 5 Mkm detector), red dots AM CVn systems from the DDWD channel, and blue dots AM CVn systems from the He-star channel. See Sections 4.3 and 5.1 for a discussion of individually resolvable sources. The noticeable low-frequency cutoff of AM CVn systems is due to the age of the universe.

(A color version of this figure is available in the online journal.)

Table 1
Total Number and Local Density of Compact-binary Systems of Different Classes (Detached, AM CVn from the DDWD Channel, AM CVn from the He-star-WD Channel) in Our Simulated Catalogs for Cases 1–5

| | Number in Catalog/Local Space Density (pc^{-3}) | | |
|--------|--|----------------------------------|----------------------------------|
| | DDWD | AM CVn (DDWD ch.) | AM CVn (He-star-WD ch.) |
| Case 1 | 26,084,411/ 1.2×10^{-5} | 23,025,764/ 1.9×10^{-5} | 11,197,735/ 9.0×10^{-6} |
| Case 2 | 26,084,411/ 1.2×10^{-5} | 261,840/ 2.1×10^{-7} | 6,643,091/ 5.9×10^{-6} |
| Case 3 | 26,084,411/ 1.2×10^{-5} | 3,178,553/ 1.1×10^{-6} | None/0 |
| Case 4 | 5,217,866/ 2.4×10^{-6} | 994,194/ 2.6×10^{-7} | 1,328,909/ 1.2×10^{-6} |
| Case 5 | 26,084,509/ 1.2×10^{-5} | 23,025,764/ 3.8×10^{-6} | None/0 |

Case 5: modified Galactic-disk model. In this scenario, we place older systems in a thicker disk. With the caveat of small-number statistics, AM CVn systems identified by emission lines in SDSS have inferred projected velocities $>70 \text{ km s}^{-1}$, hinting that AM CVn systems with longer periods may originate from a disk with a thick component, or at least from a thin disk with a larger scale height. Indeed, such a distribution of AM CVns at birth is consistent with our current (if everchanging) understanding of the spatial and temporal evolution of thin and thick galaxy disks (Scannapieco et al. 2011; Jurić et al. 2008; Schönrich & Binney 2009a, 2009b; Siebert et al. 2011; Bovy et al. 2012). Observational constraints are unfortunately limited (see Napiwotzki 2009 for a discussion on the thin-disk, thick-disk, and halo populations of single WDs). As an illustrative example, we altered the disk component of Equation (3) to have $Z_{\text{disk}} = 1250 \text{ pc}$ for systems older than 7 Gyr, $Z_{\text{disk}} = 300 \text{ pc}$ for systems younger than 7 Gyr, and $r_{\text{bulge}} = 0.5 \text{ kpc}$ as for cases 1–4. This effectively places all systems older than 7 Gyr in a thick disk, which includes the great majority of longer period systems. In addition, we assumed that AM CVn systems only formed via the DDWD channel. The DDWD and AM CVn local densities are 1.2×10^{-5} and $3.8 \times 10^{-6} \text{ pc}^{-3}$.

For each scenario, Table 1 shows the numbers of AM CVn and DDWD systems in the catalogs, as well as their local density ρ_0

(i.e., the Galactic mid-plane density at 8.5 kpc from the Galactic center).

4.2. Detecting Gravitational Waves from WD Binaries

Compact Galactic binaries in the low-frequency band emit quasi-monochromatic GWs, with frequency drifts of (at most) several frequency bins for year-long observation. In this paper, we model compact-binary waveforms by including only the dominant quadrupolar emission (Peters & Mathews 1963), with instantaneous strain amplitude

$$h = 10^{-21} \left(\frac{\mathcal{M}_z}{M_\odot} \right)^{5/3} \left(\frac{P_{\text{orb}}}{\text{hr}} \right)^{-2/3} \left(\frac{d}{\text{kpc}} \right)^{-1}, \quad (5)$$

where $\mathcal{M}_z = (1+z)\mu^{3/5}M^{2/5} = (1+z)\mathcal{M}$ is the redshifted chirp mass (with M the total mass, μ the reduced mass, \mathcal{M} the chirp mass, and z the redshift), d is the luminosity distance to the source, and P_{orb} is the orbital period of the binary.

Figure 1 shows the number density as a function of h (the GW strain of Equation (5)), and $f = 2/P_{\text{orb}}$ (the GW frequency), for the different DDWD and AM CVn catalogs of each of our scenarios. The colored dots indicate detected binaries (see Section 4.3): green shows DDWDs, red shows AM CVn systems (from the DDWD channel), and blue shows AM CVn systems (from the He-star-WD channel).

Binaries with orbital periods shorter than approximately 20 minutes will appear as isolated signals in year-long data

sets of space-based GW observatories, and will be detectable when the signals are sufficiently strong compared to instrument noise at the same frequency. By contrast, most binaries with longer orbital periods will not be detected individually, because they will be too dense in frequency space to tell apart. These will produce a noise-like confusion foreground that will affect the detection and parameter estimation of extra-Galactic GW sources such as massive black hole binaries. Nevertheless, the brightest binaries will be detected above the confusion foreground.

Looking at Figure 1, we see that DDWD systems have larger densities than AM CVn systems, and larger GW strains (because they have greater chirp masses). Thus, it is DDWD systems that are principally responsible for the shape and strength of the GW foreground from WD binaries (see Section 5.2). We note also that all AM CVn populations show an overall low-frequency cutoff at $\log_{10} f = -3.5$, due to the age of the universe.

A simplified (but roughly correct) description of GW data analysis is that GW signals will be detectable when their signal-to-noise ratio (S/N) is greater than a detection threshold S/N_{thr} chosen to yield a minimal number of false alarms. S/N is the ratio of signal strength to rms instrument noise (plus confusion noise, if present) at the same frequency. For *LISA*-like interferometric detectors, the yearly orbit and rotation of the spacecraft constellation imprint the signals with frequency and amplitude modulations that encode the sky position of the source. Detector response is further affected in frequency-dependent fashion by Time-Delay Interferometry (TDI), the technique used to subtract the otherwise overwhelming laser frequency noise (thus, S/N would be evaluated for the TDI signal with respect to TDI noise). See Appendix B for an overview of *LISA*-like detectors, their response to GWs, and the theory of probabilistic signal detection in noise.

4.3. Simulations of Detection and Subtraction

To estimate the number of Galactic-binary systems that we will be able to detect individually with our reference detectors, as well as the confusion noise from the residual unsubtracted systems, we performed simulations of detection and subtraction using a variant of the method introduced by (Timpano et al. 2006; see Section 5.1). To wit:

1. For each of our scenarios, we compute the frequency-domain TDI response for each system in the corresponding catalog, using an optimized variant (Vallisneri 2011) of the Mock *LISA* Data Challenge fast-binary code (Cornish & Littenberg 2007); we then superpose the responses to produce a full year-long data set.
2. We run through the catalog and evaluate the S/N of each system (see Appendix B) with respect to a background of instrumental noise plus confusion noise from all nearby systems, estimated as a running median of their power spectral density. We flag as detected the systems with $S/N > S/N_{\text{thr}}$.
3. We subtract the signals detected in step 2 from the data set. We then go back to step 2 and iterate, using the resulting lower estimate of confusion noise.
4. The process converges after a few iterations, with the number of detected systems decreasing better than exponentially. We then evaluate and fit the power spectral density from the residual confusion foreground.

This process represents a rather optimistic model of source detection and subtraction that neglects two important effects:

Table 2
Detectable Sources in Our Simulations (5 Mkm Detector)

| | DDWD | AM CVn (DDWD) | AM CVn (He-WD) |
|--------|--|---|---|
| Case 1 | $11,898 \times (\rho_{\text{eff}}/5)^{-1.4}$ | $2,093 \times (\rho_{\text{eff}}/5)^{-1.8}$ | $122 \times (\rho_{\text{eff}}/5)^{-2.8}$ |
| Case 2 | $13,957 \times (\rho_{\text{eff}}/5)^{-1.4}$ | $16 \times (\rho_{\text{eff}}/5)^{-1.7}$ | $72 \times (\rho_{\text{eff}}/5)^{-2.9}$ |
| Case 3 | $13,708 \times (\rho_{\text{eff}}/5)^{-1.4}$ | $397 \times (\rho_{\text{eff}}/5)^{-2.0}$ | none |
| Case 4 | $5,252 \times (\rho_{\text{eff}}/5)^{-1.2}$ | $161 \times (\rho_{\text{eff}}/5)^{-1.7}$ | $79 \times (\rho_{\text{eff}}/5)^{-1.8}$ |
| Case 5 | $11,972 \times (\rho_{\text{eff}}/5)^{-1.4}$ | $2,107 \times (\rho_{\text{eff}}/5)^{-1.7}$ | none |

Notes. Values are shown for $S/N_{\text{thr}} = 5$, one interferometric observable, and one year of observation, with approximate scalings as a function of $\rho_{\text{eff}} = S/N_{\text{thr}}(T_{\text{obs}}/\text{yr})^{-1/2}(N_{\text{obs}})^{-1/2}$.

source detection will be confused by the presence of nearby signals beyond what is predicted by the increased noise level, and subtraction will be degraded by the imperfect estimation of parameters in noise (see, e.g., Crowder & Cornish 2004). Nevertheless, we can adjust the level of optimism by varying our assumptions regarding the duration of observation T_{obs} , the detection threshold S/N_{thr} , and the number N_{obs} of TDI observables that are available (two, in effect, for the standard *LISA* configuration with five or six operating links; one for a two-arm configuration).

It turns out that our results for the number of detected sources and for the residual confusion foregrounds are fit remarkably well by power laws of a single *effective-S/N* parameter $\rho_{\text{eff}} = S/N_{\text{thr}}(T_{\text{obs}}/\text{yr})^{-1/2}(N_{\text{obs}})^{-1/2}$. As a baseline, we take a standard assumption, consistent with prior literature, of one year, one observable (the standard unequal-arm Michelson *X*), and $S/N_{\text{thr}} = 5$. A conservative assumption would correspond to taking one year, one observable, and $S/N_{\text{thr}} = 10$, yielding $\rho_{\text{eff}} = 10$; an optimistic assumption to five years, two independent observables, and $S/N_{\text{thr}} = 5$, yielding $\rho_{\text{eff}} = 1.58$. Results are not sensitive to details such as the choice of running-median window, and whether signals are subtracted immediately or after each full catalog run-through.

5. RESULTS

In this section, we present our results on the impact that different Galactic populations of DDWD and AM CVn systems have on the number of individually detectable sources (Section 5.1) and on the total and confusion GW foregrounds (Section 5.2), for our two representative space-based GW detectors and for each of our five population scenarios.

Figure 2 recapitulates the findings of our simulations: it shows the frequency-space density of catalog sources; the power spectral density of the unsubtracted (blue) and subtracted (red) confusion foreground, compared to the instrument noise (black, all in the bottom panel); and the location and numbers of detected sources (dots in the bottom panel, histograms in the top panels). The detected sources lie approximately a factor of S/N_{thr}^2 above the composite of instrument noise and subtracted confusion noise.

5.1. Individually Detected Sources

Tables 2 (for the 5 Mkm detector) and 3 (for the 1 Mkm detector) show the number of recovered systems of each type in our five scenarios. The numbers quoted correspond to simulation results for one year, one interferometric observable, and $S/N_{\text{thr}} = 5$; the power laws give approximate fits as functions of $\rho_{\text{eff}} = S/N_{\text{thr}}(T_{\text{obs}}/\text{yr})^{-1/2}(N_{\text{obs}})^{-1/2}$.

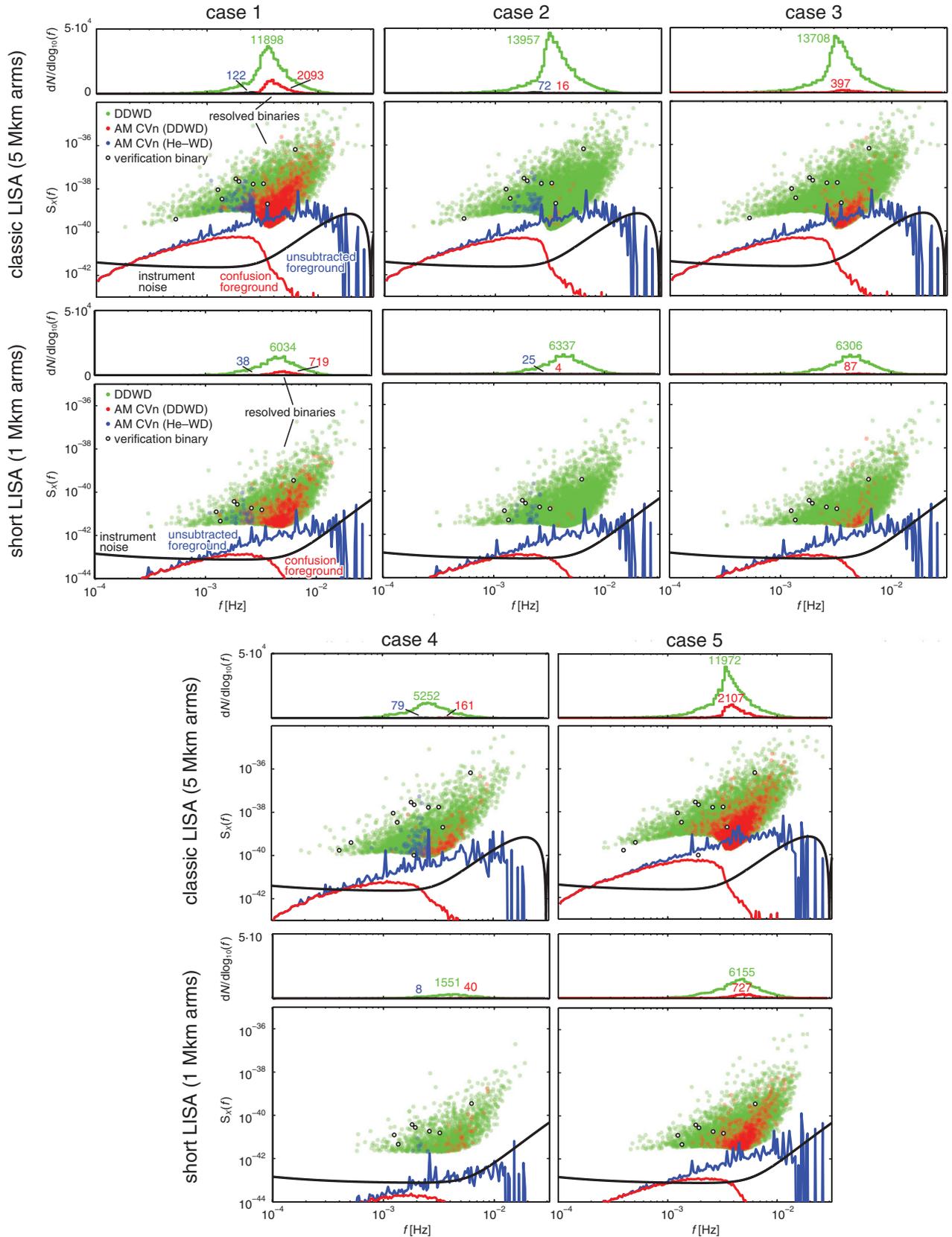


Figure 2. Frequency-space density and GW foreground of DDWD and AM CVn systems for the five scenarios examined in this paper. In each subplot, the bottom panel shows the power spectral density of the unabstracted (blue) and partially subtracted (red) foreground, compared to the instrument noise (black). The open white circles indicate the frequency and amplitude of the so-called verification binaries (see Nelemans 2012). These comprise known DDWD and AM CVn systems; a handful have known distances from *HST* FGS parallax observations, while we assume that the rest are at a distance of 1 kpc. The green dots show the individually detectable DDWD systems, the red dots show the detectable AM CVn systems formed from DDWD progenitors, and the blue dots show the detectable AM CVn systems that arise from the He-star–WD channel. The top panel shows histograms of the detected sources in frequency space.

(A color version of this figure is available in the online journal.)

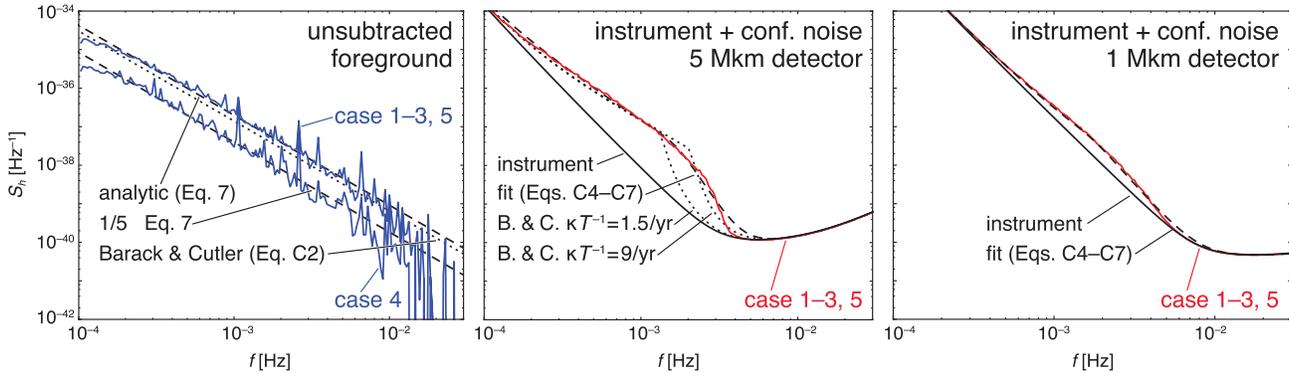


Figure 3. Left panel: power spectral density of GW strain for the unsubtracted Galactic foreground, as measured in our simulations for cases 1–3 and 5 (top continuous curve) and case 4 (bottom continuous curve); the foreground is fit well by our Equation (6), with a 1/5 smaller coefficient for case 4 (dashed curves). For comparison, the dotted curves shows the Barack and Cutler estimate, Equation (C2). At high frequencies, the simulated spectra drop because frequency bins are unevenly occupied, and because of the onset of mass transfer; at low frequency, they drop because some binaries are born at frequencies within the detector’s bandwidth, contrary to the assumptions that yield the $f^{-7/3}$ profile. (These spectra of GW strain do not include the instrument response to GWs; to compare with the TDI X spectra of Figure 2, one would multiply by the frequency-dependent response of Equation (C8).) Middle and right panels: total equivalent GW-strain noise (instrument noise plus residual Galactic-binary confusion noise) for the 5 Mkm and 1 Mkm detectors of our study, as obtained in our simulations for cases 1–3 and 5 and for $\rho_{\text{eff}} = 5$ (red curves); case 4 is not shown to avoid visual clutter. The total noise is fit well (dashed curves) by our expressions (7)–(9) (see also Equations (C4)–(C7)) and less well (dotted curves) by Barack and Cutler’s standard prescription (C1)–(C3), which improves by setting $\kappa T^{-1} \simeq 9 \text{ yr}^{-1}$.

(A color version of this figure is available in the online journal.)

Table 3
Detectable Sources in Our Simulations (1 Mkm Detector)

| | DDWD | AM CVn (DDWD) | AM CVn (He-WD) |
|--------|--|---|--|
| Case 1 | $6034 \times (\rho_{\text{eff}}/5)^{-1.2}$ | $719 \times (\rho_{\text{eff}}/5)^{-1.4}$ | $38 \times (\rho_{\text{eff}}/5)^{-2.6}$ |
| Case 2 | $6337 \times (\rho_{\text{eff}}/5)^{-1.2}$ | $4 \times (\rho_{\text{eff}}/5)^{-1.4}$ | $25 \times (\rho_{\text{eff}}/5)^{-2.2}$ |
| Case 3 | $6306 \times (\rho_{\text{eff}}/5)^{-1.2}$ | $87 \times (\rho_{\text{eff}}/5)^{-1.8}$ | None |
| Case 4 | $1551 \times (\rho_{\text{eff}}/5)^{-1.1}$ | $40 \times (\rho_{\text{eff}}/5)^{-1.3}$ | $8 \times (\rho_{\text{eff}}/5)^{-2.2}$ |
| Case 5 | $6155 \times (\rho_{\text{eff}}/5)^{-1.2}$ | $727 \times (\rho_{\text{eff}}/5)^{-1.4}$ | None |

Note. Values are shown for $S/N_{\text{thr}} = 5$, one interferometric observable, and one year of observation, with approximate scalings as a function of $\rho_{\text{eff}} = S/N_{\text{thr}}(T_{\text{obs}}/\text{yr})^{-1/2}(N_{\text{obs}})^{-1/2}$.

Consider first the number of detectable sources as seen by the longer, standard LISA-like detector. In all cases except the “fewer progenitors” scenario 4, the number of detected DDWDs remains comparable to prior optimistic projections ($\sim 12,000$ – $14,000$). Even in case 4, where the DDWD systems in the catalog are reduced by a factor of five, detections drop only by half, thanks to a weaker confusion foreground. By contrast, the number of detected AM CVn varies considerably depending on their density and formation scenarios (~ 15 – 2000). In cases 2 and 3, more DDWDs are detectable at relatively high frequencies because of the reduced confusion noise from fewer AM CVn.

Case 1 was chosen to be directly comparable to the standard population catalogs of earlier works (Nelemans et al. 2001a, 2001b, 2001c, 2004), but the claimed number of detections for AM CVn systems is significantly lower than those earlier estimates, derived with a simple resolvability criterion of a single source per frequency bin with instrument-noise $S/N > 1$ (Nelemans et al. 2001b, 2004). This is unsurprising, given that in our analysis we model detection by identifying and subtracting bright sources iteratively, and that we require a higher S/N , defined with respect to both instrument and confusion noise. Furthermore, in this paper, we use proper interferometric observables instead of “raw” GW strain.

Downscaling the detector’s armlength to 1 Mkm approximately halves the number of DDWD systems detected for $\rho_{\text{eff}} = 5$, except for case 4, where the decrease is ~ 3 . The number of AM CVn system detections is likewise reduced by factors ~ 3 – 4 . Note however that the ρ_{eff} scalings of Tables 2

and 3 are slightly different. The ESA eLISA/NGO study (Amaro-Seoane et al. 2012) finds numbers compatible with ours, once we take into account our noise assumptions for the 1 Mkm detector, which are somewhat more favorable than NGO’s.

5.2. Unsubtracted and Residual Compact-binary Foregrounds

In Appendix A, we apply a familiar analytical treatment of the compact-binary GW foreground to a broad range of Galactic models examined in the literature, and we conclude that the foreground depends strongly on the Galactic-binary merger rate and on their characteristic chirp masses, but only weakly on other parameters. Namely,

$$S_h(f) \simeq 1.9 \times 10^{-44} (f/\text{Hz})^{-7/3} \text{ Hz}^{-1} \times \left(\frac{D_{\text{char}}}{6.4 \text{ kpc}} \right)^{-2} \left(\frac{\mathcal{R}_{\text{gal}}}{0.015 \text{ yr}^{-1}} \right) \left(\frac{\mathcal{M}_{z,\text{char}}}{0.35 M_{\odot}} \right)^{5/3}, \quad (6)$$

where D_{char} is the characteristic distance from Earth to Galactic binaries (a weak function of the model), \mathcal{R}_{gal} is the binary merger rate in the Galaxy, and $\mathcal{M}_{z,\text{char}}$ is the characteristic chirp mass $(\mathcal{M}_z^{5/3})^{-5/3}$. The $f^{-7/3}$ slope describes the power emitted at GW frequency f by a *single* binary evolving due to GWs, and it applies to a stationary *ensemble* where new binaries are born with a constant rate (equal to \mathcal{R}_{gal}) at frequencies below the detector bandwidth. Equation (6) is normalized to values representative of our assumptions; as shown in the left panel of Figure 3, it is in good agreement with the results of our simulations for cases 1–3 and 5, while the coefficient is reduced by a factor of five for case 4.

The residual confusion foreground consists of the unresolvable DDWD and AM CVn systems that are left once relatively brighter or isolated sources have been subtracted iteratively from the original population catalog. The residual foregrounds found in our simulations are very similar among cases 1–3 and 5, while they are a factor of five weaker for case 4. We note that the overall number of DDWD systems in cases 1–3 and 5 differs by a factor of five, thereby confirming earlier work and our predictions that the DDWD population provides the dominant component to the strength of the confusion foregrounds (both unsubtracted and subtracted). As shown in the middle and right

panels of Figure 3, for both the 5 Mkm and 1 Mkm detectors, the total equivalent strain noise (instrument noise plus confusion foreground) is fit well by the expression

$$S_h^{\text{total}}(f) = S_h^{\text{inst}} + S_h^{\text{conf}} \times \tanh^\alpha \left(\frac{1}{2} \beta dN/df \right), \quad (7)$$

where $\alpha \simeq 1$, $\beta \simeq 1 \text{ yr}^{-1}$,

$$S_h^{\text{conf}}(f) = 1.4 \times 10^{-45} (f/\text{Hz})^{-8/3} \text{ Hz}^{-1}, \quad (8)$$

and where

$$\frac{dN}{df} = 5 \times 10^{-3} (f/\text{Hz})^{-11/3} \text{ Hz}^{-1} \quad (9)$$

is the frequency-domain density of binary systems, which modulates the transition from the confused to the resolved regime. Equation (7) is formulated in analogy to the standard expression Equation (C1) (see Barack & Cutler 2004a, 2004b), with a different functional form that fits our simulations better. In Appendix C, we give more precise α and β fits as functions of ρ_{eff} , and we provide expressions for the total TDI X noise and for the seasonal modulation of the GW foreground due to detector motion (Giampieri & Polnarev 1997; Seto 2004; Edlund et al. 2005).

6. CONCLUSIONS

In this work, we have examined the impact that different observationally inferred populations of Galactic WD binaries have on the number of individually detectable sources and on the confusion foreground for future low-frequency GW observations. Earlier studies of Galactic WD binaries as GW sources relied primarily on population synthesis, because of the small number of observed systems, in particular mass-transferring ones (i.e., AM CVn systems). However, the past several years have seen a flurry of new observations, quintupling the known detached systems that will merge within a Hubble time, doubling the known AM CVn systems, and leading to new, lower estimates for the local densities of the latter.

To understand the consequences of these new estimates on GW detection, we have generated a range of plausible populations of WD binaries, assuming different AM CVn formation scenarios. For example, in one model (case 3), we postulate that fewer AM CVn systems are observed by SDSS than predicted because the tidal coupling between the spin of the accretor and the orbit is not as efficient as expected, so more systems are lost to mergers. We have then investigated the resulting numbers of individually resolvable GW sources and the magnitude of the total and residual GW foregrounds.

In four out of five models, we find that the *number of detectable detached systems* is $\sim 12,000$ – $14,000$, for one year of observation by a LISA-like GW detector with 5 Mkm arms (using an optimistic detection threshold, but considering a single interferometric observable). These four models leave unchanged the number of detached binaries used in earlier estimates, but make various assumptions about the number of systems lost to mergers or to ELDs, or (in case 5) about the spatial distribution of binaries. In one model (case 4) that instead explains the observed local AM CVn density with five times fewer detached progenitor systems, we find that ~ 6000 of the latter should be detectable. For the 1 Mkm arm detector, DDWD detections would be ~ 6000 and 1500 for cases 1–3 and 5, and case 4, respectively. The *number of detectable AM CVn systems* ranges

from several tens to a few hundred (for cases 2–4), or even as high ~ 2000 (for cases 1 and 5), again for the 5 Mkm detector. For the 1 Mkm detector, these numbers are smaller by approximately a factor of three to four. See Tables 2 and 3 for details.

Now, why is it that even in our upper-bound model (case 1), where the total number of AM CVn systems is comparable to earlier estimates, we project far fewer AM CVn detections than the 10,000 quoted by Nelemans et al. (2001b, 2004)? Conversely, in four out of five models we project *more* detached-binary detections (12,000–14,000 versus 10,000) than Nelemans et al. (2001b). Such differences arise because we define detection by way of an iterative identify-and-subtract process, where a source’s S/N is evaluated with respect to noise from both the instrument and the partially subtracted foreground. Our approach is similar to that used by Timpano et al. (2006). By contrast, according to Nelemans et al. (2001b, 2004), a source is detectable if it has an S/N (relative to instrument noise alone) greater than five, or an S/N greater than one with no other source in the same frequency bin. Our more stringent definition impacts the number of detected AM CVn systems more than their detached counterparts at the same frequencies, because AM CVn systems have smaller chirp masses, and therefore relatively weaker GW-strain amplitudes.

Indeed, we confirm that the dominant contribution to the *diffuse GW foreground* comes from detached binaries, which have larger chirp masses. In Appendix A, we show that the spectrum of the unsubtracted foreground can be derived robustly by simple analytical arguments, and depends mainly on the Galactic merger rate and the characteristic binary chirp mass (see Equation (6)). These analytical estimates agree well with our numerical results. In addition, for both the 5 Mkm and 1 Mkm detectors, we provide fitting expressions (Equations (7)–(9)) for the residual confusion foreground due to unresolvable sources. Our estimates are comparable to (but measurably different from) the standard formulae discussed by Barack & Cutler (2004a) and used by many authors in the past.

The projected detection numbers and residual confusion foregrounds change significantly if we use different assumptions on the detection threshold, the duration of observation, and the number of available interferometric observables. We have simulated the identify-and-subtract process for several such assumptions, and we find empirically that the detection numbers and confusion-foreground fitting parameters scale as power laws of a single *effective threshold* $\rho_{\text{eff}} = S/N_{\text{thr}} (T_{\text{obs}}/\text{yr})^{-1/2} (N_{\text{obs}})^{-1/2}$.

In one of our models (case 5), we have examined (to our knowledge, for the first time) how GW observations would change if we modify the evolution and structure of the Galactic disk to match the observed local AM CVn density, while keeping overall binary numbers as in case 1. We find no significant impact on the number of detectable systems, nor on the strength or shape of the confusion foreground. However, on average, the detected systems will have measurably smaller GW amplitudes, because of their larger distances from the solar system. We conjecture that such distance measurements could be used to infer the structure of the Galactic disk.

Finally, we expect that the advent of wide-field synoptic and targeted surveys (such as the Palomar Transient Factory and the ELM survey respectively), as well as ESA’s *Gaia* mission, will continue to provide new data to this field of research, and will enable more stringent constraints on binary populations. So far, the observed density of detached systems appears to be consistent with theoretical estimates, but a detailed

Table 4
Estimates of GW from Populations of Compact Binaries

| Reference | Model | Source Classes | Input Population |
|-----------------------------------|---------|--------------------------------|---|
| This work, Nelemans et al. (2004) | RS + CB | DDWD + AMCVn | SeBa ^a |
| Nelemans et al. (2001b) | RS + CB | DDWD + AMCVn | SeBa ^a |
| Edlund et al. (2005) | RS + CB | DDWD + AMCVn | SeBa ^a |
| Timpano et al. (2006) | RS + CB | DDWD | SeBa ^a , Webbink (1984) |
| Seto (2002) | RS + CB | DDWD | Webbink (1984) |
| Bender & Hils (1997) | RS + CB | DDWD | Webbink (1984) |
| Hils et al. (1990) | CB | DDWD | Webbink (1984) |
| Hils & Bender (2000) | CB | AMCVn | Iben & Tutukov (1984); Tutukov & Yungelson (1996) |
| Hils (1998) | RS + CB | AMCVn | Iben & Tutukov (1984); Tutukov & Yungelson (1996) |
| Yu & Jeffery (2010) | RS + CB | DDWD (+halo) | BSE ^b + Han (1998) |
| Liu (2009); Liu et al. (2010) | RS + CB | DDWD | BSE ^b + Han (1998) |
| Farmer & Phinney (2003) | CB | Extragalactic DDWD + AM CVn | BSE ^b |
| Webbink & Han (1998) | CB | DDWD | Han (1998) |
| Ruiter et al. (2010) | RS + CB | DDWD + AMCVn | StarTrack, Belczynski et al. (2002, 2008) |
| Ruiter et al. (2009) | | (halo) | |
| Benacquista et al. (2004) | RB + CB | DDWD (+globular) | Specified in paper |
| Postnov & Prokhorov (1998) | CB | DDWD | See Lipunov et al. (1996) |
| Lipunov et al. (1995) | CB | DDWD (+halo) | Specified in paper |
| Lipunov & Postnov (1987) | CB | Compact binaries | Specified in paper |
| Lipunov et al. (1987) | | | |
| Giampieri & Polnarev (1997) | CB | Generic | Generic |
| Evans et al. (1987) | CB | DDWD | Specified in paper |

Notes. RS: resolved sources; CB: confusion background.

^a SeBa: Portegies Zwart & Verbunt (1996); Portegies Zwart & Yungelson (1998); Nelemans et al. (2001c); Toonen et al. (2011).

^b BSE: Hurley et al. (2000, 2002).

population analysis based on the ELM survey has yet to be undertaken. Furthermore, all estimates of AM CVn densities based on surveys are made uncertain by the intricate selection effects. Because of these uncertainties, in this paper, we chose to examine a variety of population models that span the range of likely outcomes. Our overall conclusion is that the Galactic-binary science enabled by low-frequency GW detectors is very robust for a broad range of possible binary populations and detector sensitivities.

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APPENDIX A

PREVIOUS ESTIMATES OF GW EMISSION FROM GALACTIC BINARIES

More than two dozen papers have made estimates of the GW signal from individually detectable Galactic compact binaries and from the continuum of unresolvable sources at low frequen-

cies (variously called “confusion background” or “Galactic foreground”) for a *LISA*-like GW detector with 5 Mkm arms. An earlier informative review of the literature is given by Ruiter et al. (2010). We note that for a 1 Mkm detector, the foreground decreases substantially, to a level comparable to instrument noise (see Figure 3 in Section 5.2).

Table 4 compares a subset (large, but possibly not complete) of relevant published estimates, grouped by population model. For each entry, we show whether the estimates include resolved sources (RSs), the continuum background (CB), or both; we show also which classes of binaries are included. In Table 5, we provide relevant scale lengths for the Galactic disk and bulge distributions, where available; these are indicative of the range of assumed Galactic models, but they do not provide a complete picture since different density laws also appear. We do not give quantitative information on possible halo components, which are also frequently used. The major differences between the various estimates arise from two sets of assumptions:

1. Assumptions about the relative numbers of systems in terms of total mass, mass ratio, and separation. These arise from different treatments of compact-binary progenitor evolution and of the stability of mass-transferring systems. Approaches include both analytic estimates and Monte Carlo population-synthesis calculations.
2. Assumptions about Galactic structure and population space densities. These can affect estimates in two ways: first, the magnitude of GW signals will depend on the scale size as-

Table 5
Estimates of GW from Populations of Compact Binaries—Spatial Distributions

| Reference | Model | Z_{disk} (kpc) | R_{disk} (kpc) | $\mathcal{D}_{\text{char}}$ (Disk) (kpc) | r_{bulge} (kpc) |
|-----------------------------------|---|----------------------------|----------------------------|---|-----------------------------|
| This work, Nelemans et al. (2004) | See Section 4.1 | 0.3 | 2.5 ^a | 6.4 ^b | 0.5 |
| This work (thick disk) | | 1.25 | 2.5 ^a | 6.8 ^b | 0.5 |
| Nelemans et al. (2001b) | $e^{-R/R_{\text{disk}}} \text{sech}^2(Z/Z_{\text{disk}})$ | 0.2 | 2.5 | 5.3 | ... |
| Edlund et al. (2005) | | | | | |
| Timpano et al. (2006) | See Hills & Bender (2000) and Nelemans et al. (2004) | | | | |
| Seto (2002) | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 0.09 | 3.5 | 4.8 | ... |
| Bender & Hills (1997) | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 0.09 | 3.5 | 4.8 | ... |
| Hils et al. (1990) | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 0.09 | 3.5 | 4.8 | ... |
| Hils & Bender (2000) | ns | ns | ns | ... | ns |
| Hils (1998) | ns | ns | ns | ... | ns |
| Yu & Jeffery (2010) | $e^{-R/R_{\text{disk}}} \text{sech}^2(Z/Z_{\text{disk}})$ | 0.352 | 2.5 | 5.6 | 0.5 |
| | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 1.158 | 2.5 | 6.5 | ... |
| Liu et al. (2010); Liu (2009) | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 0.09 | 2.5 | 5.0 | ... |
| Farmer & Phinney (2003) | ... | ... | ... | ... | ... |
| Webbink & Han (1998) | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 0.09 | 4.0 | 4.7 | ... |
| Ruiter et al. (2010) | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 0.2 | 2.5 | 5.4 | 0.5 |
| Ruiter et al. (2009) | (halo) | | | | |
| Benacquista et al. (2004) | $e^{-R/R_{\text{disk}}} \text{sech}^2(Z/Z_{\text{disk}})$ | 0.2 | 2.5 | 5.3 | ... |
| Postnov & Prokhorov (1998) | $e^{-R/R_{\text{disk}}} e^{-(Z/Z_{\text{disk}})^2}$ | 4.2 | 5 | 7.2 | ... |
| Lipunov et al. (1995) | $e^{-R/R_{\text{disk}}} e^{-Z/Z_{\text{disk}}}$ | 0.46 | 4.6 | 5.7 | See paper |
| Lipunov & Postnov (1987) | $e^{-R/R_{\text{disk}}} e^{-(Z/Z_{\text{disk}})^2}$ | 4.2 | 10 | 9.6 | ... |
| Lipunov et al. (1987) | | | | | |
| Giampieri & Polnarev (1997) | $e^{-R/R_{\text{disk}}} e^{-(Z/Z_{\text{disk}})^2}$ | 0.1 | 3.5 | 4.5 | ns |
| Evans et al. (1987) | Constant density | 0.2 | 10 | | ... |

Notes. ns: not specified. Distance to the Galactic center is assumed to be 8.5 kpc for all models. Exponential squared bulge unless otherwise specified:

$e^{-(r/r_{\text{bulge}})^2}$ with $\mathcal{D}_{\text{char}}(\text{bulge}) = 8.4$ kpc.

^a Numerical fit, DDWD systems only.

^b $\mathcal{D}_{\text{char}}$ computed for disk + bulge for this case.

summed for the Galactic population; second, the “calibration” of the local number density of sources using observations of a particular source type will depend sensitively on the assumptions about Galactic structure.

We now present a semi-analytical derivation of the amplitude and shape of the GW continuum. Similar models have been used extensively in GW data analysis, see, for example, works by Hughes (2002) and Barack & Cutler (2004a, 2004b). Our results show that the magnitude and spectral shape of the continuum are a robust function of the merger rate and characteristic chirp mass of Galactic-binary systems, and depend only weakly on Galactic structure.

We follow the arguments presented in Evans et al. (1987) and Giampieri & Polnarev (1997). The GW luminosity of a circular binary and the change in its orbital energy as a function of GW frequency $f = 2f_{\text{orb}}$ are given by (Peters & Mathews 1963)

$$\begin{aligned} \frac{dE}{dt} &= \frac{32}{5} \pi^{10/3} \frac{G^{7/3}}{c^5} \mathcal{M}_z^{10/3} f^{10/3}, \\ \frac{dE}{df} &= \frac{1}{3} \pi^{2/3} G^{2/3} \mathcal{M}_z^{5/3} f^{-1/3}, \end{aligned} \quad (\text{A1})$$

where E is the energy of the system and f is the GW frequency. Combining these expressions yields the frequency evolution of the system,

$$\frac{df}{dt} = \frac{96}{5} \pi^{8/3} \frac{G^{5/3}}{c^5} \mathcal{M}_z^{5/3} f^{11/3}. \quad (\text{A2})$$

If $\mathcal{R}_{\text{birth}}(f, \mathbf{r})$ is the birth rate per unit frequency and volume of systems with $f_{\text{orb}} < f/2$, then in a *stationary regime* the density of sources must be

$$\begin{aligned} \frac{d^2 N}{df d\mathbf{r}} &= \frac{\mathcal{R}_{\text{birth}}(f, \mathbf{r})}{df/dt} \\ &= \frac{5}{96} \pi^{-8/3} \frac{c^5}{G^{5/3}} \mathcal{M}_z^{-5/3} \mathcal{R}_{\text{birth}}(f, \mathbf{r}) f^{-11/3}. \end{aligned} \quad (\text{A3})$$

By continuity, $\mathcal{R}_{\text{birth}}(f, \mathbf{r})$ must also be equal to the merger rate, $\mathcal{R}_{\text{mrg}}(f, \mathbf{r})$, of systems which merge at $f_{\text{orb}} > f/2$. DDWD systems, which generate most of the GW foreground, merge at $f > 10^{-2}$ Hz, and most of them are born below a few 10^{-4} Hz (although estimates vary). Thus, if we restrict Equation (A3) between those two frequencies, then we may just set $\mathcal{R}_{\text{birth}}(f, \mathbf{r}) = \mathcal{R}_{\text{mrg}}(\mathbf{r})$, defined as the total merger rate above 10^{-2} Hz.

Now, the direction-averaged GW strain from a circular binary is given by

$$\begin{aligned} h^2 &= \langle h_+^2 + h_\times^2 \rangle = \frac{1}{\pi^2} \frac{1}{d^2} \frac{G}{c^3} f^{-2} \frac{dE}{dt} \\ &= \frac{32}{5} \pi^{4/3} \frac{G^{10/3}}{c^8} \mathcal{M}_z^{10/3} f^{4/3}, \end{aligned} \quad (\text{A4})$$

see for instance (Giampieri & Polnarev 1997), where d is the luminosity distance to the binary; the corresponding detector response is

$$h_l^2 = \langle h_+^2 F_+^2 + h_\times^2 F_\times^2 \rangle \equiv h^2 F^2(\hat{\mathbf{r}}, t), \quad (\text{A5})$$

where F_+ and F_\times are the antenna-pattern functions (see, e.g., Maggiore 2007) and $F^2(\hat{\mathbf{r}}, t)$ is the polarization- and inclination-averaged antenna pattern ($\langle F_+^2 \rangle + \langle F_\times^2 \rangle$)/2 in the direction $\hat{\mathbf{r}}$.

We combine Equations (A3)–(A5) to obtain the one-sided detector response density S_{h_l} . The yearly orbital motion of *LISA*-like detectors modulates the amplitude of signals to individual sources; this effect averages out for isotropic source distributions, but not for the anisotropic Galactic-binary foreground (Giampieri & Polnarev 1997; Edlund et al. 2005). We therefore average over a year (while assuming for now that all binaries have the same chirp mass \mathcal{M}_z), to obtain

$$\begin{aligned} S_{h_l}(f) &= \left\langle \int h^2(d(\mathbf{r}), f) F^2(\hat{\mathbf{r}}, t) \frac{d^2 N}{df d\mathbf{r}} d\mathbf{r} \right\rangle \\ &= \frac{1}{3} \pi^{-4/3} \frac{G^{5/3}}{c^3} \mathcal{M}_z^{5/3} f^{-7/3} \int \frac{\langle F^2(\hat{\mathbf{r}}, t) \rangle_t \mathcal{R}_{\text{mrg}}(\mathbf{r})}{d(\mathbf{r})^2} d\mathbf{r} \\ &\equiv \frac{1}{3} \pi^{-4/3} \frac{G^{5/3}}{c^3} \mathcal{M}_z^{5/3} f^{-7/3} \langle F^2 \rangle_{\hat{\mathbf{r}}, t} \langle \mathcal{R}_{\text{mrg}}/d^2 \rangle_{\mathbf{r}}. \end{aligned} \quad (\text{A6})$$

Of the two quantities defined in the last line of Equation (A6), $\langle F^2 \rangle_{\hat{\mathbf{r}}, t}$ accounts for the time-, merger-rate- and direction-averaged instrument response, and it depends on the orbit and geometry of the detector. For an isotropic distribution of sources and a triangular interferometer with 60° angles, $F^2(\hat{\mathbf{r}}, t) = 1/5 \times \sin^2(60^\circ) = 3/20$, which is often taken as the average isotropic response for *LISA*-like detectors (Giampieri & Polnarev 1997). The factor $1/5$ is the average of ($\langle F_+^2 \rangle + \langle F_\times^2 \rangle$)/2 over source locations and polarizations (again, e.g., in Maggiore 2007). Again for a *LISA*-like detector, but for typical disk populations, $F^2(\hat{\mathbf{r}}, t)$ is found empirically to vary by $\sim 10\%$ from $3/20$.

The quantity $\langle \mathcal{R}_{\text{mrg}}/d^2 \rangle_{\mathbf{r}}$ is the merger-rate-averaged inverse squared distance. For a Galactic disk, it depends on the scale height and radial scale length, and of course on the position of the detector within the disk. A typical disk density distribution considered in many papers is the squared-hyperbolic-secant disk model,

$$\mathcal{R}_{\text{mrg}}(\mathbf{r}) = \frac{\mathcal{R}_{\text{gal}}}{V_{\text{gal}}} e^{-R/R_{\text{disk}}} \text{sech}^2(Z/Z_{\text{disk}}), \quad (\text{A7})$$

where \mathcal{R}_{gal} is the total merger rate in the Galaxy and V_{gal} is the volume of the Galaxy. We find empirically that $\langle \mathcal{R}_{\text{mrg}}/d^2 \rangle_{\mathbf{r}}$ varies slowly for $2.5 \text{ kpc} \leq R_{\text{disk}} \leq 10 \text{ kpc}$ and $0.2 \text{ kpc} \leq Z_{\text{disk}} \leq 1 \text{ kpc}$. Table 5 summarizes the wide range of Galactic models used in the literature. For most disk models, the characteristic distance to Galactic sources $\mathcal{D}_{\text{char}} \equiv (V_{\text{gal}}/\mathcal{R}_{\text{gal}})^{-1/2} \langle \mathcal{R}_{\text{mrg}}/d^2 \rangle_{\mathbf{r}}^{-1/2}$ is approximately 5–6 kpc, and

$\mathcal{D}_{\text{char}} + R_{\text{disk}}$ is close to 8.5 kpc, the assumed distance to the Galactic center.

Inserting numerical values appropriate for our simulations (cases 1–3) in Equation (A6) yields

$$\begin{aligned} S_{h_l}(f) &\simeq 1.9 \times 10^{-44} (f/\text{Hz})^{-7/3} \text{Hz}^{-1} \times \langle F^2 \rangle_{\hat{\mathbf{r}}, t} \\ &\times \left(\frac{\mathcal{D}_{\text{char}}}{6.4 \text{ kpc}} \right)^{-2} \left(\frac{\mathcal{R}_{\text{gal}}}{0.015 \text{ yr}^{-1}} \right) \left(\frac{\mathcal{M}_{z, \text{char}}}{0.35 M_\odot} \right)^{5/3}, \end{aligned} \quad (\text{A8})$$

where $\mathcal{M}_{z, \text{char}}$ is the characteristic chirp mass ($\mathcal{M}_z^{5/3}$) $^{-5/3}$. The power spectral density $S_h(f)$ of the GW strain is obtained by dropping the factor $\langle F^2 \rangle_{\hat{\mathbf{r}}, t}$ (which is close to $3/20$ for *LISA*-like detector). Equation (A8) is in good agreement with the results of our simulations. Note however that the spectra of Figure 2 are plotted for the TDI observable X (rather than for strain h or strain response h_l), so they must be compared with

$$S_X(f) = \frac{16 x^2 \sin^2 x}{1 + 0.6 x^2} S_{h_l}(f), \quad (\text{A9})$$

where $x = 2\pi f L/c$, and L is the detector armlength.

Our analysis indicates that $S_{h_l}(f)$ and $S_h(f)$ are relatively insensitive to the structure of the Galactic disk, consistent with the results of our simulations, and instead depend strongly on the merger rate \mathcal{R}_{gal} and the characteristic chirp mass $\mathcal{M}_{z, \text{char}}$. By contrast, we expect that the local population density of sources, as well as the number of resolvable sources as a function of their GW strain, will depend roughly linearly on the disk's scale height.

The transition between the confused and resolved detection regimes is characterized by dN/df , which is determined by the evolution of the chirp frequency and by the absolute merger-rate density of sources. Integrating Equation (A3) over \mathbf{r} , we find

$$\begin{aligned} \frac{dN}{df} &= 5 \times 10^{-3} (f/\text{Hz})^{-11/3} \text{Hz}^{-1} \left(\frac{\mathcal{R}_{\text{gal}}}{0.015 \text{ yr}^{-1}} \right) \\ &\times \left(\frac{\mathcal{M}_{z, \text{char}}}{0.35 M_\odot} \right)^{-5/3}, \end{aligned} \quad (\text{A10})$$

consistent with our simulations.

This simple analytical derivation of the GW continuum shows that the major differences between the various estimates listed in Table 4 arise primarily from two sources: the total merger rate of systems \mathcal{R}_{gal} and the composition of the source population, leading to a different characteristic chirp mass $\mathcal{M}_{z, \text{char}}$. $\mathcal{D}_{\text{char}}$ is also relevant, but less important. (Remember however that our derivation is only predictive for frequencies high enough that most binaries form at longer periods—for *LISA*-like detectors, this is true above 10^{-4} Hz. For lower frequencies, the shape of the continuum depends on the detailed accounting of birth rate versus period.)

APPENDIX B

LOW-FREQUENCY GW INTERFEROMETERS AND THEIR RESPONSE TO BINARY WAVES

LISA (Prince et al. 2009; Jennrich 2009), the space-based GW observatory studied jointly by NASA and ESA until early 2011, comprises three spacecraft orbiting the Sun in a quasi-equilateral configuration with armlengths $\sim 5 \times 10^6$ km. *LISA* measures GW strain by monitoring the oscillating distance between freely falling test masses, using heterodyne laser interferometry. The

spacecraft hover around the test masses to protect them from external perturbations, performing slight orbital corrections with extremely precise micro-Newton thrusters.

Although *LISA* is sometimes described as a large Michelson interferometer in space, the lasers exchanged by spacecraft are neither split nor reflected. Rather, two lasers on each spacecraft are used to establish six one-way links; each link provides a test-mass-referenced interferometric measurement y_{ij} of the phase (or equivalently, the frequency) of the incoming laser, compared to the local, outgoing beam. In terms of fractional laser frequency, the effect of a plane GW on the links is given by the two-pulse response (Estabrook & Wahlquist 1975)

$$y_{ij}(t) = \frac{1}{2} \frac{\hat{n}_{ij} \cdot [\mathbf{h}(p_i, t - L_{ij}) - \mathbf{h}(p_j, t)] \cdot \hat{n}_{ij}}{1 - \hat{n}_{ij} \cdot \hat{k}}, \quad (\text{B1})$$

where \hat{n}_{ij}^i is the unit vector from spacecraft i to j , \hat{k} is the GW propagation vector, $\mathbf{h}(p_k, t)$ is the TT-gauge GW strain (see, e.g., Maggiore 2007) at the spacecraft position p_k and at time t , and L_{ij} is the photon time of flight along the arm. Thus, a GW pulse is registered twice in y_{ij} : once when it impinges on spacecraft j , and a time L_{ij} after it has impinged on spacecraft i . In the limit of long GW wavelengths $\lambda \gg L$, the y_{ij} response to plane GWs is therefore proportional to L .

Each y_{ij} measurement is also affected by *displacement* noise from residual test-mass accelerations, *position* noise from optical path and shot noise in the heterodyne measurements, and, most importantly, by oscillations in the central laser frequency, which are many orders of magnitude stronger than expected GW imprints, and which appear as $y_{ij} = C_{ji}(t) - C_{ij}(t - L_{ij})$, with C_{ij} as the frequency noises for the six lasers. Laser frequency noise is removed by combining multiple one-way measurements with appropriate time delays so that each laser appears in canceling pairs, as in the *unequal-arm Michelson* observable

$$\begin{aligned} X(t) = & [y_{31}(t) + y_{13}(t - L_{31}) + y_{21}(t - L_{31} - L_{13}) \\ & + y_{12}(t - L_{31} - L_{13} - L_{21})] \\ & - [y_{21}(t) + y_{12}(t - L_{21}) + y_{31}(t - L_{21} - L_{12}) \\ & + y_{13}(t - L_{21} - L_{12} - L_{31})]. \end{aligned} \quad (\text{B2})$$

This noise-suppression technique, specific to *LISA*-like interferometers, is known as TDI (see, e.g., Vallisneri 2005 for a review). TDI observables can be seen as *synthesized* interferometers, since their y_{ij} components trace the paths (and reproduce the phase delays) of light across recognizable interferometric configurations (hence X 's ‘‘Michelson’’ designation). Many different observables are possible with *LISA*'s six laser links, but all can be reconstructed from a set of three observables, such as the unequal-arm Michelson X , Y , and Z centered on the three spacecraft (Vallisneri et al. 2008).

In this paper, we assume an rms displacement noise

$$\begin{aligned} S_{\text{pm}}^{1/2}(f) = & 3 \times 10^{-15} \text{ m s}^{-2} \text{ Hz}^{-1/2} \left(1 + \left(\frac{f}{10^{-4} \text{ Hz}} \right)^{-1} \right)^{1/2} \\ & \times \left(1 + \left(\frac{f}{0.008 \text{ Hz}} \right)^4 \right)^{1/2}, \end{aligned} \quad (\text{B3})$$

for each of the proof masses, and a total rms position noise

$$S_{\text{op}}^{1/2}(f) = 18 \times 10^{-12} \text{ m Hz}^{-1/2} \left(1 + \left(\frac{f}{0.002 \text{ Hz}} \right)^{-4} \right)^{1/2} \quad (\text{B4})$$

for each y_{ij} measurement.

The GW mission currently under study as an ESA-led project (Amaro-Seoane et al. 2012) would decrease cost primarily by reducing spacecraft and propellant mass, allowing for launch on smaller rockets than envisaged for *LISA*. Propellant is saved by placing the spacecraft closer together and closer to Earth. At the low-frequency end, the shorter armlengths reduce the response to GWs proportionally; for the same displacement noise, sensitivity also decreases by the same ratio. At the high-frequency end, the laser power available for position measurement increases as L^{-2} , since beams are broadly defocused at millions of kilometers, improving shot noise (but not other optical noises) by an rms factor L^{-1} . As a consequence, the sweet spot of the *LISA* sensitivity shifts to higher frequencies.

Spacecraft mass may also be saved by adopting a two-arm ‘‘mother–daughters’’ configuration with only four laser links, two each between the middle ‘‘mother’’ spacecraft and one of the ‘‘daughters.’’ With four links, only one independent TDI observable can be formed, with a loss in sensitivity averaging $\sqrt{2}$ at compact-binary frequencies. Because no trade studies have yet been performed to optimize the payload for the new orbits, in this paper we consider a ‘‘short *LISA*’’ configuration with aggressively reduced 1×10^6 km armlengths, and with the *LISA* displacement and position noises (except for the power rescaling of the position-noise component, which we assume as five times smaller in rms). The actual sensitivity of a cost-constrained space-based GW observatory is likely to fall somewhere between this short *LISA* and the classic configuration.

Searches for GWs of known shape proceed by correlating detector data with theoretical waveform *templates* computed for the expected range of source parameters. The statistical confidence of detection for a GW signal h immersed in Gaussian additive noise n is described by the S/N of the data $s = h + n$ after filtering by the best-fitting *normalized* template \hat{t} . For a single TDI observable X , this is given by (Cutler & Flanagan 1994)

$$\rho(s; \hat{t}) = 4 \text{Re} \int_0^\infty \frac{\tilde{X}_s(f)^* \tilde{X}_{\hat{t}}(f)}{S_X(f)} df, \quad (\text{B5})$$

where $\tilde{X}_s(f)$ and $\tilde{X}_{\hat{t}}(f)$ are the Fourier transforms of the TDI data and of the TDI response to waveform \hat{t} , respectively; ‘‘*’’ denotes the complex conjugate; and $S_X(f)$ is the noise power spectral density of TDI observable X . Normalization amounts to requiring $\rho(\hat{t}; \hat{t}) = 1$.

With this noise weighting, the false-alarm probability of S/N exceeding a chosen detection threshold ρ_{thr} is proportional to $\exp(-\rho_{\text{thr}}^2/2)$. Because the many templates used in typical searches amount to repeated independent trials, ρ_{thr} is routinely set to relatively large values (e.g., 5–10) to yield one false alarm per many years of data. The detectability of a signal h is characterized by its *optimal* S/N $\rho(h; h)$, which is the S/N that would be obtained on average over noise realizations for a perfectly matching template.

The noise $S_X(f)$ includes the confusion-noise foreground $S_X^{\text{conf}}(f)$ from unresolved sources (see Section 5), as well as instrument noise S_X^{inst} . For the unequal-arm Michelson observable X in the (rather reasonable) limit of equal interferometer armlengths, S_X^{inst} is given by (Vallisneri et al. 2008)

$$S_X^{\text{inst}} = 16 \sin^2(2\pi Lf) [2(1 + \cos^2(2\pi Lf)) S_{\text{pm}}^{\Delta f/f}(f) + S_{\text{op}}^{\Delta f/f}(f)]. \quad (\text{B6})$$

In these units $S_{\text{pm}}^{\Delta f/f} = S_{\text{pm}}(f)/(2\pi c f)$, $S_{\text{op}}^{\Delta f/f}(f) = S_{\text{op}}(f) \times (2\pi f/c)$.

To obtain the TDI response to a GW signal (5), we insert Equation (5), multiplied by the appropriate phasing function and sky-position-dependent polarization tensors, into Equation (B2). The response is modulated further by the heliocentric motion of the detector (which yields a time-dependent Doppler shift) and its yearly rotation in a plane inclined 60° from the ecliptic (which generates sidebands at frequency multiples of 1 yr^{-1}).⁷ Both effects imprint sky-position and polarization angles into the TDI response; this structure makes it possible to resolve sufficiently strong GW sources that overlap in frequency space, and to determine their approximate sky position (see, e.g., Królak et al. 2004). The TDI response to a population of sources is given simply by the linear superposition of the responses to individual sources.

APPENDIX C

FITS OF THE PARTIALLY SUBTRACTED CONFUSION-NOISE FOREGROUND

Barack and Cutler (2004a, 2004b) codified the lore about the *total noise* of a *LISA*-like detector (i.e., the instrument noise plus Galactic-binary confusion noise) by providing a heuristic derivation for the well-known expression

$$S_h^{\text{total,BC}}(f) = \min[S_h^{\text{inst}}(f)e^{\kappa T^{-1}dN/df}, S_h^{\text{inst}}(f) + S_h^{\text{conf,BC}}(f)], \quad (\text{C1})$$

where they estimate

$$S_h^{\text{conf,BC}}(f) = 1.4 \times 10^{-44} (f/\text{Hz})^{-7/3} \text{ Hz}^{-1}, \quad (\text{C2})$$

$$\frac{dN}{df} = 2 \times 10^{-3} (f/\text{Hz})^{-11/3} \text{ Hz}^{-1}, \quad (\text{C3})$$

and $\kappa T^{-1} \simeq 1.5 \text{ yr}^{-1}$. As we discuss in Appendix A, under generic assumptions, the *unsubtracted* Galactic-binary foreground will follow a $f^{-7/3}$ spectrum; this is confirmed in our simulations, with a coefficient of 2.6×10^{-44} for cases 1–3 and 5, and five times smaller for case 4 (see Figure 3). After the subtraction of detectable sources, the total noise found in our simulations is fit better by a smaller amplitude, a slightly different exponent, and a smoother transition function:

$$S_h^{\text{total,sim}}(f) = S_h^{\text{inst}} + S_h^{\text{conf,sim}} \times \tanh^\alpha\left(\frac{1}{2}\beta dN/df\right), \quad (\text{C4})$$

with

$$S_h^{\text{conf,sim}}(f) = 1.4 \times 10^{-45} (f/\text{Hz})^{-8/3} \text{ Hz}^{-1}, \quad (\text{C5})$$

$$\frac{dN}{df} = 5 \times 10^{-3} (f/\text{Hz})^{-11/3} \text{ Hz}^{-1}. \quad (\text{C6})$$

For cases 1–3 and 5, and for our reference subtraction run (one year, one interferometric observable, $\rho_{\text{thr}} = 5$), taking $\alpha = 1$, $\beta = 1 \text{ yr}^{-1}$ yields a good fit for a 5 Mkm *LISA*-like detector, while $\alpha = 0.7$, $\beta = 1.2 \text{ yr}^{-1}$ is appropriate for 1 Mkm arms.

For the entire range of ρ_{eff} that we probed, good fits are obtained by setting

$$\begin{aligned} \alpha &= 2 - 1.2 (\rho_{\text{eff}}/5) + 0.2 (\rho_{\text{eff}}/5)^2, \\ \beta &= [-0.2 + 1.5 (\rho_{\text{eff}}/5) - 0.3 (\rho_{\text{eff}}/5)^2] \text{ yr}^{-1} \quad (5 \text{ Mkm}), \\ \alpha &= 1.6 - 1.2 (\rho_{\text{eff}}/5) + 0.3 (\rho_{\text{eff}}/5)^2, \\ \beta &= [0.3 + 1.3 (\rho_{\text{eff}}/5) - 0.4 (\rho_{\text{eff}}/5)^2] \text{ yr}^{-1} \quad (1 \text{ Mkm}). \end{aligned} \quad (\text{C7})$$

For case 4, an acceptable fit follows from reducing $S_h^{\text{conf,sim}}$ by a factor of five.

The equivalent confusion noise in the TDI X combination (expressed as fractional-frequency fluctuations) is given by

$$S_X^{\text{total,sim}}(f) = \frac{3}{20} \frac{16x^2 \sin^2 x}{1 + 0.6x^2} S_h^{\text{total,sim}}(f), \quad \text{with } x = 2\pi f L/c. \quad (\text{C8})$$

Here, $3/20$ is the all-sky averaging $\langle F^2 \rangle_{\text{f},t}$ discussed above, $1 + 0.6x^2$ is an approximation to the frequency-dependent interferometer response function, and $16x^2 \sin^2 x$ is the TDI X transfer function. (In Barack and Cutler’s convention (2004a), which we follow here, the average response function is included in the definition of the equivalent strain noise S_h^{total} , so that the sky-averaged S/N can be computed directly as $4 \text{Re} \int |\tilde{h}|^2 / S_h^{\text{total}}(f) df$; therefore S_h^{conf} , rather than $S_{h_i}^{\text{conf}}$, appears in S_h^{total} . By contrast, TDI observables already include the position-dependent instrument response, so it is appropriate to define the TDI noise S_X^{total} without the sky averaging.)

Due to the evolving orientation of the *LISA*-like formation with respect to the Galaxy, the Galactic foreground (before and after subtraction) has a strong seasonal variation in amplitude, almost constant across frequencies (Giampieri & Polnarev 1997; Seto 2004; Edlund et al. 2005). For a single TDI X combination, for both our detector configurations, for all scenarios, and for our assumptions about the relative orientation of detector and Galaxy at $t = 0$ (consistent with the standard MLDC *LISA* orbits, Arnaud et al. 2006), this variation is fit well by

$$\begin{aligned} S_h^{\text{conf,sim}}(f, t) &= S_h^{\text{conf,sim}}(f) \times [1 - 0.21 \cos(2\pi t/\text{yr}) \\ &\quad - 0.45 \cos(4\pi t/\text{yr}) - 0.05 \sin(2\pi t/\text{yr}) \\ &\quad + 0.01 \sin(4\pi t/\text{yr})], \end{aligned} \quad (\text{C9})$$

where $S_h^{\text{conf,sim}}(f, t)$ denotes the power spectral density computed for data segments shorter than one year, but longer than the typical measurement timescale of a few hours. An equation analogous to Equation (C9), with the same coefficients, holds also for the unsubtracted background $S_h(f, t)$.

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⁷ Because “short *LISA*” orbits are very similar to those planned for classic *LISA*, this discussion is valid for both. Different orbits, such as halo trajectories around Lagrange points or Earth-centric orbits, would yield different signal modulations.

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