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A quantum radiation pressure noise-free optical spring

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Recent advances in micro- and nanofabrication have spawned great interest in the field of cavity optomechanics, which provides a promising avenue towards quantum-limited metrology and studies of quantum behaviors of macroscopic mechanical objects. One major impediment to reaching the quantum regime is thermal excitation, which can be overcome for sufficiently high mechanical quality factor Q. Here, we propose a method for increasing the effective Q of a mechanical resonator by stiffening it via the optical spring effect exhibited by linear optomechanical systems, and show how the associated quantum radiation pressure noise can be evaded by sensing and feedback control. In a parameter regime that is within current technology, this method allows for realistic quantum cavity optomechanics in a frequency band well below that which has been realized thus far.

Introduction.—Catalyzed by vast improvements in micro- and nanofabrication processes, the field of cavity optomechanics has seen a boom in interest [1–3]. In addition to providing a means for quantum-limited force measurements [4], e.g., in gravitational-wave detection [5] and scanning probe microscopy, optomechanical devices can also be used to probe the quantum behavior of mechanical systems. Recently, several experiments have demonstrated the cooling of a resonator down to its quantum ground state via cryogenics or optomechanical interaction [6–8]. In addition, more than one group [9, 10] has demonstrated the so-called optomechanically induced transparency (OMIT) effect, an analog of the electromagnetically induced transparency (EIT) [11, 12] effect observed in atomic systems. This can be used as a narrow-band quantum filter, e.g., to effect the frequency-dependent phase rotation of squeezed light injected to enhance the sensitivity of quantum noise-limited interferometric gravitational wave detectors [13, 14]. This also opens up the possibility for the processing and storing of nonclassical states of light through coherent transfer of quantum states between light and a mechanical oscillator, a technique that would find much use in the emergent field of quantum information processing.

The ubiquitous bath of thermal energy presents a major obstacle to these efforts, randomly exciting a system and masking its underlying quantum nature. A characteristic figure of merit for quantifying this thermal decoherence effect is given by the ratio of the thermal occupation number $\bar{n}_{\rm th}$ and the mechanical quality factor Q:

$$\frac{\bar{n}_{\rm th}}{Q} = \frac{k_B T}{\hbar \omega_m Q} \propto (Qf)^{-1}, \tag{1}$$

where k_B is the Boltzmann's constant and $f = \omega_m/2\pi$ is the mechanical frequency. When this ratio becomes smaller than one, the oscillator quantum state will survive longer than one oscillation period before the thermal effect destroys it.

As is apparent from Eq. (1), the quantum state lifetime is ultimately limited by the product of the quality factor Q mechanical frequency f. A significant body of research has focused on increasing this product for a wide range of mechanical systems. If Q were truly a frequency-independent quantity—as in the "structural damping" model proposed by Saulson [15]—then moving to higher eigenfrequencies would lead to an immediate improvement. In the opposite direction, there are many experiments that would benefit from the use of low-frequency (sub-kHz) resonators. A number of large bulk systems have been found to exhibit extremely high Q in this frequency range [16, 17]; unfortunately, these bulk systems tend to have relatively large (gram- to kg-scale) effective masses, making them unsuitable for typical optomechanics experiments. The realization of sub-microgram effective masses requires the use of microfabricated resonators. In practice, excess damping from surface effects [18], phonon tunneling loss [19] or intrinsic mechanisms such as thermoelastic [20] and Akhiezer damping [21] limits the achievable Q and thus the Qf product for low-frequency resonators. In addition, we add the further requirement that the desired system exhibit excellent optical quality (i.e., high reflectivity owing to low scatter loss and absorption), which limits the resonator options considerably, especially in light of the fact that typical dielectric materials used to create multi-layer optical coatings (e.g., SiO_2/Ta_2O_5) exhibit low mechanical quality factors [22]. Here, we propose a method for using the optical-spring effect in linear optomechanical devices [23–27] to increase the effective Q of a given mechanical resonator, while simultaneously eliminating the excess fluctuation due to quantum radiation pressure noise imparted by the optical fields. This technique should facilitate the creation of an oscillator with a Qf product considerably higher than those available today, enabling useful applications in quantum metrology and also creation of long-lived quantum states at lower frequencies than were practical before.

Our idea uses the fact when a strong optical spring is linearly coupled to a mechanical resonator, the resonator's hamiltonian becomes augmented or even dominated by contributions from the radiation pressure forces of the optical fields. In this way, the bare resonator's thermal noise is "diluted" by the ratio of the intrinsic elastic energy to that stored in the optical field [27]. Typically, the modification of a resonator's dynamics via linear coupling is accompanied by excess noise from quantum back-action—the quantum fluctuation of the radiation pressure, in our case. This has been identified as a serious issue in the strong dilution regime by Chang *et al.* [28] and Ni *et al.* [29], who instead propose to achieve optical dilution by using a nonlinear quadratic optical potential to trap a partially reflective membrane [30], which would be immune to linear quantum back-action. The device we propose evades such parasitic quantum back-action—i.e., the quantum radiation pressure noise—by performing homodyne detection on a proper outgoing quadrature and actively feeding back to the oscillator, resulting in a nearly noise-free optical spring. Since this method allows for easy coupling of the diluted mechanical resonator to an external optical system from the other side of the resonator, it can be used as a black-box effective mechanical resonator of exceptionally high mechanical quality.

Optical spring.—The canonical optomechanical system is shown in the dashed box in Fig. 1. In such a system, the "optical spring" effect arises from dynamical back-action of the optical cavity field on the mechanical oscillator forming one cavity boundary. The mechanical oscillator displacement $\hat{x}$ is coupled to the cavity field $\hat{a}$ via radiation pressure, as described by the following interaction Hamiltonian [31]:

$$\mathscr{H}_{\rm int} = \hbar\bar{G}_0\hat{x}(\hat{a}^\dagger+\hat{a}) \equiv \hat{x}\hat{F}_{\rm rad}. \tag{2}$$

The coupling constant is $\bar{G}_0 = \bar{a}\omega_c/L$; $\bar{a}$ is the classical mean amplitude of $\hat{a}$ due to coherent driving of an external laser; ω_0 is the cavity resonant frequency; L is the cavity length. When the frequency of the external laser ω_l that drives the cavity field is detuned from ω_c, $\hat{F}_{\rm rad}$ depends on the oscillator displacement, creating a mechanical response that mimics a spring. More specifically, $\hat{F}_{\rm rad}$ in the frequency domain can be written as (the details of the derivation are in the Appendix):

$$\hat{F}_{\rm rad}(\omega) = -K_{\rm os}(\omega)\hat{x}(\omega) + \hat{F}_{\rm noise}(\omega), \tag{3}$$

where the optical spring coefficient $K_{\rm os}$ is approximately given by

$$K_{\rm os}(\omega) \approx -\frac{2\hbar\bar{G}_0^2\Delta}{\Delta^2+\gamma^2} - \frac{4i\hbar\bar{G}_0^2\gamma\Delta\omega}{(\Delta^2+\gamma^2)^2} \equiv m\omega_{\rm os}^2 - im\Gamma_{\rm os}\omega \tag{4}$$

with the cavity detuning $\Delta \equiv \omega_0 - \omega_l$ and γ being the cavity bandwidth. Here, the approximation is taken for the case of large detuning and cavity bandwidth, which we will show to be the relevant parameter regime for realization of this idea. In addition, we have introduced the optical spring frequency $\omega_{\rm os}$ and the optical damping $\Gamma_{\rm os}$. As we can see, when the detuning is negative, i.e., $\Delta < 0$, the optical rigidity is real and positive, and the optical damping is negative $\Gamma_{\rm os}$ (heating), and vice versa. By introducing an additional driving field with a different detuning frequency, we can create the so-called stable double optical spring [26], which can make both the rigidity and the damping positive (we will elaborate on this issue later). The optical spring modifies the mechanical response $R_0(\omega)$, defined through $R_0(\omega) \equiv \hat{x}(\omega)/\hat{F}(\omega)$,

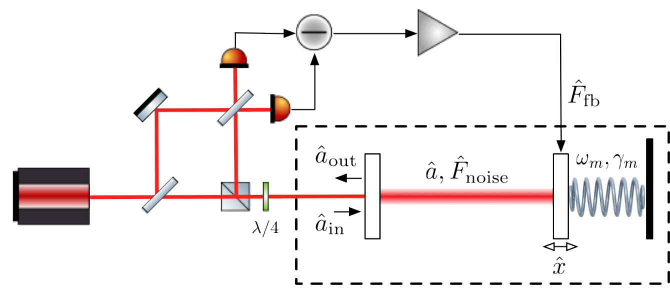

FIG. 1: Simplified experimental layout, with the canonical optomechanical system shown within the dashed box. Input vacuum fluctuations drive the cavity mode, which in turn exerts radiation pressure forces on the mechanical resonator forming the cavity boundary. The output mode of the cavity is sensed via homodyne detection, and, by measuring the appropriate quadrature that is mentioned in the text, an error signal may be derived for feeding back to the resonator position to cancel the radiation pressure noise.

from its original value

$$R_0^{-1}(\omega) = -m(\omega^2 + i\gamma_m\omega - \omega_m^2) \tag{5}$$

to an effective one:

$$R_{\rm eff}^{-1}(\omega) = -m[\omega^2 + i(\gamma_m+\Gamma_{\rm os})\omega - (\omega_m^2+\omega_{\rm os}^2)]. \tag{6}$$

For a strong optical spring $\omega_{\rm os} \gg \omega_m$, we can significantly stiffen the mechanical oscillator with the restoring energy from the optical field.

One immediate issue with this approach comes from the quantum radiation pressure noise $\hat{F}_{\rm noise}(\omega)$ in Eq. (3) which arises from quantum fluctuation of the optical field:

$$\hat{F}_{\rm noise}(\omega) = \frac{2\hbar\bar{G}_0\sqrt{\gamma}[(\gamma-i\omega)\hat{v}_1(\omega)+\Delta\hat{v}_2(\omega)]}{(\omega+\Delta+i\gamma)(\omega-\Delta+i\gamma)}, \tag{7}$$

where $\hat{v}_1 \equiv (\hat{a}_{\rm in}+\hat{a}_{\rm in}^\dagger)/\sqrt{2}$ and $\hat{v}_2 \equiv (\hat{a}_{\rm in}-\hat{a}_{\rm in}^\dagger)/\sqrt{2}i$ are the amplitude and phase quadratures of the input optical field. This additional noise term will increase the effective temperature of the thermal bath, and drive the mechanical oscillator away from the quantum regime, as pointed out by Chang *et al.* [28]. Its strength can be quantified by the spectral density

$$S_F(\omega) = \frac{4\hbar^2\bar{G}_0^2\gamma(\gamma^2+\omega^2+\Delta^2)}{[(\omega-\Delta)^2+\gamma^2][(\omega+\Delta)^2+\gamma^2]} \approx \frac{4\hbar^2\bar{G}_0^2\gamma}{\gamma^2+\Delta^2}, \tag{8}$$

where we have again assumed large bandwidth and detuning. As we can see from the above expression and Eq. (4), the optical rigidity (real part of $K_{\rm os}$) scales with the optomechanical coupling strength in the same way as the quantum radiation pressure noise:

$$K_{\rm os}, S_F \propto \bar{G}_0^2. \tag{9}$$

Essentially, this means that an increase in the optical spring frequency is accompanied by an increase in the radiation pressure noise when we scale up the optical power of the driving laser.

Evading quantum radiation pressure noise.—To solve the aforementioned issue, we make use of the fact that the output field emerging from the cavity contains information about the quantum radiation pressure noise that has been imposed onto the mechanical oscillator. By choosing an appropriate quadrature of the output field, we can measure most of the radiation pressure noise without being sensitive to the oscillator displacement. By feeding this measurement outcome back to the mechanical oscillator with the proper filter, we can evade the quantum radiation pressure noise and achieve a nearly noise-free optical spring. Note that this does not violate the fundamental principle of quantum measurement—any linear continuous measurement of any dynamical variable that does not commute at different times (non-conservative) is associated with quantum back-action on that variable [4]. Here we only sense the quantum radiation pressure noise and have essentially no sensitivity to the mechanical displacement, and that is why we can evade such back-action noise.

We elaborate on this idea using the input-output relation for this system

$$\hat{a}_{\rm out}(\omega) = -\hat{a}_{\rm in}(\omega) + \sqrt{2\gamma}\hat{a}(\omega). \tag{10}$$

In terms of amplitude and phase quadratures: $\hat{y}_1 \equiv (\hat{a}_{\rm out} + \hat{a}^\dagger_{\rm out})/\sqrt{2}$ and $\hat{y}_2 \equiv (\hat{a}_{\rm out} - \hat{a}^\dagger_{\rm out})/\sqrt{2}i$, we have (again for large detuning and bandwidth):

$$\hat{y}_1(\omega) \approx \frac{(\gamma^2-\Delta^2)\hat{v}_1(\omega)+2\gamma\Delta\hat{v}_2(\omega)}{\Delta^2+\gamma^2} - \frac{2\sqrt{\gamma}\bar{G}_0\Delta}{\Delta^2+\gamma^2}\hat{x}(\omega), \tag{11}$$

$$\hat{y}_2(\omega) \approx \frac{(\gamma^2-\Delta^2)\hat{v}_2(\omega)-2\gamma\Delta\hat{v}_1(\omega)}{\Delta^2+\gamma^2} - \frac{2\sqrt{\gamma}\bar{G}_0\gamma}{\Delta^2+\gamma^2}\hat{x}(\omega). \tag{12}$$

By choosing the appropriate local oscillator phase, we can perform homodyne detection on the following quadrature, which is a linear combination of the above output amplitude and phase quadratures ($\hat{y}_\zeta \equiv \hat{y}_1\sin\zeta + \hat{y}_2\cos\zeta$ with $\tan\zeta = -\gamma/\Delta$):

$$\hat{y}_\zeta(\omega) = \frac{\gamma\hat{v}_1(\omega)+\Delta\hat{v}_2(\omega)}{(\Delta^2+\gamma^2)^{1/2}} \equiv \hat{v}_{\zeta'}(\omega). \tag{13}$$

This quadrature has nearly no response to the oscillator displacement $\hat{x}$, while it contains a particular quadrature $\hat{v}_{\zeta'}(\omega)$ of the quantum fluctuation:

$$\hat{v}_{\zeta'}(\omega) \equiv \hat{v}_1(\omega)\sin\zeta' + \hat{v}_2(\omega)\cos\zeta', \tag{14}$$

with $\tan\zeta' = \gamma/\Delta$. This turns out to be the same quadrature responsible for the quantum radiation pressure noise, which can be seen from the expression for $\hat{F}_{\rm noise}$ [cf. Eq. (7)] and in the large detuning and bandwidth case:

$$\hat{F}_{\rm noise}(\omega) \approx \frac{2\hbar\bar{G}_0\sqrt{\gamma}[\gamma\hat{v}_1(\omega)+\Delta\hat{v}_2(\omega)]}{(\Delta^2+\gamma^2)} \propto \hat{v}_{\zeta'}(\omega). \tag{15}$$

Therefore, by feeding the measurement result of $\hat{y}_\zeta$ back to the mechanical oscillator with the correct linear filter, we can evade the quantum radiation pressure noise, as shown schematically in Fig. 1.

Residual radiation pressure noise.—While strong radiation pressure noise cancellation can be achieved using this technique, a small fraction cannot be canceled owing to two effects: (i) optical loss within the cavity or from non-unity quantum efficiency in photodetection, which will introduce vacuum noise that is uncorrelated with $\hat{v}_1$ and $\hat{v}_2$; (ii) frequency dependence of the radiation pressure noise, which we have thus far ignored by assuming a large detuning and cavity bandwidth. In real experiments, there is always certain amount of optical loss, and the bandwidth and detuning are all finite. We can make an estimate of the magnitude of the residual noise; supposing we cancel the leading-order term [c.f. Eq. (15)] from the full radiation pressure force [c.f. Eq. (7)], there remain terms of $O(\omega)$ and higher:

$$\hat{F}^{\rm res}_{\rm noise}(\omega) = \frac{2i\hbar\bar{G}_0\sqrt{\gamma}[(\gamma^2-\Delta^2)\hat{v}_1(\omega)+2\gamma\Delta\hat{v}_2(\omega)]}{(\gamma^2+\Delta^2)^2}\omega + O(\omega^2). \tag{16}$$

Accounting for this, and for the imperfect cancellation of the first term due to the finite loss ε, the power spectrum of the residual radiation pressure noise is

$$S^{\rm res}_F(\omega) = \frac{4\hbar^2\bar{G}_0^2\gamma}{\gamma^2+\Delta^2}\left(\varepsilon + \frac{\omega^2}{\gamma^2+\Delta^2} + O(\omega^4)\right). \tag{17}$$

Here, ε is the sum of the intracavity roundtrip loss and the overall loss in photodetection due to imperfect modematching and non-unity quantum efficiency of the photodiode (given the quality of currently available optical elements, it will likely be limited by quantum efficiency at a level of $\varepsilon \approx 0.1-1\%$). Since the frequency we are interested in is around the shifted mechanical resonant frequency, $\omega_{\rm os}$, the effect of the finite bandwidth can be negligibly small.

By comparison with the thermal force spectrum from a viscous damping model, $S^{\rm th}_F = 4m\gamma_m k_B T$, we can assign an effective temperature to such residual noise, as

$$T^{\rm res}_{\rm eff} \equiv \frac{S^{\rm res}_F}{4m\gamma_m k_B}. \tag{18}$$

This effective temperature must be below the required bath temperature for a given application.

Experimental realization with double optical spring.—As mentioned earlier, a single optical spring is unstable by nature [cf. Eq. (4)]—in the case of a blue-detuned field, the system exhibits a positive restoring force but a negative damping force, whereas, for a red-detuned field, the forces are damping but anti-restoring. Thus, a system whose dynamics are dominated by an optical spring will be inherently unstable.

A novel approach proposed in Ref. [26] uses a second optical spring field to create a passively stable system. The linear combination of two $K_{\rm os}$s, with one red-detuned and the other blue, can be made to exhibit both positive restoring and damping, resulting in a passively stable spring. The sum of the two optical spring contributions is thus:

$$K_{\rm os}^{\rm tot} \approx im\omega \left[\frac{\Delta_B \omega_{\rm os_B}^2}{(\gamma_B^2 + \Delta_B^2)} + \frac{\Delta_R \omega_{\rm os_R}^2}{(\gamma_R^2 + \Delta_R^2)} \right] + m\omega_{\rm os_B}^2 - m\omega_{\rm os_R}^2 \tag{19}$$

where γ_B, γ_R and Δ_B, Δ_R are the cavity bandwidth and detuning as seen by the blue and red fields, respectively (note that $\Delta_B < 0$). For a proper choice of these parameters as a function of the ratio $\omega_{\rm os_B}/\omega_{\rm os_A} > 1$, the expression in the brackets can be made to vanish, and the effective resonator is stiffened without instability or excess damping[1]

A set of sample parameters is given in Table I. Under these conditions, an oscillator with a resonant frequency of $\omega_{\rm os}/2\pi \approx 100$ kHz and an effective Q of 10^9 is formed[2]. Such a device can in principle be cooled to its ground state from an environmental temperature of $T \approx 4800$ K (Clearly, this should not be attempted, but it serves to illustrate what this technique implies in the context of quantum experiments)! From Eq. (18), we can also calculate the effective temperatures of the residual quantum radiation pressure noise from the two optical spring fields as $T_{\rm eff}^{\rm res,B} \approx 0.5$ K and $T_{\rm eff}^{\rm res,R} \approx 12$ K, in the lossless case, or $T_{\rm eff}^{\rm res,B} \approx 10$ K and $T_{\rm eff}^{\rm res,R} \approx 20$ K, for $\varepsilon = 0.001$. Even in the lossy case, the residual noise temperatures are considerably lower than a reasonable target environment temperature.

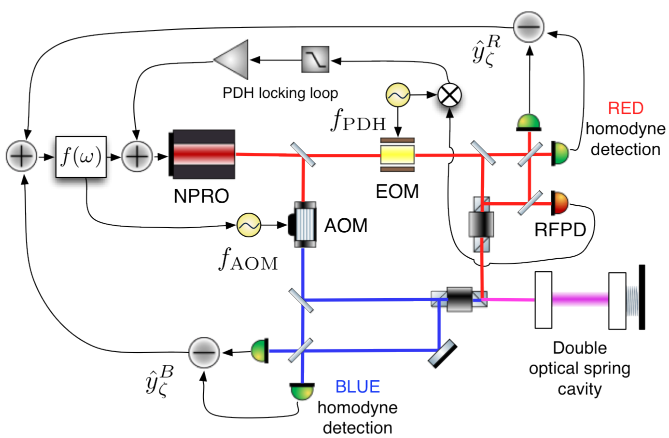

FIG. 2: The proposed experimental setup. A laser is Pound-Drever-Hall (PDH) locked to the optical spring cavity via feedback to the laser frequency, and then an offset $-|\Delta_R|/2\pi$ is injected to detune it to the red side of resonance. A pickoff of this beam is upshifted with an acousto-optic modulator (AOM) by $f_{AOM} = (|\Delta_R| + |\Delta_B|)/2\pi$, and then fed into the cavity with an orthogonal polarization, forming the blue-detuned field. Homodyne detection is performed on each reflected field at the appropriate angle, and a filtered combination of these two signals is applied to each the laser frequency and the AOM frequency, providing active cancellation of radiation pressure noise.

Experimental setup.—A possible experimental layout is shown in Fig. 2. A laser's frequency is stabilized to the optical spring cavity length using the Pound-Drever-Hall (PDH) locking technique. An offset is injected into the loop such that the optical frequency is red-detuned from the cavity resonance by $|\Delta_R|$. A pickoff beam is passed through an acousto-optic modulator (AOM), which upshifts its frequency by $\Delta_R + \Delta_B$, such that it is blue-detuned from the cavity resonance by $|\Delta_B|$. The two beams are resonant in orthogonal polarizations for simple isolation.

Further pickoffs are taken from each path to be used as optical local oscillators for homodyne detection of their respective reflected beams. The photodetector signals are differenced at the appropriate homodyne angle, and these signals are summed and filtered for feedback to the laser and AOM frequencies, which is done in such a way as to cancel the measured radiation pressure noise. In practice, the transfer function from laser frequency to force on the resonator can be quite complicated. In this case, a more direct amplitude modulation technique might be employed (e.g., by placing AOMs in AM configuration on the beams entering the cavity).

Conclusion.—We have proposed a method for creating an effective mechanical resonator with far higher Qf product than any available today. In addition, these resonators can be made to operate in lower frequency bands than current ones of competitive quality. While the use of optical dilution to mitigate thermal noise has been proposed and demonstrated in the past, we have considered a parameter regime in which the deleterious effects of quantum radiation pressure noise from the strong optical spring fields can be all but eliminated, allowing in principle for dilution to arbitrarily high quality. We feel that the application of this technique holds great promise for any field requiring very-high-Q resonators, including, but not limited to, those of quantum optomechanics and sensitive force measurement.

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[1] Note that the expression need not vanish, but only be positive for the resultant resonator to be stable. Furthermore, any positive damping from the optical fields is cold, and therefore does not contribute noise or degrade SNR. We specifically consider the case of zero additional damping, however, since it leads to an effective resonator whose Q is determined solely by the intrinsic damping of the bare mechanical system.

[2] This $Q_{\rm eff}$ value is calculated assuming a viscous damping model; the mechanical damping, γ_m, is fixed, and so, since the optical spring adds no damping, the improvement is given by $Q_{\rm eff} = (\omega_{\rm os}/\omega_m)Q$. Several candidate mechanical resonators are predicted to be better approximated by a structural damping model, in which case the improvement in Q is potentially much greater.

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TABLE I: A sample set of parameters. These values generate an optical spring with $\omega_{os}/2\pi \approx 100$ kHz and $Q_{eff} \approx 10^9$. The laser powers P_B and P_R refer to the circulating powers, and Q refers to the quality of the bare mechanical system. For the specified geometry, the required finesses are of order $\mathscr{F} \approx 10,000$, compatible with the optical quality of resonators in production today.

parameter	m	L	$\omega_m/2\pi$	Q	$\gamma_B/2\pi$	$\Delta_B/2\pi$	P_B	$\gamma_R/2\pi$	$\Delta_R/2\pi$	P_R
value	250 ng	1 mm	100 Hz	10^6	20 MHz	-20 MHz	390 mW	4 MHz	4 MHz	16 mW

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Appendix

In this appendix, we will show some additional details for the derivation of the formulas presented in the main text, which have been mostly covered in the literature [31–35]. Here, we use a notation nearly identical to those in Ref. [34]. We start with the standard Hamiltonian for the canonical optomechanical device, shown in the dashed box in Fig. 1:

$$\begin{aligned}\mathscr{H} =& \frac{\hat{p}^2}{2m}+\frac{1}{2}m\omega_m^2\hat{x}^2+\hbar\omega_c\hat{a}^\dagger\hat{a}+\hbar G_0\hat{x}\hat{a}^\dagger\hat{a}\\ &+i\hbar\sqrt{2\gamma}[\hat{a}_{\rm in}(t)e^{-i\omega_0 t}\hat{a}^\dagger-\hat{a}_{\rm in}^\dagger(t)e^{i\omega_0 t}\hat{a}]\,. \end{aligned} \tag{20}$$

Here, the first two terms are the free Hamiltonian for the oscillator with ω_m being the mechanical frequency; the third term is the free Hamiltonian for the cavity mode, ω_c is the cavity resonant frequency, and $\hat{a}$ is its annihilation operator satisfying $[\hat{a},\hat{a}^\dagger]=1$; the fourth term describes the interaction between the oscillator and the cavity mode, with $G_0=\omega_c/L$ being the coupling strength and L the cavity length; the remaining part is the coupling between the cavity mode with the external continuum $\hat{a}_{\rm in}(t)$, with coupling rate γ and $[\hat{a}_{\rm in}(t),\hat{a}_{\rm in}^\dagger(t')]=\delta(t-t')$. We have ignored those terms accounting for the dissipation mechanism of the mechanical oscillator coupling to its thermal environment. Later we will include their effects in the equation of motion for the oscillator.

Linearized Hamiltonian.—In the experiment, the cavity mode is driven coherently by a laser with a large amplitude at frequency ω_0. We can therefore study the linearized dynamics perturbing around the steady state. In the rotating frame of the laser frequency ω_0, the corresponding linearized Hamiltonian for the system reads:

$$\begin{aligned}\hat{\mathscr{H}} =& \frac{\hat{p}^2}{2m}+\frac{1}{2}m\omega_m^2\hat{x}^2+\hbar\Delta\hat{a}^\dagger\hat{a}+\hbar\bar{G}_0\hat{x}(\hat{a}^\dagger+\hat{a})\\ &+i\hbar\sqrt{2\gamma}[\hat{a}_{\rm in}(t)\hat{a}^\dagger-\hat{a}_{\rm in}^\dagger(t)\hat{a}]\,, \end{aligned} \tag{21}$$

where the cavity detuning is the difference between the cavity resonant frequency and the laser frequency, i.e., $\Delta\equiv\omega_c-\omega_0$, the linear coupling strength is $\bar{G}_0=G_0\bar{a}$ with $\bar{a}$ being the steady-state amplitude of the cavity mode. These operators should be viewed as perturbed parts of the original ones and the quantum state they act on is also transformed correspondingly. For instance, the input state for $\hat{a}_{\rm in}$ is originally a coherent state (for an ideal laser), and now it is the vacuum state $|0\rangle$ with $\langle 0|\hat{a}_{\rm in}(t)\hat{a}_{\rm in}^\dagger(t')|0\rangle=\delta(t-t')$.

Equations of motion.—Given the above Hamiltonian, the cavity mode satisfies the following Heisenberg equation of motion:

$$\dot{\hat{a}}(t)+(\gamma+i\Delta)\hat{a}(t)=-i\bar{G}_0\hat{x}(t)+\sqrt{2\gamma}\hat{a}_{\rm in}(t)\,. \tag{22}$$

and it is related to the cavity output $\hat{a}_{\rm out}$ by the standard input-output relation:

$$\hat{a}_{\rm out}(t)=-\hat{a}_{\rm in}(t)+\sqrt{2\gamma}\hat{a}(t). \tag{23}$$

Similarly, we can read off the equation of motion for the oscillator:

$$m[\ddot{\hat{x}}(t)+\gamma_m\dot{\hat{x}}(t)+\omega_m^2\hat{x}(t)]=\hat{F}_{\rm rad}(t)+\hat{F}_{\rm th}(t)\,. \tag{24}$$

Here we have defined the radiation pressure

$$\hat{F}_{\rm rad}(t)\equiv-\hbar\bar{G}_0[\hat{a}(t)+\hat{a}^\dagger(t)]\,. \tag{25}$$

In addition we have added the damping term $m\gamma_m\dot{\hat{x}}(t)$ and the associated thermal fluctuation force $\hat{F}_{\rm th}$ into the equation of motion, of which the correlation function is $\langle\hat{F}_{\rm th}(t)\hat{F}_{\rm th}(t')\rangle=4m\gamma_m k_BT\,\delta(t-t')$ in the high-temperature limit $k_BT\gg\hbar\omega_m$.

Solution to the cavity mode.—The above linear equations of motion can be solved in the frequency domain. The solution for the cavity mode reads:

$$\hat{a}(\omega)=\frac{\bar{G}_0\hat{x}(\omega)+i\sqrt{2\gamma}\hat{a}_{\rm in}(\omega)}{\omega-\Delta+i\gamma}\,. \tag{26}$$

From this, we can obtain the expression for the radiation pressure:

$$\hat{F}_{\rm rad}(\omega)=-K_{\rm os}(\omega)\hat{x}(\omega)+\hat{F}_{\rm noise}(\omega)\,. \tag{27}$$

We introduce the optical spring coefficient $K_{\rm os}$ as:

$$K_{\rm os}(\omega)\equiv\frac{2\hbar\bar{G}_0^2\Delta}{(\omega-\Delta+i\gamma)(\omega+\Delta+i\gamma)}\,, \tag{28}$$

and the quantum radiation pressure noise term as:

$$\hat{F}_{\rm noise}(\omega)\equiv\frac{2\hbar\bar{G}_0\sqrt{\gamma}[(\gamma-i\omega)\hat{v}_1(\omega)+\Delta\hat{v}_2(\omega)]}{(\omega-\Delta+i\gamma)(\omega+\Delta+i\gamma)} \tag{29}$$

with $\hat{v}_1\equiv(\hat{a}_{\rm in}+\hat{a}_{\rm in}^\dagger)/\sqrt{2}$ and $\hat{v}_2\equiv(\hat{a}_{\rm in}-\hat{a}_{\rm in}^\dagger)/\sqrt{2}i$ being the vacuum fluctuation of the input amplitude and phase quadratures. The strength of the radiation-pressure noise can be quantified by its power spectrum which is defined through

$$\langle 0|\hat{F}_{\rm noise}^\dagger(\omega)\hat{F}_{\rm noise}(\omega')|0\rangle_{\rm sym}\equiv\pi S_F(\omega)\delta(\omega-\omega'), \tag{30}$$

where the subscript 'sym' denotes for symmetrization and the spectrum is a single-sided one. Notice that for vacuum input state $\langle 0|\hat{v}_k^\dagger(\omega)\hat{v}_l(\omega')|0\rangle_{\rm sym}=\pi\,\delta_{kl}\,\delta(\omega-\omega')$, and therefore

$$S_F(\omega)=\frac{4\hbar^2\bar{G}_0^2\gamma(\gamma^2+\omega^2+\Delta^2)}{[(\omega-\Delta)^2+\gamma^2][(\omega+\Delta)^2+\gamma^2]}\,. \tag{31}$$

For the case of large bandwidth and detuning that we are interested in, the above radiation pressure noise can be approximated as (up to zeroth order of ω):

$$\hat{F}_{\rm noise}(\omega)\approx\frac{2\hbar\bar{G}_0\sqrt{\gamma}[\gamma\hat{v}_1(\omega)+\Delta\hat{v}_2(\omega)]}{\Delta^2+\gamma^2}. \tag{32}$$

This indicates that only a particular linear combination — $\gamma\hat{v}_1+\Delta\hat{v}_2$ — of the amplitude and phase quadrature fluctuation is responsible for the quantum radiation-pressure noise. It turns out that we such a combination is measurable in the cavity output, and that is why we can evade the quantum radiation pressure noise by feeding back the measurement result with an appropriate linear filter, which is the main idea of this work.

Solution to the mechanical oscillator.—Given the expression for the radiation pressure, we can write down the solution for the mechanical displacement $\hat{x}$ as:

$$\hat{x}(\omega) = \frac{\hat{F}_{\rm noise}(\omega) + \hat{F}_{\rm th}(\omega)}{-m[\omega^2 - \omega_m^2 + i\gamma_m\omega] + K_{\rm os}(\omega)}\,. \tag{33}$$

As we can see, the mechanical response is modified into an effective one due to the optical-spring effect. Since we are focusing in large cavity bandwidth and detuning case, the optical spring $K_{\rm os}$ can be expanded as:

$$K_{\rm os} \approx -\frac{2\hbar\bar{G}_0^2\Delta}{\Delta^2+\gamma^2} - \frac{4i\hbar\bar{G}_0^2\gamma\Delta\omega}{(\Delta^2+\gamma^2)^2} \equiv m\omega_{\rm os}^2 - i m\Gamma_{\rm os}\omega\,, \tag{34}$$

where $\omega_{\rm os}$ is the optical-spring frequency and $\Gamma_{\rm os}$ is the optical damping coefficient. We can then rewrite the mechanical displacement $\hat{x}$ as

$$\hat{x}(\omega) = R_{\rm eff}^{-1}(\omega)[\hat{F}_{\rm noise}(\omega) + \hat{F}_{\rm th}(\omega)]\,, \tag{35}$$

where the effective mechanical response function $R_{\rm eff}$ is defined through:

$$R_{\rm eff}^{-1}(\omega) \equiv -m[\omega^2 + i(\gamma_m + \Gamma_{\rm os})\omega - (\omega_m^2 + \omega_{\rm os}^2)]\,. \tag{36}$$

In the negative detuning case $\Delta < 0$, $\omega_{\rm os}$ is positive and real, and the damping $\Gamma_{\rm os}$ is negative; while in the positive detuning case $\Delta > 0$, $\omega_{\rm os}$ is pure imaginary and the damping $\Gamma_{\rm os}$ is positive. In both cases, the mechanical system is potentially unstable, especially when the intrinsic damping γ_m is small as in our proposed parameter regime. By introducing an additional laser that has a different detuning frequency, we can combine two optical spring and achieve both positive frequency and damping — the so-called double optical spring. Such a scheme has been realized experimentally by Corbitt *et al.* [26]. We can therefore significantly upshift the mechanical resonant frequency while keeping the oscillator stable.

Solution to the cavity output.—From the input-output relation, the cavity output is given by:

$$\hat{a}_{\rm out}(\omega) = -\frac{\omega-\Delta-i\gamma}{\omega-\Delta+i\gamma}\hat{a}_{\rm in}(\omega) + \frac{\sqrt{2\gamma}\bar{G}_0}{\omega-\Delta+i\gamma}\hat{x}(\omega). \tag{37}$$

It is convenient to reexpress it in terms of amplitude and phase quadratures, of which the linear combination is measured by using a homodyne detection scheme. For the output amplitude quadrature, we have

$$\begin{aligned}\hat{y}_1(\omega) \equiv& \frac{\hat{a}_{\rm out}(\omega) + \hat{a}_{\rm out}^\dagger(-\omega)}{\sqrt{2}}\\ =& -\frac{(\omega^2-\Delta^2+\gamma^2)\hat{v}_1(\omega) + 2\gamma\Delta\hat{v}_2(\omega)}{(\omega-\Delta+i\gamma)(\omega+\Delta+i\gamma)}\\ &+ \frac{2\sqrt{\gamma}\bar{G}_0\Delta\hat{x}(\omega)}{(\omega-\Delta+i\gamma)(\omega+\Delta+i\gamma)}\,.\end{aligned} \tag{38}$$

For the phase quadrature, we have

$$\begin{aligned}\hat{y}_2(\omega) \equiv& \frac{\hat{a}_{\rm out}(\omega) - \hat{a}_{\rm out}^\dagger(-\omega)}{\sqrt{2}i}\\ =& -\frac{(\omega^2-\Delta^2+\gamma^2)\hat{v}_2(\omega) - 2\gamma\Delta\hat{v}_1(\omega)}{(\omega-\Delta+i\gamma)(\omega+\Delta+i\gamma)}\\ &+ \frac{2\sqrt{\gamma}\bar{G}_0(\gamma-i\omega)\hat{x}(\omega)}{(\omega-\Delta+i\gamma)(\omega+\Delta+i\gamma)}\,.\end{aligned} \tag{39}$$

Given the large bandwidth and detuning, we can approximate them as:

$$\hat{y}_1(\omega) \approx \frac{(\gamma^2-\Delta^2)\hat{v}_1(\omega) + 2\gamma\Delta\hat{v}_2(\omega)}{\Delta^2+\gamma^2} - \frac{2\sqrt{\gamma}\bar{G}_0\Delta}{\Delta^2+\gamma^2}\hat{x}(\omega), \tag{40}$$

$$\hat{y}_2(\omega) \approx \frac{(\gamma^2-\Delta^2)\hat{v}_2(\omega) - 2\gamma\Delta\hat{v}_1(\omega)}{\Delta^2+\gamma^2} - \frac{2\sqrt{\gamma}\bar{G}_0\gamma}{\Delta^2+\gamma^2}\hat{x}(\omega). \tag{41}$$

The linear combination of the above quadratures that contains the fluctuation responsible for quantum radiation pressure noise [cf. Eq. (29)] is:

$$\hat{y}_\zeta(\omega) \equiv \sin\zeta\,\hat{y}_1(\omega) + \cos\zeta\,\hat{y}_2(\omega), \tag{42}$$

with $\tan\zeta = -\gamma/\Delta$, which gives:

$$\hat{y}_\zeta(\omega) = \frac{\gamma\hat{v}_1(\omega) + \Delta\hat{v}_2(\omega)}{(\Delta^2+\gamma^2)^{1/2}}\,. \tag{43}$$

By measuring this quadrature, we will be able to evade the quantum radiation pressure noise.