Cosmological constraints on the very low frequency gravitational-wave background

Naoki Seto[1,2] and Asantha Cooray[2]

[1]*Theoretical Astrophysics, California Institute of Technology, MC 130-33, Pasadena, California 91125, USA*

[2]*Department of Physics and Astronomy, University of California, 4186 Frederick Reines Hall, Irvine, California 92697, USA*

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The curl modes of cosmic microwave background polarization allow one to indirectly constrain the primordial background of gravitational waves with frequencies around 10^{-18} to 10^{-16} Hz. The proposed high precision timing observations of a large sample of millisecond pulsars with the pulsar timing array or with the square kilometer array can either detect or constrain the stochastic gravitational-wave background at frequencies greater than roughly 0.1 yr^{-1}. While existing techniques are limited to either observe or constrain the gravitational-wave background across six or more orders of magnitude between 10^{-16} and 10^{-10} Hz, we suggest that the anisotropy pattern of time variation of the redshift related to a sample of high-redshift objects can be used to study the background around a frequency of 10^{-12} Hz. Useful observations to detect an anisotropy signal in the global redshift change include spectroscopic observations of the Ly-α forest in absorption towards a sample of quasars, redshifted 21 cm line observations either in absorption or emission towards a sample of neutral HI regions before or during reionization, and high-frequency (0.1 to 1 Hz) gravitational-wave analysis of a sample of neutron star–neutron star binaries detected with gravitational-wave instruments such as the Decihertz Interferometer Gravitational Wave Observatory (DECIGO). For reasonable observations expected in the future involving extragalactic sources, we find limits at the level of $\Omega_{\rm GW} < 10^{-6}$ at a frequency around 10^{-12} Hz while the ultimate limit is likely to be around $\Omega_{\rm GW} < 10^{-11}$. On the other hand, if there is a background of gravitational waves at 10^{-12} Hz with an amplitude larger than this limit, its presence will be visible as a measurable anisotropy in the time-evolving redshift of extragalactic sources.

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I. INTRODUCTION

The observation of cosmic microwave background (CMB) anisotropies allows one to indirectly constrain the primordial background of gravitational waves with wavelengths around the horizon size today, corresponding to a frequency of $\sim 10^{-18}$ Hz [1]. Recent temperature anisotropy observations at large angular scales with Wilkinson Microwave Anisotropy Probe (WMAP; [2]) yield a limit (at the 2σ level) of $\Omega_{\rm GW} < 5 \times 10^{-11}$ [3,4]. This limit is improved by a factor of 2, at most, when CMB data are combined with other large-scale structure observations [5]. Here, $\Omega_{\rm GW}$ is the fractional density contribution to the energy density of the Universe from a background of stochastic gravitational waves (GW) with the density $\rho_{\rm GW}$:

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f}, \tag{1}$$

when the closure density is $\rho_c = 3H_0^2/8\pi G$. Here, H_0 is the Hubble constant. At the superhorizon and horizon-size scales probed by CMB, a stochastic background of gravitational waves is expected from inflationary physics [1,6].

While anisotropies in the CMB intensity, or temperature, due to gravitational waves are dwarfed by the dominant scalar, or density, fluctuations, a distinct signature of these tensor fluctuations is present in CMB polarization. The signature involves the so-called B-modes of polarization or the component of polarization that can be decomposed to a curl. To the first order, curl modes are not expected to contain anisotropies generated by density fluctuations [7]. The presence of gravitational waves is inferred through the B-mode polarization component that is generated by electron scattering of the quadrupole associated with tensor metric fluctuations. This indirect detection of gravitational waves is one of the main goals of the upcoming CMB polarization anisotropy observations both from ground and space with the planned Inflationary Probe mission, or CMBpol, of NASA's Beyond Einstein Program. Relative to the current limit inferred from intensity anisotropies, a dedicated polarization mission can improve the existing limit on the amplitude of gravitational waves by at least 5 orders of magnitude. This is comparable to roughly the lower limit allowed by the cosmic variance-limited removal of the confusion arising from cosmic shear mixing of polarization modes [8]. With a separation of lensing produced curl modes, one expects the limit to improve down to $\Omega_{\rm GW} < 10^{-16}$ with the best sensitivity at a frequency around 10^{-17} Hz [9].

Pulsar timing is generally used to study the subhorizon-scale gravitational-wave background composed of both the primordial background from inflation, with modes that enter the horizon during radiation domination, or backgrounds associated with massive black hole binaries [10,11], among others. In the future, high precision timing observations of a large sample of millisecond pulsars with the pulsar timing array, or with the square kilometer array, can be used to study this stochastic gravitational-wave background at frequencies greater than roughly $1/T_{\rm obs}$

where $T_{\rm obs}$ is the total observational duration of the pulsar sample [12]. While waves with frequencies lower than $1/T_{\rm obs}$ linearly change the observed interval of the pulse, the lower frequency limit of $1/T_{\rm obs}$ comes from the fact that one cannot distinguish changes to the pulsar profile from gravitational waves with long-term intrinsic pulsar spin evolution. Since observations are likely to be restricted to less than 100 years, the pulsar timing arrays cannot constrain the gravitational-wave background below 10^{-10} Hz. Observations of 2 millisecond pulsars over a period of 17 years limit the background to be $\Omega_{\rm GW}h^2 < 2 \times 10^{-9}$ (at the 95% confidence level) [13,14]. With the pulsar timing array, using a sample of order 100 millisecond pulsars, the limit on $\Omega_{\rm GW}h^2$ is expected to reach the level of 2×10^{-13} [12].

Beyond the inflationary gravitational-wave background, at frequencies below the level probed with millisecond pulsar timing profiles, a background of stochastic gravitational waves may be generated by global cosmic strings, phase transitions in the early Universe, and cosmic turbulence [15]. While the horizon-size primordial background today can be constrained with CMB polarization observations and the background with wavelengths corresponding to the horizon size or below during the big bang nucleosynthesis can be constrained with the predicted abundance of light elements compared to observations [16], no significant constraint exists for modes with frequencies between $\sim 10^{-16}$ and 10^{-10} Hz. The binary pulsars currently limit the background between $10^{-11}\ {\rm Hz} < f < 4.4 \times 10^{-9}$ Hz to be $\Omega_{\rm GW}h^2 < 0.04$ and between $10^{-12}\ {\rm Hz} < f < 10^{-11}$ Hz to be $\Omega_{\rm GW}h^2 < 0.5$; These constraints are based on the time variation of their orbital period compared to the prediction from general relativity [14]. In the future, this limit might be improved to a level of $\Omega_{\rm GW}h^2 \sim 10^{-4}$, with the uncertainty dominated primarily by the local acceleration of these binaries [17].

Using galactic objects, it is difficult to get information on the gravitational-wave background below $f \lesssim 10^{-12}$ Hz as this frequency corresponds roughly to the typical distance between galactic objects and us. The measurement sensitivity for such low-frequency waves is reduced by the cancellation associated with the fact that long wavelength gravitational waves act in an equal manner on galactic objects as well as the observer [17]. Therefore, it is essential that we use extragalactic objects to set limits on the low-frequency gravitational-wave background below 10^{-12} Hz as the distance to such objects is greater than the wavelength. The current limit comes from observations of quasar proper motions across the sky (transverse motions) [18]. For data collected over a 10 yr span, the resulting constraint on the stochastic background at a frequency below 2×10^{-9} Hz is $\Omega_{\rm GW}h^2 < 0.11$ [19] (see also [20] for prospects with the square kilometer array). While it is, in principle, possible to constrain the low-frequency background between 10^{-10} and 10^{-16} Hz with galaxy statistics such as the galaxy correlation function [21], to derive a tight limit or to make a reliable detection of the GW background, information on galaxy clustering statistics undisturbed by gravitational waves is needed for a comparison with the data.

To probe the GW background at frequencies below 10^{-12} Hz, we propose measuring the anisotropy pattern of the rate of change of redshift of a sample of high-redshift objects. Previously, observations of the redshift variation with time have been proposed to study global cosmological parameters [22]. While cosmology can induce either an acceleration or a deceleration to the redshift of an object, the resulting change can be extracted through variations to the monopole from a sample of objects spread over the sky since all sources are equally affected by changes to the background cosmological model. The resulting modifications associated with a gravitational-wave background, at the lowest order in spherical moments of the anisotropy pattern, is present in the quadrupole moment of the time-evolving redshift [23]. The proposed use of an anisotropy pattern of time varying redshift also allows one to separate secondary effects that can induce redshift change with time, such as the peculiar acceleration of the observer which is present via a dipole at the lowest order of multiple moments. We focus on the quadrupole measurement, but the measurement of additional higher-order multiple moments of the anisotropy can be used to increase the confidence level of the derived limit.

As we will discuss, the proposed method using the time variation of redshift of a sample of objects would be performed as a by-product of a dark energy study involving the monopole component of the redshift variation pattern following Ref. [22]. Therefore, we can relate the sensitivity of detecting the gravitational-wave background to that associated with the expansion rate that captures dark energy properties. With the monopole, we will be studying an underlying cosmological model and we can expect to an order of magnitude that there is a simple relation between the sensitivity to measure the gravitational-wave background to that required to measure the cosmic acceleration due to dark energy. For example, from a simple dimensional analysis, the time variation of the redshift z caused by a background of gravitational waves around frequency $f_{\rm GW}$ is $(dz/dt) \sim h_{\rm GW} f_{\rm GW}$ and the normalized gravitational-wave energy density is $\Omega_{\rm GW} \sim (h_{\rm GW} f)^2/H_0^2$. Here, $h_{\rm GW}$ is the nondimensional strain amplitude. Therefore, the sensitivity, $\Delta(dz/dt)$, to detect the mean redshift change can be related to that involved with changes from the gravitational-wave background, $\Delta\Omega_{\rm GW}$, as $\Delta\Omega_{\rm GW} \sim (\Delta(dz/dt)/H_0)^2$. In this paper we calculate this relation exactly and we discuss potential constraints on the gravitational-wave background that we can place with observations of the anisotropic redshift variation with time of a sample of extragalactic sources.

As a practical implementation of our method, extending the discussion in Ref. [22], we will consider a measurement of the quadrupole pattern related to the anisotropy of time derivatives of Ly-α forest redshifts seen in absorption towards a sample of high-redshift quasars. Additional probes for the same purpose include low-frequency 21 cm observations either in absorption or emission towards a sample of neutral HI regions before or during reionization [24]. Another possibility is the high-frequency (0.1 to 1 Hz) gravitational-wave analysis of a sample of neutron star–neutron star binaries detected with gravitational-wave instruments such as the Decihertz Interferometer Gravitational Wave Observatory (DECIGO) [25]. These observations have been suggested for a measurement of cosmological parameters based on the phase variation induced during the propagation of gravitational waves from compact extragalactic binaries in an expanding cosmological background [25,26]. While cosmological expansion induces a global phase shift (a monopole), the low-frequency gravitational-wave background induces an anisotropy to the phase shift of high-frequency gravitational waves emitted by these binaries. Thus, the presence of a low-frequency gravitational-wave background can be extracted indirectly from a sample of binaries spread over the sky with well-modeled waveforms of their gravitational-wave signals.

In the future, assuming conservative measurements of all these possibilities, we find the best constraints at the level of $\Omega_{\rm GW} < 10^{-6}$ at a frequency around 10^{-12} Hz, though the eventual limit, using 21 cm lines, is probably around $\Omega_{\rm GW} < 10^{-11}$. Reaching this level is challenging as it requires high resolution observations of the 21 cm background to spatially resolve kiloparsec-scale halos at redshifts around 10 with follow-up spectral measurements over a 10 yr span in a bandwidth less than a kHz. In Fig. 1, we plot sensitivities of various current and future constraints for the low-frequency gravitational-wave background. This figure makes it clear why extragalactic probes are needed to study the gravitational-wave background with frequencies below 10^{-12} Hz and with frequencies above those studied by CMB experiments.

While we discuss the use of anisotropies of certain observables to constrain the low-frequency gravitational-wave background, in principle, one can also use the expansion rate to constrain the energy density of the gravitational background. The resulting correction comes from the fact that a subhorizon-scale gravitational-wave background acts as an additional radiation field in the universe and the expansion rate is modified to become $H^2(z) = H_0^2[\Omega_m(1+z)^3 + (\Omega_R + \Omega_{\rm GW})(1+z)^4 + \Omega_\Lambda]$, in a spatially flat universe with cosmological constant Ω_Λ, the matter density parameter Ω_M, and the radiation content Ω_R, all relative to the critical density. Since matter and dark energy densities dominate the expansion rate over the redshift ranges, one can probe with current observational techniques, we do not pursue a limit based on this argument further. An interesting constraint, however, could

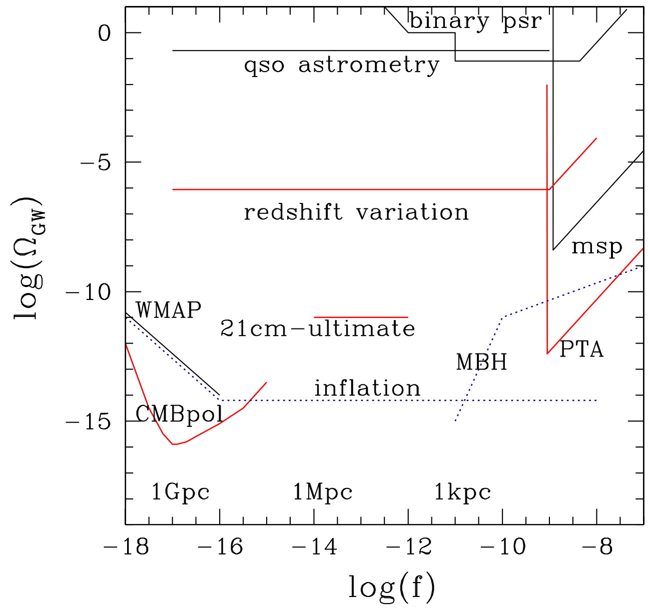

FIG. 1 (color online). Sensitivities for various current (thin lines) and future (thick lines) measurements. Curves labeled "WMAP" and "CMBpol" show the current and future constraints based on CMB anisotropies at large angular scales. The curve labeled "binary psr" is the limit based on a sample of binary pulsars, based on the time variation of their orbits, the curve labeled "msp" is the current limit based on a 17-year timing monitoring of 2 ms pulsars, while the curve labeled "PTA" is the limit based on proposed pulsar timing array network. The limit based on "QSO astrometry" involves proper motion observations of a sample of radio bright quasars while "redshift variation" involves time monitoring of redshift variations in the Lyman-α forest towards bright quasars, the 21 cm line at very low radio frequencies, and cosmological neutron star binaries with ultimate DECIGO. The curve labeled "21cm-ultimate" is the eventual ultimate limit for method with 21 cm line. The dotted line with "inflation" is the amplitude of the stochastic background constrained by WMAP and corresponds to the energy scale 3.4×10^{16} GeV [39]. The line with "MBH" is the estimated background made by merging massive black hole binaries given in [11].

potentially be derived if one were to study earliest times in the universe such as the era of big bang nucleosynthesis and using the number of massless neutrino species. The constraint applies to waves with wavelengths below about a 10 pc, and the previous limit based on this argument constraints the high-frequency background, as observed today, to be $\Omega_{\rm GW} < 5 \times 10^{-4}$ [16].

With an improved measurement of the ratio of photon-to-baryons from both the cosmic microwave background anisotropy spectrum, with physics dominating at the last scattering surface around $z \sim 1100$, and the same ratio measured independently based on nucleosynthesis arguments, one can derive an improved constraint on the subhorizon gravitational background that existed between an age for the Universe of a minute to a few hundred thousand years. Another limit can be derived based on the fact that the matter-radiation equality is now well measured to be at a redshift of ~3500 using CMB anisotropies alone [3]. If there is a significant subhorizon gravitational-wave background, the matter-radiation equality one would infer directly from an analysis of CMB data would be different from the estimate based on Ω_m and Ω_Λ at low redshifts; this limit requires a better understanding of dark energy and to what extent the Universe is spatially flat. While we do not pursue these possibilities in detail, we note that these possibilities will be explored in an upcoming paper using CMB anisotropy measurements from WMAP [27].

The discussion related to potential future constraints on the low-frequency gravitational-wave background is organized as follows: In the next section, we outline the calculation related to an anisotropy pattern in the time-evolving redshift of a sample of sources in the presence of a gravitational-wave background. In Sec. III, we discuss cosmological probes related to this effect and estimate resulting limits that one can potentially obtain in the future related to the presence of a low frequency, but subhorizon, background of gravitational waves. We conclude with a summary in Sec. IV.

II. FORMULATION

We first study how a stochastic gravitational-wave background affects the redshift of a photon, or a massless particle, propagating to an observer, present today, from a high-redshift source. We consider a spatially flat universe and write the metric that is perturbed by the gravitational-wave background as

$$ds^2 = a(\eta)^2\{-d\eta^2 + [\delta_{ij} + h_{ij}(\eta, \boldsymbol{x})]dx^i dx^j\}, \qquad (2)$$

where η is the conformal time and the scale factor $a(\eta)$ is normalized such that $a(\eta_0) = 1$ at the present epoch with $\eta = \eta_0$. We write the conformal time interval and the normal time interval today as $\Delta\eta$ and ΔT, respectively. When compared to a background universe with $h_{ij} = 0$, the propagation time of a photon changes by $\delta\eta$ when passing through gravitational waves. We can formally write down $\delta\eta$ as a line integral along the photon path:

$$\delta\eta = \frac{n_i n_j}{2} \int_{\eta_0 - L}^{\eta_0} h_{ij}[\eta, (\eta_0 - \eta)\boldsymbol{n}]d\eta, \qquad (3)$$

where $\boldsymbol{n}$ is the unit directional vector to any high-redshift luminous source and L is its comoving distance from us. This shift $\delta\eta$ will result in an apparent fluctuation Δz of the source's redshift z as

$$\Delta z = \frac{n_i n_j}{2} \int_{\eta_0 - L}^{\eta_0} \frac{\partial h_{ij}}{\partial \eta}[\eta, (\eta_0 - \eta)\boldsymbol{n}]d\eta. \qquad (4)$$

Since we are interested in wavelengths ($\lesssim 10$ Mpc) smaller than the horizon scale, we can safely express gravitational-wave modes related to this modification as

$$h_{ij}(f, \boldsymbol{p}, \eta, \boldsymbol{x}) = \exp[i2\pi f(\eta - \boldsymbol{p}\cdot\boldsymbol{x})]D(\eta, f)b_{ij}(f\boldsymbol{p}), \qquad (5)$$

where $\boldsymbol{p}$ is the unit vector along the direction in which the wave propagates, $D(\eta, f)$ represents the cosmological evolution of the wave amplitude, and $b_{ij}(f\boldsymbol{p})$ denotes each wave characterized by the vector $f\boldsymbol{p}$.

We decompose Eq. (5) as

$$\begin{aligned} h_{ij}(f, \boldsymbol{p}, \eta, \boldsymbol{x}) = {} & \exp[i2\pi f(\eta - \boldsymbol{p}\cdot\boldsymbol{x})]\{D(\eta, f) - D(\eta_0, f)\} \\ & \times b_{ij}(f\boldsymbol{p}) + \exp[i2\pi f(\eta - \boldsymbol{p}\cdot\boldsymbol{x})] \\ & \times D(\eta_0, f)b_{ij}(f\boldsymbol{p}), \end{aligned} \qquad (6)$$

and substitute this in Eq. (4). Since our aim is to extract a common effect that is seen by all sources, we can ignore the contribution from the first term in Eq. (6) when concentrating on waves with wavelengths shorter than the horizon size. The common coherent effect comes from perturbations close to us in the integral in Eq. (3). The term $D(\eta, f) - D(\eta_0, f)$, however, makes such perturbations small. As is well known [28], the integral in Eq. (3) of the second term of Eq. (6) can be expressed as a sum of two quantities involving the information from a gravitational wave at the source distance and at the observer distance separately. The former term can be ignored here since it only leads to a random effect when we consider the spatial distribution of sources on the sky.

The only relevant effect is captured by the second contribution. Note that this term is also similar to the standard method in which timing residuals of galactic millisecond pulsars are used to study a low-frequency gravitational background. Random variations in Δz for each binary due to a gravitational-wave background can be regarded as a noise when measuring the common signal. For extragalactic binaries, however, the expected magnitude of variation is smaller than sources of noise associated with local acceleration and the detector. The observed redshift variation due to a single gravitational-wave mode around us ($\boldsymbol{x} = 0$) can be written as

$$\Delta z = \tfrac{1}{2}(1 + \cos\theta)\cos 2\phi\, h(\eta, 0), \qquad (7)$$

where θ is the angle between propagation direction $\boldsymbol{p}$ and the source location $\boldsymbol{x}$, ϕ is the polarization orientation angle, and $h(\eta, 0)$ is the value of the gravitational-wave mode at $\boldsymbol{x} = 0$. Our basic observable is not the redshift fluctuation Δz but rather the time variation of the redshift as photons propagate through gravitational waves, dz/dt. We decompose the angular dependence of time-dependent redshift in terms of spherical harmonics as [23]

$$\dot{z}_{\rm GW}(\theta, \phi) \equiv \frac{1}{H_0}\left(\frac{dz}{dt}\right)_{\rm GW} = \sum_{l\geq 2} \dot{z}_{lm;{\rm GW}} Y_{lm}(\theta, \phi). \qquad (8)$$

We assume an isotropic gravitational-wave background. As the contribution from each wave to the sum $\sum_m \langle \dot{z}_{lm;{\rm GW}} \dot{z}^*_{lm;{\rm GW}} \rangle$ does not depend on the orientation of the coordinate system, we can sum up all modes with a specific coordinate choice, such as the one used to describe Eq. (7).

After some straightforward simplifications, the angular power spectrum of coefficient $\dot{z}_{lm\mathrm{GW}}$ can be written as

$$\langle \dot{z}_{lm;\mathrm{GW}} \dot{z}^*_{l'm';\mathrm{GW}} \rangle = \frac{12\pi^2 \Omega_{\mathrm{GW}}}{(l+2)(l+1)l(l-1)} \delta_{ll'} \delta_{mm'}, \quad (9)$$

where the parameter Ω_{GW} is defined using the normalized energy density $\Omega_{\mathrm{GW}}(f)$ per unit log frequency interval as

$$\Omega_{\mathrm{GW}} = \int_{f_{\mathrm{cutoff}}}^{(\Delta T)^{-1}} \Omega_{\mathrm{GW}}(f) \frac{df}{f}. \quad (10)$$

At the low-frequency regime of $f \lesssim 10^{-15}$ Hz, the magnitude of the gravitational-wave background $\Omega_{\mathrm{GW}}(f)$ is well constrained with CMB observations at a level better than that based on the redshift-time variation method (see Fig. 1). Above this frequency CMB does not allow one to place a strong constraint on the amplitude of the gravitational-wave background. If the frequency-integrated gravitational-wave background Ω_{GW} is detected with a time-dependent redshift variation, we expect the integral to be dominated in a frequency range higher than that studied with CMB data. Therefore, we include an artificial low-frequency cutoff $f_{\mathrm{cutoff}} \sim 10^{-15}$ Hz in the integral (10) as this is roughly the high end of gravitational-wave frequencies probed by CMB anisotropy measurements.

For a statistical study using a sufficiently large sample of extragalactic sources, we also define the angular power spectrum C_l of the anisotropy distribution of time-dependent redshift as

$$C_l = \frac{1}{2l+1} \sum_m |\dot{z}_{lm}|^2 = \frac{12\pi^2 \Omega_{\mathrm{GW}}}{(l+2)(l+1)l(l-1)}. \quad (11)$$

In Eq. (8), the summation starts from the $l = 2$ quadrupole moment and is independent of the exact source redshift z. The global redshift change Δz due to cosmic acceleration and deceleration is the monopole ($l = 0$) corresponding to the coefficient $\dot{z}_{00}$ as

$$\dot{z}_{00;\mathrm{acc}} = -\sqrt{4\pi}\{[\Omega_M(1+z) + \Omega_R(1+z)^2 + \Omega_\Lambda(1+z)^{-2}]^{1/2} - 1\}(1+z).$$

While the quadrupole is independent of the redshift, this term depends on the redshift z of extragalactic sources [22] We define a parameter X that characterizes the second order correction to the relation between time intervals at the observer, Δt, and time intervals at the source, Δt_z, resulting from an effective acceleration as

$$(1+z)\Delta t_z = \Delta t - X\Delta t^2 + O(\Delta t^3). \quad (12)$$

Here, X is related to the rate at which redshift varies

$$\frac{dz}{dt} = 2X(1+z). \quad (13)$$

The parameter X can have a finite value due to the dark energy related cosmological acceleration, due to background of gravitational waves, and local acceleration at the observer and the source. For each high-redshift source, one can fit models to the parameter X to statistically separate these three contributions. In the case of cosmological acceleration, one can write

$$X_{\mathrm{acc}}(z) = \frac{1}{2}\left[H_0 - \frac{H(z)}{1+z}\right] = \frac{H_0 \dot{z}_{00;\mathrm{acc}}}{4\sqrt{\pi}(1+z)}, \quad (14)$$

where $H(z)$ is the Hubble parameter at a redshift z. The parameter $X_{\mathrm{acc}}(z)$ can also be written as $X_{\mathrm{acc}}(z) = 0.5[\partial_t a(0) - \partial_t a(z)]$. At small redshifts, when $z \ll 1$, $X_{\mathrm{acc}}(z) = -0.5 H_0 q_0 z$, where $q_0 = \Omega_m - \Omega_\Lambda/2$ is the global deceleration parameter.

For gravitational waves emitted by a chirping binary, the time variation of the source redshift appears as an additional phase change to the Fourier transformed gravitational-waveform. This phase change is proportional to $f^{-13/3}X(z)$ and has a frequency dependence largely different from the phase evolution associated with the intrinsic binary evolution under post-Newtonian predictions. This phase correction has been used as a probe of cosmological parameters in Refs. [25,26]. The anisotropy of the phase correction, or the anisotropy measurement of $X(z)$ from a large sample of binaries spread over the sky, can again be used a probe of large wavelength gravitational waves.

We introduce the ratio

$$R \equiv X_{\mathrm{acc}}(z)/H_0 \quad (15)$$

to relate cosmic acceleration and the low-frequency gravitational-wave background with anisotropies in $X(z)$; the former is measured through the monopole of time-dependent redshifts, while the latter comes from the anisotropy pattern of redshift variations. In this sense, the ratio R simply captures the fractional anisotropy relative to the monopole. Since the monopole measurement involves a redshift dependence, we analyze extragalactic sources binned over some finite width in the redshift. Using same data, but higher-order anisotropies, we also constrain Ω_{GW}. The statistical uncertainty related to a measurement of the angular power spectrum of $\dot{z}_{lm}$ is

$$\Delta C_l = \sqrt{\frac{2}{(2l+1)f_{\mathrm{sky}}}}\left[C_l + 4\pi\sqrt{f_{\mathrm{sky}}}\frac{\sigma^2}{N}\right], \quad (16)$$

where σ is the rms fluctuation in the redshift variation $H_0^{-1} dz/dt$ of each source and N is the number of sources used for the anisotropy measurement. The parameter f_{sky} is the fraction of the sky covered with proposed observations. In Eq. (16) the first term represents the cosmic variance and the second term can be considered as the Poisson fluctuation.

The minimum energy density of the gravitational-wave background measurable by any of the possibilities we will soon discuss is

$$\sigma_{\Omega_{\rm GW}}^{-2} = \sum_{l>180/\theta} \frac{1}{(\Delta C_l)^2}\left(\frac{\partial C_l}{\partial \Omega_{\rm GW}}\right)^2, \tag{17}$$

where $\theta \approx 203 f_{\rm sky}^{1/2}$ is roughly the width (in degrees) of the sky area covered by observations. In restricting the sum to $l > 180/\theta$, we have assumed that no information from modes with wavelengths larger than the survey size can be obtained. For example, to measure the quadrupole, we require observations over a substantially large area of the sky. In the context of CMB anisotropies, issues related to partial-sky coverage are discussed in Ref. [29]; With a π steradian patch of the sky, one can hope to constrain at least a couple of the 5 independent moments of the quadrupole. We consider this as a minimal requirement on the sky area needed for the proposed measurement. The sky area requirement mostly applies when applying this method to quasistellar object (QSO) absorption lines or 21 cm lines from neutral hydrogen clouds detectable with low radio frequency interferometers. When considering a measurement of phase shift associated with extragalactic compact binary gravitational-waveforms, gravitational-wave interferometers are nearly omnidirectional and the requirement related to whole sky observations are met by design (see e.g. [30]).

Hereafter we assume that the source sampling is over the whole sky ($f_{\rm sky} = 1$) and discuss the sensitivity to detect a gravitational-wave background in terms of the uncertainty ΔR associated with the monopole signal arising from cosmic acceleration R as

$$\Delta R = \frac{1}{2(1+z)}\frac{\sigma}{\sqrt{N}}. \tag{18}$$

Note that here there is no equivalent of the cosmic variance term in Eq. (16), as the monopole itself is not a statistical quantity in contrast to the higher-order modes as variances.

To estimate the limit, one can place on $\Omega_{\rm GW}$, under the null hypothesis of a no gravitational-wave background, we set $C_l = 0$ in Eq. (16). Instead of using the whole power spectrum related to redshift derivatives, or $X(z)$, to constrain $\Omega_{\rm GW}$, we concentrate only on using the quadrupole, C_2. Using Eqs. (16) and (18), the uncertainty in the quadrupole estimate can be written in term of the error ΔR associated with the monopole measurement. In Fig. 2, we plot the angular spectrum C_l when $\Omega_{\rm GW} = 10^{-4}$. We also put the measurement error for each C_l if the sensitivity for measuring the monopole of R is 4×10^{-3} in a single redshift shell centered at $z = 1$.

Making use of the quadrupole anisotropy only, using Eqs. (16) and (18), we obtain the 1-σ constrain for the low-

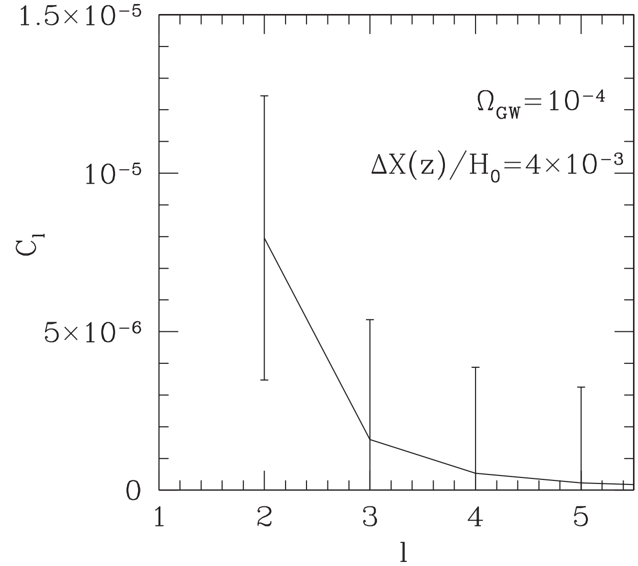

FIG. 2. The angular power spectrum C_l in time-dependent redshift variations due to a low frequency gravitational-wave background with $\Omega_{\rm GW} = 10^{-4}$. The error bars assume the case when we can measure the monopole mode R with sensitivity 4×10^{-3} in a shell of sources center in redshift at $z = 1$.

frequency gravitational-wave background as

$$\Omega_{\rm GW} < 4\pi \times 8\pi^{-2}(1+z)^2\sqrt{2/5}(\Delta R)^2_{\rm bin} \simeq 2.6\times 10^1\{(1+z)/2\}^2(\Delta R)^2_{\rm bin} \tag{19}$$

from probes binned in redshift where $(\Delta R)_{\rm bin}$ is the error to which $R(z)$ can be established in an individual redshift bin. When we use anisotropy information up to $l = 5$, the coefficient 2.6 is reduced to 2.5, which is insignificant; statistical power to constrain the gravitational-wave background predominantly exists only in the quadrupole. As we commented earlier, the anisotropy signal due to the gravitational-wave background does not depend on the redshift. This suggests that the overall signal-to-noise ratio can be improved when one combines additional bins at different redshifts than the above result which considered a single bin centered at a redshift of unity.

For a cosmological model with $h = 0.7$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$, R is $\sim$0.05 for $z \sim 1$. To study the cosmic acceleration with a signal-to-noise ratio level of 10 requires reaching an accuracy level in R of $\Delta R \sim 0.005$ in a redshift interval $dz \sim 0.1$. With such a sensitivity also involved with anisotropy measurements of Δz, we can set the upper limit on $\Omega_{\rm GW}$ as

$$\Omega_{\rm GW} \sim 2.7\times 10^1 \times 10^{-1} \times (5\times 10^{-3})^2 \sim 7\times 10^{-5}, \tag{20}$$

where the factor 10^{-1} represents an average estimate on the expected improvement associated with increasing the effective number of bins used. If bins in redshifts between

1 and 3 can be divided at intervals of 0.1, there is an additional factor of a few improvements on the Ω_{GW} limit.

III. COSMOLOGICAL PROBES

A. Redshift variations in the line emission

The use of time variation associated with the redshift of distant sources as a probe of global cosmological parameters is described in Ref. [22] (see also [31]). The resulting change is an overall common shift to the redshift distribution and this can be observed with redshift data of extragalactic sources separated in time. For this purpose, the most useful observations are the ones that involve narrow lines either in emission or absorption since the precise redshift can easily be ascertained from spectroscopic observations. In Ref. [22], the use of the Lyman-α forest in absorption towards luminous quasars is suggested. The expected mean shift in the Lyman-α forest, at a redshift $\sim$3, is roughly 2 m s^{-1} over a time period of 100 years for presently favorable cosmological parameters. While this is a small shift in wavelength, spectroscopic variations at the level of $\sim$1 m s^{-1} are now routinely measured with 10-meter class telescopes towards bright stars in extrasolar planetary searches using radial velocity measurements [32]. The advent of 30-meter class telescopes over the next decade or more, and the expected increase in spectral resolution, will improve searches for small velocity shifts in high-redshift quasar spectra. We also note that one of the concept instruments of the proposed 50-meter class telescopes under consideration by the European Space Observatory is a multiobject spectrograph that can reach sub-m s^{-1} sensitivity in absorption line measurements and whose primary goal is the global cosmic acceleration with time-dependent redshift variations.

While the precise measurement of an overall variation in the redshift a single quasar will require observations that span many tens of years, a detection of the global change can be obtained through a statistical study of a large number of quasars over a short time interval. While such a study will allow a measurement of cosmological parameters [22], the effect due to a gravitational-wave background requires a study related to the quadrupolar pattern of time-dependent redshift variation. The quadrupolar pattern allows one to separate the effect related to cosmological acceleration, which is present in the monopole, and that due to local motion (such peculiar velocities) that is present in the dipole, with that expected from a background of gravitational waves.

Using estimates in Ref. [22] for observations of 1000 quasar sight lines with improved spectroscopic data with a pixel scale of 0.5 km s^{-1}) over a 10 yr span, and assuming no systematic uncertainties in the calibration of the spectrum, we find that one can constrain the monopole to an accuracy of $\sim$1%. The resulting constraint on a low-frequency background of gravitational waves is at the level $\Omega_{GW} < 10^{-4}$ and corresponds to waves whose wavelengths are out to a distance corresponding to a redshift of 3. While this limit is not significant, it is still useful to obtain this directly from the data as this will be one of the few methods to constrain the energy density of gravitational waves around 10^{-12} Hz.

In addition to the Lyman-α forest, one can constrain the energy density of gravitational waves using the redshift distribution of 21 cm emitters. Here, the line emission is at low radio wavelengths and statistics are expected to significantly improve given the expected narrow width of these lines and the narrow band observations one can achieve at low radio frequencies with interferometric observations. Expected statistics of HI emitters before reionization are not well known and the complication here will be more related to the foreground contamination [33] more than the number of objects one can use to make these measurements. Improvements in foreground removal techniques suggest that the contamination can be controlled to some extent, though we will understand this better with first generation low-frequency radio interferometers [34].

We follow estimates in Ref. [35] which assume a model for the cosmological distribution of neutral gas before reionization based on the halo approach [36] and using so-called ''mini-halos'' [37]. These halos contain adiabatically cooled gas with a temperature below that of the CMB thermal radiation at those redshifts. We estimate the linewidth of each signal to be around 3 km s^{-1} and to be dominated by thermal motions within each halo. This compares to linewidths of order 20 km s^{-1} in the Lyman-α spectrum. Since the characteristic mass scale of each of these mini halos is around 10^5 M_{sun} to 10^6 M_{sun}, one expects a total of, in principle, 10^{18} halos across the whole sky with a size of 1 kpc or a projected size of 30 milliarcseconds. While the whole set of mini halos are not needed, using a sample of 10^6 brightest 21 cm lines over the whole sky, and assuming observations with an observational bandwidth of 0.2 kHz, which corresponds to a velocity of $\sim$0.4 km s^{-1}, we find that one can constrain the background to be below $\Omega_{GW} < 10^{-6}$. The ultimate limit related to 21 cm anisotropies, using the whole sample of 10^{18} halos and assuming sufficient resolution to resolve them in redshift space and ignoring source confusion, is $\Omega_{GW} < 10^{-11}$ over a 10 yr span of observations. This is comparable to the current limit from CMB anisotropies at frequencies between 10^{-18} and 10^{-16} Hz.

B. Gravitational waves from neutron star binaries around 1 Hz

Now we discuss prospects for measuring Ω_{GW} at the very low-frequency regime ($\lesssim 10^{-12}$ Hz) with proposed space-based gravitational-wave missions such as the big bang observer (BBO) or DECIGO. The sensitivity of these proposed interferometers peaks between 0.1 to 1 Hz and covers the gap in frequency between LISA and ground based detectors [38]. The most important aim of these detectors is the direct detection of the gravitational-wave background from early universe with $\Omega_{GW} \lesssim 10^{-15}$ level at 0.1 to 1 Hz [39].

For this measurement, the foreground gravitational waves from various astrophysical sources must be subtracted from the data streams down to some appropriate level. The cosmological neutron star binaries are expected to produce a significant foreground and we need an extensive fitting analysis with accurate templates to remove these sources from the data. In this fitting procedure we can measure not only intrinsic binary parameters but also extrinsic effects made by the cosmic or local acceleration through phase variation related to X. In the presence of a low-frequency gravitational-wave background, extracted X values for a sample of binaries spread over the sky and in a certain redshift bin are expected to show anisotropies. As discussed, the quadrupolar pattern can be used as a probe of the low-frequency gravitational-wave background. Thus, gravitational-wave observations at frequencies between 0.1 to 1 Hz is used to limit the gravitational-wave background at very low frequencies with $f \lesssim 10^{-12}$ Hz.

In addition to the low-frequency gravitational-wave background, cosmological neutron star binaries can be regarded as a probe of the dark energy, through the monopole variation related to $X(z)$ [25,26]. While the discussion is related to neutron star binaries detectable in the 0.1 to 1 Hz band, we can also make a similar argument for stellar mass black hole binaries in the same band, though their intrinsic waveforms are more complicated and the extraction of $X(z)$, through changes to the phase of a gravitational wave, would be more difficult. The local acceleration for such black hole binaries might also be larger than that of neutron star binaries.

At present, it is not clear how well we can actually resolve and subtract the gravitational-wave contribution of each merging neutron star binary in data streams of BBO or DECIGO (see [40] for a recent study). For example, the confusion effect is more important for a binary that takes a long time to merge, as its gravitational waves will be in the low-frequency regime where the number density of binaries per unit frequency can be high. In addition to binaries, the background made by sources such as supernovae may lead to an additional stochastic background [41].

Now, we study how well we might be able to set a constraint on Ω_{GW} at $f \lesssim 10^{-12}$ Hz based on a recent estimate on the parameter $R \equiv X(z)/H_0$ for the cosmic acceleration [26] from phase shifts in compact binary waveforms. Following their paper, we use the coalescence rate of neutron star binaries at $10^{-6}/\mathrm{yr}/\mathrm{Mpc}^3$ assuming it is constant with time. The total number of binaries in a redshift shell $z \sim z + dz$ is $\sim 10^4/\mathrm{yr}$ for a width $dz \sim 0.1$ at $z \sim 1$. As we have mentioned, in addition to the measurement error ΔX_{det} caused by the detector noise, the random local acceleration becomes an effective noise $\Delta X_{\mathrm{local}}$ for estimating the cosmic acceleration signal or the gravitational-wave background. In the case of a neutron star binary, we expect $\Delta X_{\mathrm{local}}/H_0 \sim 10^{-1}$ in terms of the noise ΔR for each binary [42]. Therefore, for a redshift bin $dz \sim 0.1$, the accuracy to which the cosmic acceleration can be measured is limited by fluctuations due to local acceleration around $(\Delta R)_{\mathrm{bin}} \sim 10^{-1} \times \sqrt{10^{-4}} \sim 10^{-3}$ for an observational period of around a year. The final fluctuation level, after using all redshift bins, is $\Delta R \sim (\Delta R)_{\mathrm{bin}} \times \sqrt{N_{\mathrm{shell}}} \sim 10^{-3.5}$. With an effective number of bins of $N_{\mathrm{bins}} = 10$, we can set an upper limit of $\Omega_{GW} \sim 3 \times 10^{-6}$ using the relation in Eq. (19).

As for the measurement error ΔX_{det} determined by the detector noise, we can reach a level of $(\Delta R)_{\mathrm{shell}} \sim 3 \times 10^{-4}$ (corresponding to $\Delta X_{\mathrm{det}}/H_0 \sim 3 \times 10^{-2}$ for each binary) with a 3 yr integration using the ultimate DECIGO noise curve (see Figs. 3 and 4 of Ref. [26]). With a 1 yr integration, however, the measurement error becomes $(\Delta R)_{\mathrm{shell}} \sim 3 \times 10^{-3}$ and dominates fluctuations caused by local acceleration. In these estimates, note that we have not included additional astrophysical confusion noise associated with unresolved sources.

Given these considerations, it is likely that the best constraint one can place on the low-frequency gravitational-wave background using neutron star binaries in future space-based gravitational-wave interferometers is at a level of $\Omega_{GW} \sim 3 \times 10^{-6}$.

IV. SUMMARY

We discussed the possibility of constraining the low-frequency gravitational-wave background below 10^{-12} Hz by monitoring the anisotropy pattern of time varying redshifts of extragalactic sources. As the direct measurement of the global time variation in redshift is one of the potential methods to study cosmic acceleration associated with dark energy, we studied the prospects for a measurement of the gravitational-wave background given the expected accuracies for a detection of the mean redshift change with time. Useful observations for this purpose include spectroscopic measurement of the Ly-α forest in absorption towards a sample of quasars, redshifted 21 cm line observations towards a sample of neutral hydrogen either in absorption or emission, and high-frequency (0.1 to 1 Hz) gravitational-wave analysis of a sample of neutron star–neutron star binaries detected with gravitational-wave instruments such as the Decihertz Interferometer Gravitational Wave Observatory (DECIGO). For reasonable observations in the future, we find best limits at the level of $\Omega_{GW} < 10^{-6}$ at a frequency around 10^{-12} Hz.

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