Wave damping by MHD turbulence and its effect upon cosmic ray propagation in the ISM

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ABSTRACT

Cosmic rays scatter off magnetic irregularities (Alfvén waves) with which they are resonant, that is waves of wavelength comparable to their gyroradii. These waves may be generated either by the cosmic rays themselves, if they stream faster than the Alfvén speed, or by sources of MHD turbulence. Waves excited by streaming cosmic rays are ideally shaped for scattering, whereas the scattering efficiency of MHD turbulence is severely diminished by its anisotropy. We show that MHD turbulence has an indirect effect on cosmic ray propagation by acting as a damping mechanism for cosmic ray generated waves. The hot ("coronal") phase of the interstellar medium is the best candidate location for cosmic ray confinement by scattering from self-generated waves. We relate the streaming velocity of cosmic rays to the rate of turbulent dissipation in this medium, for the case in which turbulent damping is the dominant damping mechanism. We conclude that cosmic rays with up to 10^2 GeV could not stream much faster than the Alfvén speed, but that 10^6 GeV cosmic rays would stream unimpeded by self-generated waves unless the coronal gas were remarkably turbulence-free.

Subject headings: MHD — turbulence — cosmic rays

1. Introduction

Cosmic ray (CR) scattering by resonant Alfvén waves has been proposed to be essential to their acceleration by shocks (e.g. Bell 1978) and to their confinement within the Galaxy (e.g. Kulsrud & Pearce 1969).

Much of the interstellar medium (ISM) is thought to be turbulent, providing a ready source of Alfvén waves. However, MHD turbulence has the property that, as energy cascades from large to small scales, power concentrates in modes with increasingly transverse

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wavevectors, i.e. perpendicular to the background magnetic field direction (Goldreich & Sridhar 1995, 1997). Cosmic rays scatter best off waves that have little transverse variation, so CR scattering by MHD turbulence is necessarily extremely weak, leading to very long CR mean free paths (e.g. Chandran 2000a; Yan & Lazarian 2002).

If cosmic rays stream faster than the Alfvén speed, they can amplify waves (naturally of the correct shape for scattering) through the resonant streaming instability (see Wentzel 1974). As the waves amplify, the scattering strength increases and the streaming velocity is reduced. For this process of self-confinement to operate, the excitation rate of the waves by streaming cosmic rays must exceed the sum of all rates of wave damping.

Wave damping depends upon the properties of the medium in which the CRs propagate. Important mechanisms include ion-neutral collisions in regions of partial ionization, and non-linear Landau damping in the collisionless limit. In this paper we introduce another mechanism, wave damping by background MHD turbulence. As cosmic ray-generated waves propagate along magnetic field lines, they are distorted in collisions with oppositely-directed turbulent wavepackets. As a result, the wave energy cascades to smaller scales and is ultimately dissipated. This process, which is best viewed geometrically, is described in § 2.2. MHD turbulence thus becomes an impediment to the scattering of cosmic rays, as opposed to just an ineffective scatterer of them. This mechanism was mentioned briefly by Cho, Lazarian & Vishniac (2003), Lazarian, Cho & Yan (2002) and Yan & Lazarian (2002).

The paper is arranged as follows. Relevant properties of the MHD cascade are described in §2.1, followed by an explanation of the turbulent damping rate in §2.2. In §3 we describe the competition between growth and damping of waves due to CR streaming. We apply these ideas to the problem of Galactic CR self-confinement in §4, and use this to place limits on the cascade rate of the turbulence in the coronal gas, assuming that the observed streaming velocities are due to self-confinement in this medium. In §4.1 we compare with other work in this area and in §5 we conclude.

2. The MHD cascade as a damping mechanism

2.1. Relevant properties of the cascade

The strong incompressible MHD cascade proposed by Goldreich & Sridhar (1995, 1997) has the property that as the cascade proceeds to smaller scales, power becomes increasingly concentrated in waves with wavevectors almost perpendicular to the local mean magnetic field. We envisage a situation in which turbulence is excited isotropically at an MHD outer scale $L_{\rm mhd}$ with RMS velocity fluctuations $v \sim v_{\rm A}$ and magnetic field fluctuations $\delta B \sim B_0$, where B_0 is the magnitude of the background magnetic field.[3] Well inside the cascade, the variations parallel to the magnetic field are much more gradual than those perpendicular to it, i.e. $v(\boldsymbol{\lambda}) \simeq v_{\lambda_\perp} \gg v_{\lambda_\parallel}$ if $\lambda_\perp \sim \lambda_\parallel$.[4] Equivalent relations hold for magnetic field fluctuations. For fluctuations of a given amplitude, therefore, the correlation length (defined so that $v_{\lambda_\perp} \sim v_{\Lambda_\parallel}$) parallel to magnetic field lines, $\Lambda_\parallel$, is much greater than that perpendicular to them, $\lambda_\perp$. Turbulent eddies are highly elongated parallel to magnetic field lines.

Strong MHD turbulence is characterized by "critical balance". In other words, a wavepacket shears at a rate which is comparable to its frequency $\omega = v_A k_\parallel \sim v_A/\Lambda_\parallel$ and is also of order $v_{\lambda_\perp}/\lambda_\perp$. Thus

$$\frac{v_{\lambda_\perp}}{\lambda_\perp} \sim \frac{v_A}{\lambda_\parallel}\,. \tag{1}$$

Application of the Kolmogorov argument for the constancy of the energy cascade rate ϵ per unit mass yields

$$\epsilon \sim \frac{v^2}{t_{\rm cascade}} \sim \frac{v^3_{\lambda_\perp}}{\lambda_\perp} \sim \frac{v_A^3}{L_{\rm mhd}}, \tag{2}$$

from which we obtain the fluctuation amplitude on perpendicular scale $\lambda_\perp$,

$$v_{\lambda_\perp} \sim v_A \left(\frac{\lambda_\perp}{L_{\rm mhd}}\right)^{1/3} \sim (\epsilon\lambda_\perp)^{1/3}. \tag{3}$$

An analogous relation holds for magnetic field perturbations. Well inside the cascade, $v \ll v_A$ and $\delta B \ll B_0$. We combine equations (1) and (3) to obtain the eddy shape:

$$\Lambda_\parallel(\lambda_\perp) \sim L_{\rm mhd}^{1/3}\lambda_\perp^{2/3} > \lambda_\perp\,. \tag{4}$$

2.2. The turbulent damping rate

The energy cascade from large to small scales in MHD turbulence is due to distortions produced in collisions between oppositely-directed Alfvén wavepackets. This is best visualized geometrically as being due to the shearing of wavepackets as they travel along wandering magnetic field lines. A good description is given in Lithwick & Goldreich (2001).

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[3]Turbulence can also be injected at smaller velocities on smaller scales, in which case $L_{\rm mhd}$ should be considered an extrapolation beyond the actual outer scale of the cascade.

[4]Throughout this paper, "perpendicular" and "parallel" wavelengths refer respectively to the inverse of the wavevector components perpendicular and parallel to the background magnetic field direction.

Consider the fate of a wavepacket with initial perpendicular and parallel wavelengths $\lambda_\perp$ and $\lambda_\parallel$. It suffers an order unity shear after traveling over a distance along which the fields lines that guide it spread by order $\lambda_\perp$. By then the energy it carries has cascaded to smaller scales, ultimately to be dissipated as heat at the inner scale. This process occurs not only for waves that are part of the turbulent cascade, but also for any other Alfvén waves in the medium. As these waves travel along the field lines, they are distorted in collisions with oppositely-directed turbulent wavepackets.

On a perpendicular scale $\lambda_\perp$, the field lines spread by order unity over a parallel distance $\Lambda_\parallel$, where $\Lambda_\parallel(\lambda_\perp)$ is a property of the background turbulence and is given by equation (4). Therefore any wavepacket of perpendicular scale $\lambda_\perp$ cascades once it travels this distance. Because of the nature of the MHD cascade, this corresponds to many wave periods for a wave with $\lambda_\parallel \lesssim \lambda_\perp \ll \Lambda_\parallel$ (but to one wave period for waves shaped like those in the turbulent cascade, as described by critical balance). The damping rate is a function of $\lambda_\perp$:

$$\Gamma_{\rm turb} \sim \frac{1}{t_{\rm cascade}(\lambda_\perp)} \sim \frac{v_{\lambda_\perp}}{\lambda_\perp} \sim \frac{v_{\rm A}}{L_{\rm mhd}^{1/3}\lambda_\perp^{2/3}} \sim \frac{\epsilon^{1/3}}{\lambda_\perp^{2/3}}. \tag{5}$$

This damping rate applies to any wave with perpendicular wavelength $\lambda_\perp$ propagating in a background of strong MHD turbulence, so long as $L_{\rm mhd} \gg \lambda_\perp \gg l_{\rm dissipation}$. The appropriate value of $\lambda_\perp$ to use for CR-generated waves will be considered in §3.2.

3. Competition between growth and damping

3.1. Resonant scattering of cosmic rays

As cosmic rays stream along magnetic field lines, they are scattered in pitch angle by magnetic irregularities (Alfvén waves, of appropriate shape; see below), and thus exchange momentum (and energy) with particular waves. If cosmic rays stream faster than the Alfvén speed, they can excite Alfvén waves traveling in the same direction. Provided the excitation rate exceeds the total damping rate due to other processes, the waves amplify exponentially. Initial perturbations too weak to significantly scatter CRs can strengthen until the scattering reduces the CR streaming velocity. Even thermal fluctuations could provide seed waves in the absence of other sources. The reduction of the streaming velocity by cosmic ray-amplified waves is known as self-confinement. Next we describe which random fluctuations are selectively amplified by cosmic ray protons with energy γ GeV.

3.1.1. Parallel lengthscale

Cosmic rays spiraling along a mean magnetic field $\mathbf{B_0} = B_0\hat{\mathbf{z}}$ scatter in pitch angle off Alfvén waves with which they are parallel-resonant, i.e. waves for which

$$k_{\parallel} = \frac{1}{\mu r_L}, \tag{6}$$

where μ is the cosine of the particle's pitch angle, and r_L is its gyroradius (Kulsrud & Pearce 1969; Wentzel 1974). On the timescale of the CR's passage, the wave is almost static since the CR is relativistic and $v_{\rm A} \ll c$. Thus the wave's time dependence is neglected in the above resonance condition. When resonance holds, the cosmic ray experiences a steady direction-changing force.

3.1.2. Perpendicular lengthscale

A cosmic ray is most efficiently scattered by parallel-propagating waves, $\lambda_\perp \gg \lambda_\parallel \sim r_L$, because in these the direction changing force maintains a steady direction in one gyroperiod. Moving through waves that have significant perpendicular components, $\lambda_\perp \ll \lambda_\parallel$, the cosmic ray traverses many perpendicular wavelengths, leading to oscillations of the direction-changing force and inefficient scattering. This explains why cosmic rays are weakly scattered by MHD turbulence (see e.g. Chandran 2000a; Yan & Lazarian 2002), and also why the waves in the turbulent cascade damp faster than cosmic rays can excite them.

The closer to parallel waves propagate, the faster streaming cosmic rays can excite them. The growth rate for waves that are parallel-resonant and reasonably close to parallel-propagating ($\lambda_\parallel \lesssim \lambda_\perp$) is given by (see Kulsrud & Pearce 1969)

$$\Gamma_{\rm cr}(k_\parallel) \sim \Omega_0 \frac{n_{\rm cr}(>\gamma)}{n_{\rm i}} \left(\frac{v_{\rm stream}}{v_{\rm A}} - 1 \right), \tag{7}$$

where $v_{\rm stream}$ is the net streaming velocity of the cosmic rays measured in the rest frame of the ISM, $\Omega_0 = eB_0/mc$ is the CR cyclotron frequency in the mean field, $n_{\rm i}$ is the ion number density in the ISM, and $n_{\rm cr}(>\gamma)$ is the number density of cosmic rays with gyroradius $r_L > \gamma mc^2/eB_0 = 1/k_\parallel$, i.e. those particles which can, for the appropriate value of μ, be resonant with waves of parallel wavevector $k_\parallel$. Because the cosmic ray energy spectrum is steep, the energies of most resonant particles are close to the lowest energy that permits resonance with the wave. Therefore we associate $k_\parallel \sim 1/r_L(\gamma)$ and $n_{\rm cr}(>\gamma) \simeq \gamma n_{\rm cr}(\gamma)$.[5]

[5]Particles with close to 90-degree pitch angles ($\mu \ll 1$) are scattered mainly by mirror interactions (Felice & Kulsrud 2001).

3.2. Growth and damping

Growth rates are highest, and damping rates lowest, for the most closely parallel-propagating waves, that is, those waves with largest $\lambda_\perp$. Therefore, we consider the limiting case of the most parallel-propagating wave that can be excited. This most-parallel wave sets the minimum streaming velocity required for the instability to operate. The limit to parallel propagation is set by the turbulent background magnetic field: the largest wave aspect ratio possible is fixed by the straightness of the field lines. In the presence of MHD turbulence, the field direction depends on position. The change in direction across scale $\lambda_\perp$ is set by turbulent field fluctuations on this scale. It is not meaningful to talk about waves propagating at an angle less than $\delta B(\lambda_\perp)/B_0$ away from parallel, because the field direction changes by this much across the wavepacket. We can therefore only have waves with

$$\frac{\lambda_\parallel}{\lambda_\perp} > \frac{\delta B(\lambda_\perp)}{B_0} \sim \left(\frac{\lambda_\perp}{L_{\rm mhd}}\right)^{1/3} \sim \left(\frac{\epsilon r_L}{v_{\rm A}^3}\right)^{1/4}, \tag{8}$$

where we have used $\lambda_\parallel \sim r_L$, the resonance condition.

To obtain the damping rate of the most closely parallel-propagating wave, we substitute equation (8) into equation (5), which yields

$$\Gamma_{\rm turb,min} \sim \left(\frac{\epsilon}{r_L v_{\rm A}}\right)^{1/2}; \tag{9}$$

all other waves damp faster than this one, for given r_L.

We can view the damping as being due to the introduction of perpendicular wavevector components to the CR-generated wave. This is how the background turbulence cascades, and the CR-generated wave is being integrated into the cascade. We can decompose the modified wave into components with almost-perpendicular and almost-parallel wavevectors. The perpendicular part is not excited and is more strongly damped, but the almost-parallel-propagating component continues to be amplified by resonant cosmic rays.

For the instability to operate, we require the maximum possible growth rate (eq. [7]) to be larger than the minimum damping rate (eq. [9]):

$$\Gamma_{\rm cr}[F_{\rm A}(\gamma)] > \Gamma_{\rm turb,min}(\gamma). \tag{10}$$

where $F_{\rm A}(\gamma) \sim (v_{\rm stream} - v_{\rm A}) n_{\rm cr}(> \gamma)$ is the cosmic ray flux measured in the frame moving with the waves. Equation (10) can be written in the form $F_{\rm A}(\gamma) > F_{\rm crit}(\gamma)$. If $F_{\rm A}$ is less than $F_{\rm crit}$ then wave amplification does not occur and the cosmic rays are not significantly scattered. Equivalently, the resonant streaming instability cannot reduce $F_{\rm A}$ below $F_{\rm crit}$.

If the instability is to confine cosmic rays to regions of shock acceleration, or to the Galaxy (which we discuss in the next section), then the level of background turbulence must be low enough to permit the growth of resonant waves.

4. Application to cosmic ray self-confinement in the ISM

Cosmic rays are preferentially produced in the denser regions of the Galaxy, and they escape from its edges. Two lines of evidence imply that they do not stream freely out of the Galaxy: the CR flux in the solar neighborhood is observed to be isotropic to within ~0.1% at energies less than $\sim 10^6$ GeV, and the abundance of the unstable nucleus ^{10}Be produced by spallation establishes that CRs are confined within the Galaxy for $\sim 10^7$ years (Schlickeiser 2002).

Scattering by Alfvén waves has been viewed as the leading mechanism for confinement. Both waves associated with background MHD turbulence and those resonantly excited by cosmic rays have been considered in this regard. Prior to the recognition that MHD turbulence is anisotropic, the former were generally favored. Now self-confinement appears to be the more viable option.

The most promising location for the operation of the streaming instability is the hot ISM (HISM), a.k.a. the coronal gas (Cesarsky & Kulsrud 1981; Felice & Kulsrud 2001). The abundances of cosmic ray nuclei produced by spallation suggest that cosmic rays spend about two-thirds of their time in this medium (see e.g. Schlickeiser 2002). Ion-neutral damping of waves is ineffective in the HISM. The coronal gas is hot ($T \sim 10^6$ K) and tenuous ($n_{\rm i} \sim 10^{-3}$ cm^{-3}), with an Alfvén velocity, assuming $B_0 \sim 3\ \mu$G, of $v_{\rm A} \sim 2 \times 10^7$ cm s^{-1}. The gyroradius of a relativistic proton in this field, $r_L \sim 10^{12}\gamma$ cm, lies within the inertial range of the MHD cascade.

Assuming the cosmic ray density in the HISM to be similar to that near the Sun,[6] $n_{\rm cr}(>\gamma) \simeq 2 \times 10^{-10}\gamma^{-1.6}$ cm^{-3} (Wentzel 1974), we can calculate the velocity above which the streaming instability in the HISM would turn on, assuming our turbulent damping to be the dominant damping mechanism. To accomplish this, we substitute equations (7) and (9) into inequality (10), treating it as an equality. We find

$$v_{\rm stream} \sim v_{\rm A}\left[1 + \frac{n_{\rm i}}{n_{\rm cr}(\gamma)}\frac{\omega_0}{\Omega_0}\left(\frac{L_{\rm mhd}}{r_L}\right)^{1/2}\right]$$

[6] If $n_{\rm cr}$ is lower than it is near the Sun, then confinement will begin to be problematic at lower energies, and vice versa

$$\sim \quad v_{\rm A}\left[1+\left(\frac{\epsilon}{700 \text{ erg s}^{-1}\text{ g}^{-1}}\right)^{1/2}\gamma^{1.1}\right] \ , \tag{11}$$

where $\omega_0 = v_{\rm A}/L_{\rm mhd}$ is the turbulent decay rate on the outer scale.

The mean rate at which turbulent dissipation heats the coronal gas is unlikely to exceed its radiative cooling rate, $\epsilon \sim 0.06$ erg g^{-1} s^{-1} for solar abundances (Binney & Tremaine 1987, p580). Unfortunately, we do not know whether the heating is continuous or episodic, and what fraction is due to shocks as opposed to turbulence.[7]

Roughly one supernova explosion occurs per century in the Galaxy, or on average one per square 100 pc of the disk every 1×10^6 yr. Turbulence injected with $v \sim v_{\rm A}$ on scales $L \sim 100$ pc decays in a time $L/v_{\rm A} \sim 5 \times 10^5$ yr, so it might be replenished before decaying. However, supernovae occur predominantly in the Galactic plane and it is uncertain how effective they are in stirring the coronal gas, which has a large vertical scale height. Suppose that each supernova releases 10^{51} erg of mechanical energy that is ultimately dissipated by turbulence. This amounts to a dissipation rate of 3×10^{41} erg s^{-1} which, if evenly distributed by volume throughout a disk of radius 10 kpc and thickness 1 kpc, would provide a mean heating rate of $\bar{\epsilon} \sim 25$ erg s^{-1} g^{-1} in the HISM. This value is much greater than our estimate of the radiative cooling rate.

The cosmic ray anisotropy measured locally is $\lesssim 0.1\%$ for $\gamma \lesssim 10^6$ (Schlickeiser 2002), i.e. up to the "knee" in the CR energy spectrum. The Alfvén velocity in the HISM is of the same order as the local streaming velocity: $v_{\rm A}/c \simeq 0.1\%$. Substituting into equation (11) the value of ϵ obtained by balancing the radiative cooling of the hot gas with heating due to steady state turbulent dissipation, we obtain

$$v_{\rm stream} \sim v_{\rm A}(1 + 9 \times 10^{-3}\gamma^{1.1}). \tag{12}$$

Equation (12) suggests that self-confinement in the HISM might account for the small observed cosmic ray anisotropy up to $\gamma \lesssim 10^2$, but not much beyond. To limit the streaming velocity of protons with $\gamma \sim 10^6$ to $\sim v_{\rm A}$ would require the turbulent dissipation rate to be astonishingly low, $\epsilon \lesssim 4 \times 10^{-11}$ erg s^{-1} g^{-1}.

4.1. Comparison with previous work

That background MHD turbulence might be an impediment to the self-confinement of cosmic rays was mentioned briefly in Lazarian et al. (2002), Yan & Lazarian (2002) and Cho

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[7]It seems plausible that shocks, especially if they intersect, would efficiently excite turbulence.

et al. (2003).

Kulsrud (1978) proposed non-linear Landau damping as the dominant damping mechanism for cosmic ray-generated waves in the HISM. Wave damping occurs when plasma ions "surf" on beat waves produced by the superposition of CR-generated waves. The damping rate for this process,[8] for similar HISM parameters as adopted in this paper, gives $v_{\rm stream} \simeq v_{\rm A}(1 + 0.05\gamma^{0.85})$ (Cesarsky & Kulsrud 1981). This predicted streaming velocity is not very different from that obtained in equation (12). Both damping mechanisms are too strong to permit self-confinement to reduce the streaming velocity of high energy cosmic rays to the locally observed levels.

Chandran (2000b) proposes that magnetic mirror interactions in dense molecular clouds may provide confinement of high energy cosmic rays. The present paper provides further support for the idea that a confinement mechanism other than scattering by Alfvén waves is dominant for high energy cosmic rays.

5. Conclusions

A background of anisotropic MHD turbulence acts as a linear damping mechanism for MHD waves excited by the streaming of cosmic rays. Low energy cosmic rays are numerous enough to excite Alfvén waves in the HISM when streaming at velocities compatible with observational limits on their anisotropy. However, high energy Galactic cosmic rays could only be self-confined to stream this slowly if turbulent dissipation in the HISM accounted for only a tiny fraction of its heat input.

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