

## FOLDED FIELDS AS THE SOURCE OF EXTREME RADIO-WAVE SCATTERING IN THE GALACTIC CENTER

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### ABSTRACT

A strong case has been made that radio waves from sources within about half a degree of the Galactic center undergo extreme diffractive scattering. However, problems arise when standard (“Kolmogorov”) models of electron density fluctuations are employed to interpret the observations of scattering in conjunction with those of free-free radio emission. Specifically, the outer scale of a Kolmogorov spectrum of electron density fluctuations is constrained to be so small that it is difficult to identify an appropriate astronomical setting. Moreover, an unacceptably high turbulent heating rate results if the outer scale of the velocity field coincides with that of the density fluctuations. We propose an alternative model based on folded magnetic field structures that have been reported in numerical simulations of small-scale dynamos. Nearly isothermal density variations across thin current sheets suffice to account for the scattering. There is no problem of excess turbulent heating, because the outer scale for the velocity fluctuations is much larger than the widths of the current sheets. We speculate that interstellar magnetic fields could possess geometries that reflect their origins: fields maintained by the Galactic dynamo could have large correlation lengths, whereas those stirred by local energetic events might exhibit folded structures.

*Subject headings:* Galaxy: center — ISM: general — radio continuum: ISM — scattering

### 1. INTRODUCTION

It appears that radio waves from sources within about half a degree of the Galactic center (GC) undergo extreme diffractive scattering. Observations of Sgr A\* (Davies et al. 1976; Rogers et al. 1994) and of maser spots in several OH/IR stars (van Langevelde et al. 1992; Frail et al. 1994) have established that the angular broadening is  $\theta_{\text{obs}} \approx 1''$  at  $\nu \approx 1$  GHz. If the scattering region is located more than a few kiloparsecs from the GC, this would correspond to enhanced, but not unusually large, levels of scattering; NGC 6334B is the record holder, with a scatter-broadened disk of size  $\approx 7''$  at  $\nu \approx 1$  GHz (Moran et al. 1990). However, if the scattering region is located close to the GC (say, within  $1^\circ$ , corresponding to a distance  $R \approx 150$  pc from the GC), then the GC region would be a site of extreme scattering. Van Langevelde et al. (1992) argue that if the scattering region is also responsible for the free-free absorption toward the GC, then upper limits on the optical depth (Pedlar et al. 1989; Anantharamaiah et al. 1991) constrain  $R$  to be in the range 0.85–3 kpc. In more recent work, Lazio & Cordes (1998, hereafter LC98) make the case that  $R \approx 130$  pc. Since the Sun’s distance from the GC is  $\approx 8.5$  kpc, this would imply that the angular scattering is larger than the observed angular broadening by a factor  $8500/130 \approx 65$ :

$$\theta_{\text{scat}} \approx 65R^{-1}\theta_{\text{obs}} \sim 3 \times 10^{-4}R^{-1}\nu_9^{-2}, \quad (1)$$

where we have set  $R = 130R$  pc and  $\nu_9 \equiv \nu/(1 \text{ GHz})$ . Spergel & Blitz (1992) estimate the gas pressure in the Galactic bulge to be  $p \equiv nT \sim 5 \times 10^6 \text{ K cm}^{-3}$ . Studies based on X-ray emission from the GC (Muno et al. 2004) suggest that, even on the smaller spatial scales of interest to us, the gas pressure may not be very different. A pressure of  $5 \times 10^6 \text{ K cm}^{-3}$  is about  $10^{2.5}$  times higher than in the local interstellar medium, and hence it may not seem unreasonable to expect the GC to be

a region of extreme scattering for radio waves. However, as LC98 demonstrate, problems arise when standard models of electron density fluctuations are employed to interpret the observations of scattering in conjunction with those of free-free radio emission.<sup>3</sup>

### 2. PROBLEMS WITH THE STANDARD INTERPRETATION

LC98 estimated the brightness temperature of free-free emission toward five highly scattered OH maser sources near the GC, based on the 10 GHz survey of Handa et al. (1987). From Table 3 of LC98, we derive a mean brightness temperature  $T_b \approx 0.25 \text{ K}$  at  $\nu = 10 \text{ GHz}$ . Since the region is optically thin, the optical depth  $\tau_{\text{ff}} \approx (T_b/T) \approx 2.5 \times 10^{-5}T_4^{-1}$ , where  $T_4 \equiv T/(10^4 \text{ K})$  is the gas temperature. Expressed in terms of the emission measure (EM),  $\tau_{\text{ff}} \approx 5 \times 10^{-7}\nu_9^{-2}T_4^{-3/2}\text{EM}_1$ , where  $\text{EM}_1 \equiv \text{EM}/(1 \text{ pc cm}^{-6})$  and we have set the averaged Gaunt factor  $\bar{g}_{\text{ff}} \approx 10$ . Hence,

$$\text{EM} \sim 5 \times 10^3 T_4^{1/2} \text{ pc cm}^{-6}. \quad (2)$$

The emission measure contributed by a medium with pressure  $p = 5 \times 10^6 \mathcal{F} \text{ K cm}^{-3}$  is  $\text{EM} \sim fn^2R \approx 3.3 \times 10^7 f\mathcal{F}^2 T_4^{-2} \mathcal{R} \text{ pc cm}^{-6}$ , where  $f$  is the volume filling factor. Provided that all the gas that contributes to the EM is at temperature  $T_4$ ,  $f \sim 1.5 \times 10^{-4} T_4^{5/2} \mathcal{F}^{-2} \mathcal{R}^{-1}$  and the mass of this gas is  $M \sim 1.5 \times 10^4 T_4^{3/2} \mathcal{F}^{-1} \mathcal{R}^2 M_\odot$ . Obviously, the constraint  $f \lesssim 1$  implies  $T_4 \lesssim 30 \mathcal{F}^{4/5} \mathcal{R}^{2/5}$ .

In the standard interpretation of radio-wave scattering in the interstellar medium, the electron density fluctuations are assumed to follow a Kolmogorov spectrum,  $\Delta n_l/n \approx (l/L)^{1/3}$  for  $l_{\text{min}} \leq l \leq L$ . For Sgr A\*,  $\theta_{\text{scat}} \propto \nu^{-2}$  for  $\nu_9 \lesssim 30$ . The  $\nu^{-2}$  scaling implies that the scales contributing dominantly to strong, diffractive scattering are smaller than the inner scale:  $l_{\text{min}} > \lambda/\theta_{\text{scat}}$ , where  $\lambda = c/\nu$  is the wavelength of the radio waves.<sup>4</sup> Because

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<sup>3</sup> Similar, but less severe, problems arise for NGC 6334B, which lies behind a Galactic H II region (Moran et al. 1990).

<sup>4</sup> If scattering were dominated by scales between  $l_{\text{min}}$  and  $L$ , we would have  $\theta_{\text{scat}} \propto \nu^{-2.2}$ .

$\lambda/\theta_{\text{scat}} \propto \nu$ , we obtain a lower limit on  $l_{\text{min}}$  that is higher at higher frequencies. Using equation (1) with  $\nu_p = 30$  gives  $l_{\text{min}} \geq 3 \times 10^6 \mathcal{R}$  cm. The scattering angle  $\theta_{\text{scat}} \sim (\nu_p/\nu)^2 \times (fR/L)^{1/2} (L/l_{\text{min}})^{1/6}$ , where  $\nu_p \approx 10^4 [n/(1 \text{ cm}^{-3})]^{1/2}$  Hz is the plasma frequency. In this expression for  $\theta_{\text{scat}}$ , we can write  $fR \approx EM/n^2$ . Using equations (1) and (2) for  $\theta_{\text{scat}}$  and EM, the condition  $l_{\text{min}} \geq 3 \times 10^6 \mathcal{R}$  cm implies that

$$L \lesssim 3 \times 10^{-8} T_4^{3/4} \mathcal{R}^{5/2} \text{ pc.} \quad (3)$$

The upper limit to  $L$  is remarkably small and independent of the pressure.<sup>5</sup> Moreover, if only a fraction  $F$  of the plasma that contributes to the free-free emission is responsible for the scattering,  $L$  would be smaller by a factor  $F^{3/2}$ . This, in essence, is LC98's argument. Henceforth we set  $\mathcal{F} = \mathcal{R} = 1$ .

The choice of a small outer scale, constrained by equation (3), introduces two problems. As was realized by LC98, it implies an unacceptably high heating rate if  $L$  is also the outer scale of the velocity fluctuations. Dissipation by Kolmogorov turbulence would heat the gas at a rate  $t_{\text{turb}}^{-1} \sim c_s/L > 10^{-5} T_4^{-1/4} \text{ s}^{-1}$ , whereas radiative cooling would be much slower,  $t_{\text{cool}}^{-1} \sim 3 \times 10^{-7} T_4^{-2} \Lambda_{-21} \text{ s}^{-1}$  [here  $c_s \sim 10 T_4^{1/2} \text{ km s}^{-1}$  is the sound speed and  $\Lambda_{-21} \equiv \Lambda/(10^{-21} \text{ ergs s}^{-1} \text{ cm}^3)$ ;  $\Lambda_{-21} \approx 1$  when  $T_4 \approx 10$ ]. It is also difficult to identify an appropriate astronomical setting in which nonlinear density fluctuations on a scale as small as  $L$  might arise. We have considered radiative shocks and interfaces between neutral and ionized gas formed by ionizing radiation or hot gas incident upon the surfaces of a molecular clouds. Even at the high pressure in the GC region, each case yields a length scale that is at least a few orders of magnitude larger than  $L$ .

### 3. FOLDED FIELDS AS A SOLUTION

Folded magnetic field structures appear in numerical simulations of small-scale, turbulent dynamos (Schekochihin et al. 2004). Of relevance here are the electrical conductivity,  $\sigma \approx 10^{13} T_4^{3/2} \text{ s}^{-1}$ , and the kinematic viscosity,  $\nu_{\text{vis}} \approx 2 \times 10^{15} T_4^{7/2} \text{ cm}^2 \text{ s}^{-1}$ , for plasma with gas pressure  $p \approx 5 \times 10^6 \text{ K cm}^{-3}$ . The magnetic Prandtl number,  $\text{Pr}_m \equiv 4\pi c^{-2} \sigma \nu_{\text{vis}} \approx 3 \times 10^8 T_4^5$ , is large, so dissipation of velocity fields occurs on much larger scales than dissipation of magnetic fields.<sup>6</sup> Schekochihin et al. (2004) have proposed a model for the organization of magnetic fields that appears attractive; we describe this briefly below, before considering its implications for density fluctuations.

Let an incompressible fluid, permeated by a weak magnetic field, be stirred on an outer scale  $L$  with random velocity  $v_L$ . Within a few stirring times,  $\tau_L \sim (L/v_L)$ , the kinetic energy cascades turbulently and creates velocity fluctuations on smaller spatial scales through nonlinear hydrodynamic interactions. The rms velocity across a scale  $l$  may be expected to follow a Kolmogorov spectrum,  $v_l \sim v_L (l/L)^{1/3}$ , for  $l_{\text{vis}} \leq l \leq L$ , where the inner scale is  $l_{\text{vis}} \sim L(\nu_{\text{vis}}/Lv_L)^{3/4} \ll L$ . Eddies of scale  $l$  turn over on times  $\tau_l \sim (l/v_l) \sim \tau_L (l/L)^{2/3}$ . Hence the early evolution of the weak magnetic field will be dominated by the stretching

<sup>5</sup> The problem of a small outer scale for density fluctuations arises more generally. Anantharamaiah & Narayan (1988) estimated that enhanced scattering in the inner Galaxy required fractional density fluctuations  $\sim 10$  on the outer scale. For a Kolmogorov spectrum, the same enhancement may be achieved with order-unity fractional density fluctuations, with an outer scale that is  $10^3$  times smaller. In this case, the outer scale of the velocity fluctuations can be much larger than the outer scale of the density fluctuations.

<sup>6</sup> These scalar transport coefficients only apply in directions parallel to the local magnetic field.

action of the smallest eddies, of size  $\sim l_{\text{vis}}$ , because their turnover time is the shortest. These eddies cease to be effective at stretching the field lines when the magnetic energy density becomes comparable to their kinetic energy density, that is, when  $B^2 \sim m_p n v_{\text{vis}}^2$ . Larger and more energetic eddies then take over, and the magnetic energy density continues to grow until it achieves approximate equipartition with the kinetic energy of the largest eddies:  $B^2 \sim m_p n v_L^2$ . It is the geometry of the magnetic field that is of particular interest to us. According to Schekochihin et al. (2004), it has a folded structure, with parallel correlation length  $\sim L$ : after a distance  $L$ , a typical field line reverses direction sharply and folds back on itself. Sheets of direction-reversing folded fields of thickness  $d$  are separated by current sheets of similar thickness,<sup>7</sup>

$$d \sim \left( \frac{c^2}{4\pi\sigma} \tau_L \right)^{1/2} \sim 10^{10} T_4^{-3/4} \tau_6^{1/2} \text{ cm,} \quad (4)$$

where  $\tau_6 \equiv \tau_L/(1 \text{ Myr})$ . For an isothermal gas at fixed total pressure, the fractional density perturbation across a current sheet of thickness  $d$  is

$$\frac{\Delta n}{n} \sim \frac{B^2}{8\pi n k T} \equiv \beta^{-1}. \quad (5)$$

Next we estimate scattering of radio waves by folded fields. Consider the idealized case of an ensemble of plane-parallel current sheets, each of radius  $L$  and thickness  $d \ll L$ . Let the sheets fill space statistically homogeneously with filling factor  $f$  and be oriented randomly.<sup>8</sup> The rms phase difference across a transverse scale  $d$ , accrued along a path length  $R \gg L$ , can be estimated by imagining a ray going through  $R/L$  independently oriented plates, of which only a fraction  $d/L$  are oriented favorably enough to each contribute to the phase a path length  $\sim L$ :

$$\Delta\Phi \sim \left( \frac{\nu_p}{\nu} \right)^2 \frac{\Delta n}{n} \frac{L}{\lambda} \left( f \frac{R}{L} \frac{d}{L} \right)^{1/2} = \left( \frac{\nu_p}{\nu} \right)^2 \frac{\Delta n}{n} \left( f \frac{Rd}{\lambda^2} \right)^{1/2}. \quad (6)$$

For a more realistic case, we take the sheets to have radii of curvature  $r_c$ , perhaps caused by Alfvén waves, which can propagate along folded fields. To estimate the rms phase difference for  $r_c > L$ , there are two limits to consider. For  $L < (dr_c)^{1/2}$ , the situation is similar to that for plane-parallel sheets and  $\Delta\Phi$  is given by equation (6). For  $(dr_c)^{1/2} < L < r_c$ , a ray goes through  $R/L$  independently oriented plates, of which a larger fraction  $L/r_c$  are oriented favorably enough for each plate to contribute path length  $\sim (dr_c)^{1/2}$ . Therefore, the expression for  $\Delta\Phi$  remains unchanged so long as  $r_c > L$ . We can also arrive at the above conclusions more formally by estimating the phase structure function, due to isotropically oriented sheets (see Appendix). The angular scattering,  $\theta_{\text{scat}} \sim (\lambda/d) \Delta\Phi \sim (\nu_p/\nu)^2 (fR/d)^{1/2} \beta^{-1}$ , depends on  $L$  through  $d$  but is otherwise independent of  $L$ . We

<sup>7</sup> Folded fields in schematic form, and as produced in a simulation, are shown in Figs. 10 and 15 of Schekochihin et al. (2004).

<sup>8</sup> A modest nonrandom orientation could account for the anisotropic images of scatter broadened sources near the GC.

write  $fR \approx (EM/n^2)$  and use equation (4) for  $d$  to obtain a general expression for the angular scattering:

$$\theta_{\text{scat}} \sim \left(\frac{v_p}{v}\right)^2 \left(\frac{EM}{n^2 d}\right)^{1/2} \beta^{-1} \\ \sim 2 \times 10^{-6} EM_1^{1/2} T_4^{3/8} \tau_6^{-1/4} v_9^{-2} \beta^{-1}. \quad (7)$$

This is our main result. It is independent of the geometric distribution of the scattering material, but this deserves further attention.

The scattering material must cover most of the area as seen from outside  $R$ . Let us suppose that folded fields fill an approximately spherical shell of radius  $R$  and thickness

$$\Delta R \sim fR \sim 2 \times 10^{-2} T_4^{5/2} \mathcal{F}^{-2} \text{ pc}. \quad (8)$$

Assuming that turbulent stirring occurs at sonic speeds,  $v_L \sim 10 T_4^{1/2} \text{ km s}^{-1}$ , we find

$$\tau_6 \sim 2 \times 10^{-3} T_4^2 \left(\frac{L}{\Delta R}\right). \quad (9)$$

Substituting this value of  $\tau_6$  and equation (2) for  $EM$  into equation (7), we obtain

$$\theta_{\text{scat}} \sim 5 \times 10^{-4} \left(\frac{T_4^{1/8}}{v_9^2 \beta}\right) \left(\frac{\Delta R}{L}\right)^{1/4}, \quad (10)$$

which compares well with the value of  $\theta_{\text{scat}}$ , derived from observations, given by equation (1). The thickness of the current sheets can be estimated by substituting equation (9) for  $\tau_6$  in equation (4):

$$d \sim 6 \times 10^8 T_4^{1/4} \left(\frac{L}{\Delta R}\right)^{1/2} \text{ cm}. \quad (11)$$

The scattering is strong because

$$\Delta\Phi \sim \frac{d}{\lambda} \theta_{\text{scat}} \sim 10^4 \left(\frac{T_4^{3/8}}{v_9 \beta}\right) \left(\frac{L}{\Delta R}\right)^{1/4}. \quad (12)$$

One motivation for considering scattering from a shell of warm ionized gas is the presence of a lobe of emission surrounding the GC, discovered by Sofue & Handa (1984) in a 10 GHz survey. Comparison with the 5 GHz survey of Altenhoff et al. (1979) enabled them to decompose the emission into thermal and nonthermal components and to establish that the thermal component arises mostly from a shell-like feature (Sofue 1985). This is one of the larger of the many sources in the GC region (LaRosa et al. 2000), with an angular size exceeding a degree. Even so, its radius of  $\approx 80$  pc appears to be only a little more than half the value of  $\approx 130$  pc we assumed, based on LC98's location of the scattering region. This implies that  $\Delta R$  might be only a little larger than half the value given in equation (8). However, our estimates of  $\theta_{\text{scat}}$  and  $\Delta\Phi$  given above are hardly affected, because of their weak dependence on  $\Delta R$ .

#### 4. SUMMARY

Turbulence giving rise to folded fields may account for extreme scattering in the GC region because large fractional density fluctuations occur across the thickness of the current sheets,  $d$ , which is much smaller than the outer scale  $L$ . Although the current sheets are thin, overheating is not a problem. The ohmic dissipation rate per volume due to current sheets is  $\sim (cB/d)^2 \sigma^{-1} \sim B^2/\tau_L$ . If, as we have argued, the magnetic energy achieves near-equipartition with the kinetic energy, then  $B^2 \sim m_p n v_L^2$  and the ohmic dissipation rate is no larger than the turnover rate of the largest eddies:  $t_{\text{ohm}}^{-1} \sim \tau_L^{-1}$ , which is much smaller than  $t_{\text{cool}}^{-1}$ . The large cooling rate also ensures that the plasma behaves nearly isothermally, an assumption used in our estimate of density fluctuations across the thickness of the current sheets, given in equation (5).

In our discussion of radio-wave scattering, we assumed that the current sheets are oriented isotropically. It then proves convenient to formulate radio-wave scattering in terms of angle-averaged quantities. The effective isotropic power spectrum of electron density fluctuations turns out to be shallow, being proportional to  $k^{-2}$  (see eq. [A2]). Such a spectrum gives rise to large density fluctuations on spatial scales  $\sim d$  but contributes little to density fluctuations on large spatial scales. This difference provides an observational test of the folded-field hypothesis. Where diffractive and refractive scintillations have been detected in the same source, their relative magnitudes suggest that a Kolmogorov spectrum of density fluctuations extends up to at least the refractive scales. However, refractive scintillations are generally not detected in strongly scattered sources. This does not conflict with the Kolmogorov model, which predicts them to be both weak and slow, but it is also compatible with the folded-field hypothesis. Higher frequency observations could provide a decisive test. For a Kolmogorov spectrum, the fractional flux modulation,  $m$ , is on the order of the cube root of the ratio of the diffractive scale to the Fresnel scale. With parameters appropriate to Sgr A\*, we find  $m \sim 10^{-2} \mathcal{R}^{1/6} v_9^{1/2}$ .

Magnetic fields might possess different geometries in different regions of the interstellar medium, depending on their origin. Some regions could possess strong mean magnetic fields, correlated over large distances, generated, perhaps, by large-scale dynamos. These could be the sites of anisotropic Kolmogorov turbulence (Goldreich & Sridhar 1995), responsible for the general level of diffractive and refractive interstellar scintillation. There could be other sites in the interstellar medium, permeated with folded fields, generated by small-scale dynamos, contributing to extreme diffractive scattering, but little to refractive scattering; the Galactic center could be one such region.

We would like to inject a cautionary concluding note. Our model of scattering applies results on the growth of magnetic fields in small-scale dynamos from Schekochihin et al. (2004). These are based on MHD with scalar diffusivities. It is an open question whether these can be applied to the low-density magnetized plasma in the GC region. We expect that our estimate for magnetic diffusivity, which sets the thickness of the current sheets, is valid. But the reduction of the kinematic viscosity in directions perpendicular to the magnetic field is a concern. It is possible that folded fields would unwind so rapidly that they could not be maintained.

## APPENDIX

## DIFFRACTIVE SCATTERING BY AN ENSEMBLE OF CURRENT SHEETS

Here we offer a physical derivation of equation (6), by estimating the phase structure function due to randomly distributed and isotropically oriented current sheets. A useful intermediate step is to calculate  $C(r)$ , the effective isotropic density-density correlation function on separation  $r$ . Its Fourier transform is  $P(k)$ , the effective isotropic power spectrum of density fluctuations. Consider an ensemble of flat current sheets, each of thickness  $d$  and radius  $L \gg d$ , all oriented in the same direction with unit normal  $\hat{n}$ . The density-density correlation function is significantly nonzero only when the separation,  $\mathbf{r}$ , is such that  $|\mathbf{r} \cdot \hat{n}| < d$ , and  $r_{\perp} = |\mathbf{r} - \hat{n}(\mathbf{r} \cdot \hat{n})| < L$ . When averaged over all directions of  $\hat{n}$ , we will obtain  $C(r)$ . Equivalently, we may keep  $\hat{n}$  fixed and average over all directions of  $\mathbf{r}$ . When  $d < r < L$ , the solid angle of the region of intersection between the platelike region (of radius  $L$  and thickness  $d$ ) and a concentric sphere of radius  $r$ , is  $\sim (dr/r^2) \sim (d/r)$ . Hence,

$$C(r) \sim (\Delta n)^2 \frac{d}{r} \quad \text{for } d < r < L. \quad (\text{A1})$$

The corresponding effective isotropic power spectrum is “shallow”:

$$P(k) \sim (\Delta n)^2 \frac{d}{k^2} \quad \text{for } L^{-1} < k < d^{-1}. \quad (\text{A2})$$

Here  $P(k) \sim (\Delta n)^2 L^2 d$  when  $kL < 1$  and  $P(k) \sim 0$  for  $kd > 1$ . Let us consider the case in which the current sheets are curved, with radius of curvature  $r_c > L$ . The solid angle of the region of intersection between the curved current sheet-like region and a concentric sphere of radius  $r$  (in the space of separations)

is  $\sim (dr_{\perp}/r^2)$ . We estimate  $r_{\perp} \approx r(1 - r^2/8r_c^2) \approx r$ , because  $r/r_c < L/r_c < 1$ . Therefore the solid angle of intersection is  $d/r$ , and the expressions for  $C(r)$  and  $P(k)$  are as given above. As may be seen, the dominant contribution to density fluctuations comes from scales close to  $d$ .

In the thin-screen model of radio-wave scattering, the entire effect of the interstellar medium is specified by the (Gaussian) random phase pattern imprinted on a wave front as it passes through a “phase screen” placed between the source and the observer. The statistical properties of the screen are completely described by the phase structure function  $D(s)$ , which is defined as the mean square phase difference across transverse separation  $s$  on the screen. When the power spectrum is isotropic,  $D$  depends only on  $s = |s|$ :

$$D(s) = \frac{R}{\pi} \lambda^2 r_e^2 \int_0^{\infty} dk_{\perp} k_{\perp} [1 - J_0(k_{\perp} s)] P(k = k_{\perp}), \quad (\text{A3})$$

where  $r_e = e^2/m_e c^2$  is the classical electron radius. For the shallow spectrum of equation (A2), it is straightforward to make the estimate

$$D(s) \sim \begin{cases} (\Delta\Phi)^2 (s/d)^2, & \text{if } s < d, \\ (\Delta\Phi)^2, & \text{if } s > d, \end{cases} \quad (\text{A4})$$

where  $\Delta\Phi$  is given by equation (6). If  $\Delta\Phi < 1$ , the scattering is weak, whereas a range of scales (smaller than, and of order  $d$ ) can contribute to strong scattering when  $\Delta\Phi > 1$ .<sup>9</sup>

<sup>9</sup> We note that  $D(s) \propto s^2$  yields a Gaussian image, as observed. A Kolmogorov spectrum of density fluctuations also yields a Gaussian image, if the diffraction scale is smaller than the inner scale.

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