Gravitational self-force in the ultra-relativistic limit: The "large-N" expansion

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We study the gravitational self-force using the effective field theory formalism. We show that in the ultra-relativistic limit $\gamma \to \infty$, with γ the boost factor, many simplifications arise. Drawing parallels with the large N limit in quantum field theory, we introduce the parameter $1/N \equiv 1/\gamma^2$ and show that the effective action admits a well defined expansion in powers of $\lambda \equiv N\epsilon$ at each order in $1/N$, where $\epsilon \equiv E_m/M$ with $E_m = \gamma m$ the (kinetic) energy of the small mass. Moreover, we show that diagrams with nonlinear bulk interactions first enter at $\mathcal{O}(1/N^2)$ and only diagrams with nonlinearities in the worldline couplings, which are significantly easier to compute, survive in the large N/ultra-relativistic limit. As an example we derive the self-force to $\mathcal{O}(\lambda^4/N)$ and provide expressions for conservative quantities for circular orbits.

Introduction. During the last years a new formalism has emerged, based on effective field theory (EFT) ideas borrowed from particle physics, to study binary systems in General Relativity. Originally proposed to tackle the Post-Newtonian approximation for non-spinning [1] and spinning [2] inspirals, the EFT approach has produced a breakthrough in our understanding of gravitationally interacting extended objects within a relatively short period of time [1–19]. Meanwhile, EFT ideas were also applied (besides particle physics) to different areas, such as cosmology [20–22], electrodynamics [23], fluid dynamics [24, 25], and in particular to extreme mass ratio inspirals (EMRIs) [26–28], which is the subject of this letter.

The study of the self-force problem within the EFT approach was initiated in [26] where power-counting and leading order effects were worked out and a proof of the effacement of internal structure for EMRIs was given. As it is traditional in the EMRI community, perturbative calculations are performed in powers of $q \equiv m/M$ where m represents a small mass object orbiting a much larger black hole with mass M. To date, only second-order $\mathcal{O}(q^2)$ equations of motion are known [29, 30].

In this letter we study the ultra-relativistic limit, i.e. $\gamma \to \infty$, of the self-force problem. Inspired by an analogy with the large N limit in quantum field theory [31], we show that many simplifications arise that are not captured by the (traditional) m/M power-counting. We show that, introducing the expansion parameter $1/N \equiv \gamma^{-2}$ and defining $\lambda \equiv N\epsilon$ with $\epsilon \equiv E_m/M$ and $E_m = \gamma m$, the gravitational effective action (which yields the self-force) admits an expansion of the type

$$S_{\rm eff} = L/N\left(1 + \lambda + \lambda^2 + \dots\right) + \mathcal{O}(\lambda/N^2), \qquad (1)$$

where $L \sim E_m M (= \gamma m M)$ is the angular momentum of the small mass (in $G_N = c = 1$ units used throughout). A similar expansion applies to the one-point function, $h_{\mu\nu}(x)$, which can also be used to compute the self-force. Our goals here are: to derive the new power-counting rules in the large N limit, to show that diagrams with nonlinear bulk interactions are subleading in the $1/N$ expansion, to report the gravitational self-force to *fourth* order in λ at leading order in $1/N$, and to provide expressions for conservative quantities for the case of circular orbits. We conclude on a more formal note with some comments on the problem of finding the self-force in the exact massless limit for a photon moving in a black hole spacetime.

Power-counting rules. Our setup is the same as in the standard EMRI case except that we consider ultra-relativistic motion where the boost factor γ is large,

$$\gamma \equiv 1/\sqrt{-g_{\mu\nu}v^\mu v^\nu} \gg 1. \qquad (2)$$

Here, $v^\alpha \equiv dz^\alpha/dt$, z^α is the small mass' worldline coordinates, t is the coordinate time, and $g_{\mu\nu}$ is the background metric of the black hole with mass M. In order to achieve this condition we imagine the mass m in a bound orbit near the light ring in Schwarzschild spacetime or moving in a (very) rapid fly-by.

We next find the scaling rules of various leading order quantities. The orbital frequency is related to the wavelength of the gravitational radiation through $\omega_{\rm orb} = d\phi/dt \sim 1/\lambda_{\rm gw}$. Because $\lambda_{\rm gw} \sim M$ it follows that $dt \sim M$ (also $dx^i \sim M$). Hence, the proper time along the object's worldline scales like $d\tau \sim dt/\gamma \sim M/\gamma$ and its four-velocity is $u^\alpha \equiv dz^\alpha/d\tau = \gamma v^\alpha \sim \gamma$, for an ultra-relativistic motion. For the scaling of the metric perturbations $h_{\mu\nu}$ produced by this ultra-relativistic small mass m we use the leading order solution

$$h_{\mu\nu}(x) \sim \int_{x'} G_{\mu\nu\alpha'\beta'}(x,x')T^{\alpha\beta}(x'), \qquad (3)$$

with $T^{\alpha\beta}(x) \propto m\int d\tau\, \delta^4(x^\mu - z^\mu(\tau))u^\alpha u^\beta/\sqrt{-g}$, and $\int_x \equiv \int d^4x\sqrt{-g}$. We find $h_{\mu\nu} \sim E_m/M = \epsilon$ where we used $\nabla_\alpha \sim \partial_\alpha \sim 1/M$ and $G_{\mu\nu\alpha'\beta'} \sim 1/M^2$ for the scaling of the Green function in a curved background (this follows almost entirely from dimensional analysis). Finally, the leading order effective action scales like

$$S^0_{\rm pp}[z^\mu] = -m\int d\tau \sim mM/\gamma \sim L/N, \qquad (4)$$

as anticipated. The scaling rules are summarized below:

dz^α	∇_α	$d\tau$	u^α	$h_{\mu\nu}$	$G_{\mu\nu\alpha'\beta'}$	$S^0_{\rm pp}$
M	$1/M$	M/γ	γ	$\epsilon = E_m/M$	$1/M^2$	L/N

Because of these rules, the condition that perturbation theory is under control in the ultra-relativistic limit demands not only ϵ to be small but also $\gamma^2\epsilon \ll 1$. The reason is simple. After including the perturbation the point particle action scales like

$$S_{\rm pp}[z^\mu, h_{\mu\nu}] = -m\int d\tau\,\sqrt{1-\gamma^2 h_{\mu\nu}v^\mu v^\nu}, \qquad (5)$$

where we used (2). According to the scaling rules, $h_{\mu\nu} \sim \epsilon$, hence we must require $\lambda \equiv \epsilon\gamma^2 = \epsilon N \ll 1$ for the perturbation to be considered small with respect to the background. This is the regime of validity of our approximations. In other words, we formally take the limit $\epsilon \to 0, N \to \infty$ with λ fixed and small.

To obtain the different scalings for the possible terms that contribute to the self-force we first need to isolate the building blocks of our Feynman diagrams and power-count them. We have either worldline or bulk vertices, which we summarize next. Using our power counting rules we have for the vertex describing the interaction of the small object m with n gravitational perturbations:

$$\sim (m)\left(\frac{M}{\gamma}\right)\left(\gamma^2\right)^n = mM\gamma^{2n-1}, \qquad (6)$$

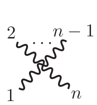
which arises from expanding the point particle action

$$S_{\rm pp} = m\sum_{n=0}^{\infty} c_n \int d\tau\, \left(h_{\alpha\beta}u^\alpha u^\beta\right)^n, \qquad (7)$$

where the $\{c_n\}$ are dimensionless numbers. Notice we *truncate* the external legs and we do not yet include the scaling for $h_{\alpha\beta}$, which ought to be contracted with worldline or bulk couplings and will introduce an extra factor of $G_{\mu\nu\alpha'\beta'} \sim M^{-2}$ for each propagator in a given diagram. Next, we need the bulk vertices that follow from expanding the (gauge-fixed) Einstein-Hilbert action in powers of $h_{\mu\nu}$ about the given background spacetime $g_{\mu\nu}$. At the $n^{\rm th}$ order this is given schematically by

$$S_{\rm EH} = \sum_{n=2}^{\infty}\int_x \nabla h\nabla h\, h^{n-2}, \qquad (8)$$

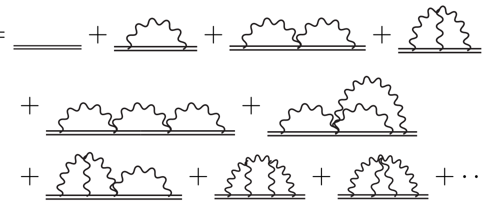
where ∇ indicates the covariant derivative. It is easy to show that the vertex for n interacting gravitational perturbations scales as:



$$\sim M^2, \qquad (9)$$

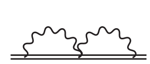
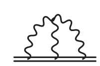
in four spacetime dimensions. This completes the power-counting rules for the building blocks of the EFT formalism. To compute the *classical* effective action we simply need to add up all possible *tree-level* diagrams. (By this we mean we do not include closed gravitational loops that represent quantum effects.) The effective action then takes the form

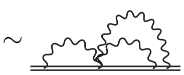
$$S_{\rm eff}[z^\mu] =$$

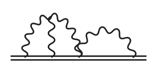
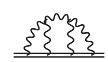
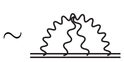


Using the rules previously derived we can power-count each diagram in the effective action, hence their contribution to the self-force. We show next that only diagrams without bulk nonlinear interactions survive in the large N limit. For that purpose it is illustrative to compare the scaling of the following diagrams, which enter to $\mathcal{O}(\lambda^3)$:

 $\sim \dfrac{L}{N}$,  $\sim \dfrac{\lambda L}{N}$ (10)

 $\sim \dfrac{\lambda^2 L}{N}$,  $\sim \dfrac{\lambda^2 L}{N^2}$ (11)

 $\sim$  $\sim \dfrac{\lambda^3 L}{N}$ (12)

 $\sim \dfrac{\lambda^3 L}{N^2}$,  $\sim$  $\sim \dfrac{\lambda^3 L}{N^3}$. (13)

We already start to see the pattern: bulk nonlinearites are suppressed in the large N limit. For a generic contribution let us consider a diagram with N_m mass insertions, N_v^k bulk vertices with k-legs and N_p propagators (including internal ones). From our power counting rules we obtain the scaling

$$(Mm/\gamma)^{N_m}\; M^{2(N_v^{\rm tot}-N_p)}\gamma^{2\left(2N_p - \sum_k kV_k\right)}, \qquad (14)$$

where $N_v^{\rm tot} = \sum_k N_v^k$ is the total number of bulk vertices. Let us first look at diagrams with $N_v^k = 0$. Using: $N_m + N_v^{\rm tot} - N_p - 1 = 0$, which follows from the topology of the diagrams that contribute in the classical limit, the expression in (14) turns into

$$\frac{L}{\gamma^2}\epsilon^{(N_m-1)}\gamma^{2(N_m-1)} = \frac{L}{N}\lambda^{(N_m-1)} \qquad (N_v^k = 0), \qquad (15)$$

and is thus a $1/N$ contribution. From a given order in λ (namely, N_m fixed) adding bulk vertices (and internal propagators) will only introduce powers of $1/N$ (see (14)) since we need at least two bulk vertices to increase the number of internal propagators and each bulk vertex has at least three legs. Intuitively this is simply because, for a fixed number of mass insertions, we lose powers of N from propagators attached to *two* worldline couplings, which are promoted to a bulk interaction. This is transparent in the terms depicted in (10)-(13).

The gravitational self-force in the large N limit. Self-force effects in EMRIs are intrinsically non-local, depending on the past history of the small object's motion around the larger black hole. Capturing these real-time dissipative interactions with an (effective) action requires a careful handling of Hamilton's variational principle of stationary action so that it is consistent with initial value data for open system dynamics (i.e., the motion of the small mass). This issue was emphasized in [32] where it was motivated by the classical limit of the "in-in" formalism [33]. A rigorous framework to handle this in a completely classical context was developed in [34] and applied to derive radiation reaction forces through 3.5 post-Newtonian order using the EFT method in [8]. We will elaborate on the details of this construction for the self-force problem in the large N limit elsewhere [35].

As we have shown, in the ultra-relativistic limit we can ignore all self-interactions of the metric perturbation that do not happen on the worldline. This means that the action for the small mass object and the metric perturbations can be taken as

$$S[z^\mu, h_{\mu\nu}] = -\frac{1}{64\pi}\int_x \left(h_{\alpha\beta;\mu}h^{\alpha\beta;\mu} - \frac{1}{2}h_{;\mu}h^{;\mu}\right) - m\int d\tau \sqrt{1 - h_{\alpha\beta}u^\alpha u^\beta}, \qquad (16)$$

where we imposed the Lorenz gauge for trace-reversed perturbations. For the reader worrying about finite size effects (e.g., $C_E \int d\tau\, \mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta}$) one can easily show they are highly suppressed in the large N expansion, first entering at $\mathcal{O}(\lambda^4 L/N^5)$. This has important consequences in the regularization of the theory because, as we shall argue, we will not encounter logarithmic divergences but only power-law, which will be handled via dimensional regularization (and set to zero since they involve scaleless integrals). We briefly discuss below the general procedure for calculating the relevant diagrams in the ultra-relativistic limit, and we will elaborate on technical issues elsewhere [35]. However, these steps follow closely those taken in a nonlinear scalar field model of EMRIs [28].

Computing the surviving diagrams in the effective action, or the diagrams for the one-point function $h_{\mu\nu}(x)$ below, involves worldline integrals over the retarded propagator,

$$I(z^{\mu'}) \equiv u^{\alpha'}u^{\beta'}\int_{-\infty}^{\infty} d\tau''\, G^{\rm ret}_{\alpha'\beta'\gamma''\delta''}(z^{\mu'}, z^{\mu''})u^{\gamma''}u^{\delta''}, \qquad (17)$$

which are in general divergent. Here, a prime on an index indicates the point or proper time that the quantity is being evaluated so that $u^{\alpha'} = u^\alpha(\tau')$, $u^{\gamma''} = u^\gamma(\tau'')$, etc. Following [36] we split this expression into a regular $G^R_{\alpha'\beta'\gamma''\delta''}$ and singular $G^S_{\alpha'\beta'\gamma''\delta''}$ piece, which allows us to isolate the part of (17) that produces the divergences. It is useful to write the singular integrals in a momentum space representation, which can be given whenever the two points on the worldline can be connected by a unique geodesic. Using the above decomposition one writes (17) as $I_S(z^{\mu'}) + I_R(z^{\mu'})$ where the singular and regular parts are, respectively, given by

$$I_S(z^{\mu'}) = 4u^{\alpha'}u^{\beta'}P_{\alpha'\beta'\gamma'\delta'}(z^{\mu'}) {\rm Re}\int_{-\infty}^{\infty} d\tau''\, u_{||}^{\gamma'} u_{||}^{\delta'} \times \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^d}\, \frac{e^{-ik^0(\tau''-\tau')}}{(k^0)^2 - \vec{k}^2 + i\epsilon} \qquad (18)$$

and

$$I_R(z^{\mu'}) = u^{\alpha'}u^{\beta'}\int d\tau''\, D^R_{\alpha'\beta'\gamma''\delta''}(z^{\mu'}, z^{\mu''})u^{\gamma''}u^{\delta''} \qquad (19)$$

where

$$\begin{aligned} &D^R_{\alpha'\beta'\gamma''\delta''}(z^{\mu'}, z^{\mu''}) \\ &\quad = \Theta(\tau'' - \tau_{\rm out})\Theta(\tau_{\rm in} - \tau'')G^{\rm ret}_{\alpha'\beta'\gamma''\delta''}(z^{\mu'}, z^{\mu''}) \\ &\qquad + \Theta(\tau_{\rm out} - \tau'')\Theta(\tau'' - \tau_{\rm in})G^R_{\alpha'\beta'\gamma''\delta''}(z^{\mu'}, z^{\mu''}). \end{aligned} \qquad (20)$$

The integral in (18) is written in d spacetime dimensions in momentum space where the momenta are dual to Fermi normal coordinates, and $u_{||}^{\gamma'} \equiv g^{\gamma'}{}_{\lambda''}(z^{\mu'}, z^{\mu''})u^{\lambda''}$ is the result of parallel propagating the velocity vector at $z^\mu(\tau'')$ to $z^\mu(\tau')$ using the propagator of parallel transport $g^{\gamma'}{}_{\lambda''}(z^{\mu'}, z^{\mu''})$. Also, $\tau_{\rm in}$ (and $\tau_{\rm out}$) are the proper time values at which the worldline enters (and leaves) the normal neighborhood of $z^\mu(\tau')$. See [37] for more details about bi-tensor calculus. As we mentioned, the singular term in (18) is easily shown to vanish in dimensional regularization because it is a (scale-independent) power-law divergent integral. As a consequence, the regularization of the theory becomes straightforward in the large N limit.

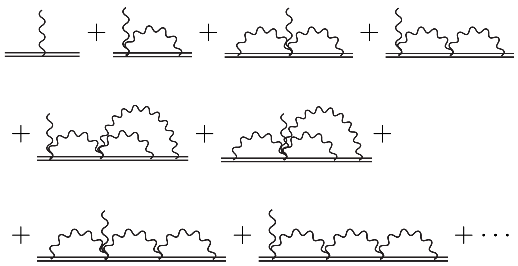
As outlined in [28], in a theory that has only worldline interactions as the relevant couplings, it is simpler to compute the metric perturbations at a field point, $h_{\mu\nu}(x)$, rather than the effective action since we can simply substitute the resulting perturbative expression into the worldline equations of motion (renormalizing parameters if necessary) to compute the self-force. We show the results next.

Metric perturbation & self-force to $\mathcal{O}(\lambda^4/N)$. In the ultra-relativistic limit the diagrams contributing to the one-point function can be represented as the convolution with a worldline *master source*,

$$h_{\mu\nu}(x) = \int d\tau'\, G^{\rm ret}_{\mu\nu\alpha'\beta'}(x, z^{\mu'})\mathcal{S}_R^{\alpha'\beta'}(z^{\mu'}), \qquad (21)$$

that is completely finite and given through $\mathcal{O}(\lambda^4 L/N)$ by computing the diagrams

$h_{\mu\nu}(x) =$



from which we find the master source to be:

$$
\begin{aligned}
\mathcal{S}_R^{\alpha'\beta'}(z^{\mu'}) &= \frac{m}{2}u^{\alpha'}u^{\beta'}\bigg\{1+\frac{m}{4}I_R(z^{\mu'})\\
&+\frac{3m^2}{32}I_R^2(z^{\mu'})+\frac{m^2}{16}u^{\gamma'}u^{\delta'}\!\int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''}u^{\epsilon''}u^{\eta''}I_R(z^{\mu''})\\
&+\frac{3m^3}{128}u^{\gamma'}u^{\delta'}\!\int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''}u^{\epsilon''}u^{\eta''}I_R^2(z^{\mu''})+\frac{5m^3}{128}I_R^3(z^{\mu'})\\
&+\frac{3m^3}{64}I_R(z^{\mu'})u^{\gamma'}u^{\delta'}\!\int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''}u^{\epsilon''}u^{\eta''}I_R(z^{\mu''})\\
&+\frac{m^3}{64}u^{\gamma'}u^{\delta'}\!\int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''}u^{\epsilon''}u^{\eta''}u^{\rho''}u^{\lambda''}\\
&\times\int d\tau''' D^R_{\rho''\lambda''\tau'''\sigma'''}u^{\tau'''}u^{\sigma'''}I_R(z^{\mu'''})+\cdots\bigg\}. \qquad (22)
\end{aligned}
$$

From the master source we can then compute the regular part of the metric perturbation evaluated on the worldline, i.e. $h^R_{\mu\nu}(z^\mu)$, by convolving (22) with $D^R_{\mu\nu\alpha'\beta'}$ in (20). In order to obtain the self-force we compute the worldline equations of motion via the variational principle on (16). We thus arrive at the desired result, in principle valid to all orders,

$$
\begin{aligned}
&\left(g_{\mu\nu}+h^R_{\mu\nu}+\frac{g_{\mu\alpha}+h^R_{\mu\alpha}}{1-h^R_{\gamma\delta}u^\gamma u^\delta}\,u^\alpha h^R_{\nu\beta}u^\beta\right)a^\nu\\
&\quad=\bigg(\frac{1}{2}u^\alpha u^\beta g_\mu{}^\nu-(g_\mu{}^\alpha u^\beta+g_\mu{}^\beta u^\alpha)u^\nu\\
&\qquad-\frac{1}{2}\frac{g_{\mu\gamma}+h^R_{\mu\gamma}}{1-h^R_{\epsilon\eta}u^\epsilon u^\eta}\,u^\alpha u^\beta u^\gamma u^\nu\bigg)\nabla_\nu h^R_{\alpha\beta}, \qquad (23)
\end{aligned}
$$

where $h^R_{\mu\nu}$ is evaluated on the worldline using the master source in (22), and we have absorbed a divergent piece into the mass m. (These divergences are set to zero in dimensional regularization; recall that there are no other counter terms at leading order in $1/N$.) The formal perturbative expression for the self-force can be easily found by expanding out (23) to the desired order (modulo some ambiguities discussed in detail in [35]).

Conservative self-force for circular orbits near light ring. On circular orbits in a Schwarzschild background the symmetry of the system implies that the regular integral in (19) is a function only of the orbital radius r_o, and is independent of time as long as we consider only the *conservative* part of the self-force, implying $I_R(z^\mu)=I_R(r_o)$. The master source in (22) then becomes, to $\mathcal{O}(\lambda^4/N)$,

$$
\mathcal{S}^{\alpha'\beta'}_{R,\frac{\lambda^4}{N}} = u^{\alpha'}u^{\beta'}\left\{\frac{m}{2}+\frac{m^2}{8}I_R+\frac{5m^3}{64}I_R^2+\frac{m^4}{16}I_R^3\right\}. \qquad (24)
$$

We can now use the above master source, and corresponding regularized metric perturbation $h^R_{\mu\nu}$ evaluated on the worldline, to obtain an (implicit) equation for the energy of the orbit, E, by solving

$$
\frac{1}{r_o}-\frac{r_o-3M}{r_o-2M}E^2=\frac{\frac{1}{2}u^\alpha u^\beta\nabla_r h^R_{\alpha\beta}}{1+\dfrac{h^R_{rr}}{r_o^2}+\dfrac{(h^R_{r\gamma}u^\gamma)^2}{r_o^2(1-h^R_{\delta\epsilon}u^\delta u^\epsilon)}}, \qquad (25)
$$

where we used the radial component of the (non-perturbative) equation of motion in (23). (The u^α and $h^R_{\alpha\beta}$ depend implicitly on E.) In principle we need to expand (25) perturbatively in powers of λ about the background energy E_0 of a circular geodesic. This will be given in detail in [35]. Unfortunately, we find that there are contributions (from $h^R_{rr}(r_o)$ and $h^R_{r\alpha}(r_o)u^\alpha$), starting already at $\mathcal{O}(\lambda^2/N)$, which have not yet been obtained numerically in the literature. To this extent, we expect our results will encourage the community to compute these terms in the future.

Concluding remarks. We have introduced the large N expansion for computing the gravitational self-force in the ultra-relativistic limit and shown that, at leading order in $1/N$, it reduces to a (mostly) combinatorial problem. As an example, we derived the self-force through fourth order and gave the (non-perturbative, implicit) expressions for the conserved energy for circular orbits near the Schwarzschild light ring. Our results are most useful the larger γ is, provided $N\epsilon=\gamma^3 q$ remains fixed and small. For example, in our computations, ignoring $1/N^2$ corrections requires $\lambda^2/N^2<\lambda^4/N$, or $1/\gamma^4<q$, while at the same time $\gamma^3 q<1$ for perturbation theory to stay under control. Therefore, our range of validity lies somewhere between $1/\gamma^4<m/M<1/\gamma^3$. This window obviously increases the less accuracy we demand.

The gravitational self-force has received significant attention lately, due to some surprising agreements with numerical results outside its range of validity (formally replacing $m/M\to mM/(m+M)^2$) [38–40]. These comparisons, however, only relied on leading order self-force effects. Our results in this letter open the door to check and improve such computations to very high orders in the large N limit. As it is often the case, these approximations may shed light on the dynamics in scenarios where γ is not significantly large, and perhaps even in cases where the mass ratio is not taken to be small. We leave this road open for future work. Our results should also be useful to further calibrate semi-analytic merger models from the ultra-relativistic regime (e.g., see [41]).

Let us finish by commenting on a more formal aspect of the ultra-relativistic limit. As it is well known, a boosted Schwarzschild black hole turns into an Aichelburg-Sexl (AS) shockwave in the ultra-relativistic limit with ϵ finite [42]. One simple way to recover this solution is using Polyakov's action ($S_{\rm Poly} = \int d\sigma \{\dot{x}(\sigma)^2/e - em^2\}$) [43], which is finite in the massless limit. A special feature of this point particle action is that it does not introduce worldline non-linearities, only bulk-type which are present through the Einstein-Hilbert action. However, all the non-linear terms cancel out for the AS solution, which is linear in G_N [42]. This is not the case in a black hole background (with finite mass M) because the shockwave can encounter its own "echoes" [44]. It would be interesting to study AS shockwave dynamics in non-trivial backgrounds as another approach to the ultra-relativistic self-force, for instance, to study the dynamics of light crossing a black hole, the merger process in binary systems [45] or to understand high-energy gravitational collisions [46, 47]. (For the case of photons, it would also be instructive to compare with the geometric-optics limit of the Einstein-Maxwell equations.) While this is not the same limit studied here, it would be interesting to understand the seemingly dual relationship between both approaches and the connections (if any) between worldline and bulk non-linearities.

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