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On the asymptotic state of high Reynolds number, smooth-wall turbulent flows

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It is argued that the extrapolation of the log-wake law for the mean turbulent velocity profile to arbitrarily large Reynolds numbers, and also the similarity scaling for the intensity of stream-wise turbulent velocity fluctuations indicated by recent experimental measurements, are consistent with the hypothesis that smooth-wall turbulence is asymptotically transitory in the sense that these fluctuations almost everywhere decay with respect to the outer velocity scale when $1/\log(Re_\tau) \ll 1$, where Re_τ is the Reynolds number based on the skin-friction velocity u_τ. The existence of one or more near-wall maxima in these turbulent velocity fluctuations whose value may grow with Re_τ, does not invalidate the main scaling arguments. At gigantic Re_τ, this paradigm suggests that nonlinear motions and "turbulent" energy production are still present immediately adjacent to the wall, but that their amplitude becomes vanishingly small compared to the outer velocity scale. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4774335]

I. INTRODUCTION

High-Reynolds number, wall-bounded flows have been studied for more than a century. Classic pipe-flow experiments begin with Osborne Reynolds and continue with the rough-wall pipe-flow measurements of Nikuradse.[1] Later the focus was on achieving ever-higher Reynolds numbers to probe possibly asymptotic behavior of the mean flow and also the structure of the wall-generated turbulence itself. Research to achieve extremely large outer-scale Reynolds numbers, $Re = U_o\,\Delta/\nu$, where U_o is an outer-scale velocity, Δ is the outer length scale, and ν is the kinematic viscosity, has either made Δ large, as in the surface layer turbulence and environmental science test (SLTEST) experiments[2] and in long working-section wind-tunnels[3] or has reduced ν while holding Δ fixed, as in the super-pipe experiments.[4] In practice, the experimental search for high-Reynolds number, wall-bounded turbulence will generally encounter wall roughness. Thus the limit $Re \to \infty$ can be expected to depend upon roughness parameters such as $\epsilon = k_s/\Delta$, where k_s may be either a geometric or equivalent sand-roughness scale. Presently we do not consider this issue but instead focus on the idealized smooth-wall case with $\epsilon = 0$ and $Re \to \infty$. This limit, while strictly "mathematical" is nonetheless of interest for historical and conceptual reasons as are other issues at infinite Reynolds number, such as the behavior of the scaled dissipation.

Nagib *et al.*[5] collate experimental data and semi-empirical laws in order to consider the high-Reynolds number, asymptotic state of the zero-pressure gradient, smooth wall, equilibrium turbulent boundary layer. They affirm support for the logarithmic-wake law for the mean velocity profile, for the Coles-Fernholz, inverse-square, log-variation (with Re) of the wall skin-friction coefficient f, and for both the Coles wake parameter Π and the Kármán parameter κ becoming asymptotically constant when Re becomes very large. The scaling of the mean-flow stream-wise velocity profile, turbulence intensities, and Reynolds stresses for large-Reynolds number, wall-bounded flows are discussed by Marusic *et al.*[6] They summarize presently available experimental evidence for the hypothesis that the presence of very-large-scale motions (VLSMs), present in the logarithmic and outer regions of canonical near-wall turbulence, could be responsible for Re dependence in either or both of two peaks in the stream-wise turbulence intensity.

Our purpose here is to point out that confidence in both classical and recent experimental and theoretical scaling results for both the mean-flow profile, and also for stream-wise turbulence intensities for canonical, smooth-wall turbulence at asymptotically large Reynolds numbers, appears to suggest the tendency that the turbulent character of these flows may be transitory in the sense that their asymptotic state is inviscid slip flow without turbulence, almost everywhere. This has been discussed previously in the context of large-eddy simulations (LES).[7,8] Presently, motivated in part by recent experimental results on the scaling of stream-wise turbulent intensities,[9] we develop these ideas in more quantitative detail. In Sec. II we consider the classical log-wake law and its implications for smooth-wall flow, while Secs. III and IV investigate some implications of recent scaling results for wall-normal, stream-wise turbulence intensity profiles, in particular the asymptotic state of stream-wise turbulent fluctuations together with the appearance of a second or outer peak in the near-wall region. We discuss the implications of some of these predictions in Sec. V and consider alternative possibilities. Concluding remarks are given in Sec. VI.

II. THE LOG-WAKE LAW

In what follows we will refer to various length scales and Reynolds numbers for the zero-pressure-gradient, flat-plate turbulent boundary layer (ZPGFPTBL), and for pipe and channel flows. For the purposes of this discussion we will assume that these flows are essentially similar with differences to be noted only where relevant. The outer velocity scale U_o is either an external stream-wise velocity U_∞ or a mean centerline velocity U_c for pipe/channel flows. The generic outer length scale Δ will be the Clauser-Rota parameter $\delta^* U_\infty/u_\tau$ for boundary layers, where δ^* is the displacement thickness, the pipe radius R for pipe flow and the channel half height for open channel flow. The inner or wall-friction velocity is $u_\tau^2 = \overline{\tau_w}/\rho$ where $\overline{\tau_w}$ is a mean wall shear stress. The inner length scale is $L_\nu \equiv \nu/u_\tau$. The friction Reynolds number is $Re_\tau \equiv \Delta\, u_\tau/\nu$, z is a wall-normal co-ordinate with $z^+ \equiv z\, u_\tau/\nu$ and the inner-scaled stream-wise velocity is $u^+ \equiv u/u_\tau$. Inner and outer length scales are related by

$$\frac{z}{\Delta} = \frac{z\, u_\tau}{\nu}\,\frac{\nu}{\Delta\, u_\tau} = z^+\, Re_\tau^{-1}. \tag{1}$$

In terms of these variables, the widely accepted log-wake law for the mean velocity can be written as

$$\frac{\overline{u(z)}}{u_\tau} = \frac{1}{\kappa}\,\log[z^+] + B + \frac{\Pi}{\kappa}\, W\left(\frac{z}{\Delta}\right), \tag{2}$$

$$= \frac{1}{\kappa}\,\log\left(\frac{z}{\Delta}\, Re_\tau\right) + B + \frac{\Pi}{\kappa}\, W\left(\frac{z}{\Delta}\right), \tag{3}$$

for $z_L^+(Re_\tau)/Re_\tau < z/\Delta \le 1$, where B is a constant, $W(\eta)$ is the Coles wake function, and $z_L^+(Re_\tau)$ is the lower limit of validity of (3). In (3) and elsewhere, except where stated otherwise, log refers to the natural logarithm and the overbar notation will be taken to be any of several well-defined definitions of an average for a statistically stationary flow, for example, a time-average, an average along one or more homogeneous directions, an ensemble average, or a suitable combination of these. Presently we will make the classical assumption that the general form of (3) remains valid for smooth-wall pipe, channel, and ZPGFPTBL flows at arbitrarily large Re_τ. We do not consider alternatives such as power law descriptions of the mean flow. Where these are not asymptotic to (3) when $Re_\tau \to \infty$, it is possible that different results may be obtained to those explored presently.

Further assumptions are as follows. First, the parameters κ, Π, and $W(\eta)$ approach finite values and a functional form, respectively, that are independent of Re_τ when $Re_\tau \to \infty$. These, however, may be different for pipe, channel, and ZPGFPTBL flows. Second, it is assumed that $z_L^+(Re_\tau)$ is either finite and independent of Re_τ or else increases no faster than $z_L^+ \sim Re_\tau^m$, $0 \le m < 1$ when $Re_\tau \to \infty$. For $m = 0$, where z_L^+ is independent of Re_τ, typical values quoted in the literature are $\kappa = 0.38$–0.41, $B \approx 4.2$, $z_L^+ = 50$, and $\Pi \approx 0.5$ for the ZPGFPTBL with lower values of Π for pipe and channel flows.

The skin-friction coefficient f is defined presently by

$$f = \frac{\overline{\tau_w}}{\frac{1}{2}\,\rho\, U_o^2} = 2\left(\frac{u_\tau}{U_o}\right)^2, \tag{4}$$

$$= \frac{2}{\left(\frac{1}{\kappa}\,\log\left(Re_\tau\right) + B + \frac{\Pi}{\kappa}\,W(1)\right)^2},$$

$$\to \frac{2\,\kappa^2}{(\log(Re_\tau))^2} + HOT, \quad Re_\tau \to \infty, \tag{5}$$

where "*HOT*" denotes higher order terms. In (5), equation (3) has been used, with $z = \Delta$, where $\overline{u(\Delta)} = U_o$. Equation (5) is the Coles-Fernholz relation[10,11] here expressed in terms of Re_τ. Nagib *et al.*[5] give numerical estimates of the constants when the expression is cast in terms of Re_θ. Their Figure 1 shows good support for (5) for the ZPGFPTBL up to $Re_\theta = 7 \times 10^4$, where $Re_\theta = \theta\, U_o/\nu$ and θ is the momentum thickness. Results consistent with (5) up to $Re_\theta = \mathcal{O}(10^{12})$ have also been obtained using LES.[7] An immediate consequence of (5) is that $u_\tau/U_o \sim 1/\log\,(Re_\tau)$ when $Re_\tau \to \infty$ while $f \sim 1/(\log\,(Re_\tau))^2$, and so both vanish asymptotically.

Re-scaling $\overline{u(z)}$ by U_o in (3) using (5) then gives

$$\frac{\overline{u(z)}}{U_o} = \frac{\frac{1}{\kappa}\,\log\left(\frac{z}{\Delta}\right) + \frac{1}{\kappa}\,\log\left(Re_\tau\right) + B + \frac{\Pi}{\kappa}\,W\left(\frac{z}{\Delta}\right)}{\frac{1}{\kappa}\,\log\left(Re_\tau\right) + B + \frac{\Pi}{\kappa}\,W(1)},$$

$$\to 1, \qquad Re_\tau \to \infty, \quad 0 < \frac{z}{\Delta} < 1, \tag{6}$$

where $z_L/\Delta \sim Re_\tau^{m-1} \to 0$, when $Re_\tau \to \infty$. It follows that belief in the log-wake law with Re_τ-independent parameters (or $z_L^+ \sim Re_\tau^m,\ 0 \le m < 1$) when $Re_\tau \to \infty$, is sufficient to conclude first that u_τ vanishes with respect to U_o, and second, that at any finite $z/\Delta \le 1$, the local mean velocity asymptotically approaches U_o. This in turn implies that the 99% mean velocity point $z = \delta_{99}$ must asymptotically move closer to the wall with respect to Δ as Re_τ increases. Indeed equation (3.3) of Nagib *et al.*[5] plotted in their Figure 5 shows $\Delta/\delta_{99} \sim Re_\theta^{0.01}$, so the approach is extremely slow. Generalizing this by assuming that the point z_{1-r} where $\overline{u}/U_0 = 1 - r$ lies in the log-region, then a straightforward calculation using (6) gives $z_{1-r}^+ \sim Re_\tau^{1-r}$ while $(z/\Delta)_{1-r} \sim Re_\tau^{-r}$. Hence for $0 \le r \le 1$ the $100(1-r)$% mean-velocity point moves to infinity on inner scaling but towards the wall on outer scaling, when $Re_\tau \to \infty$. This indicates asymptotic plug flow for the pipe or channel and strictly slip flow for the flat-plate boundary layer on outer scaling, but order one shear everywhere on inner scaling. Furthermore, since outer-scale turbulence is expected to be driven by u_τ fluctuations at the wall, then it is plausible that the former will asymptotically decline in intensity when $Re_\tau \to \infty$. This is now discussed.

III. STREAM-WISE TURBULENT INTENSITIES

The stream-wise turbulent intensity $\overline{u'^2}$ is presently interpreted as a signature of turbulence activity over the whole of the wall-bounded turbulent region. There has been much discussion in the literature on the appropriate scaling for $\overline{u'^2}$ in both the outer and inner parts of the wall layer. For the ZPGFPTBL, the experimental compilation depicted in Figure 7 of Monkewitz *et al.*[12] shows no clear preference for inner, outer, or mixed scaling of turbulent stream-wise velocity-fluctuation intensities like $\overline{u'^2}$ over the outer part of the wall layer over a broad range of Reynolds number up to $Re_\theta = 60\,000$.

Recently there has been increased support for the scaling

$$\frac{\overline{u'^2}}{u_\tau^2} = F(z/\Delta), \tag{7}$$

over the outer part of the wall layer. For pipe flow, Morrison *et al.*[13] show the second-, third-, and fourth-order moments, scaled with the outer length scale and u_τ noting "... striking collapse of all moments in the outer region for $z/R > 0.4$ ..." which improves with increasing Re. Based on ZPGFPTBL measurements at $Re_\theta \sim \mathcal{O}(10^4)$, Marusic *et al.*[14] and Marusic and Kunkel[15] build an empirical model of turbulence intensities consistent with (7) in the outer part of the boundary layer. This is supported up to $Re_\theta \sim \mathcal{O}\left(10^{12}\right)$ by LES.[7] Further evidence for this scaling is provided by the pipe-flow measurements of Hultmark *et al.*[9] using a nanoscale anemometry probe. Their Figure 2(b) shows excellent collapse for Re_τ in the range 1985–98 187. As Re_τ increases the data appear to follow (7) to smaller z/R where a peak appears and the turbulence intensity then declines closer to the wall. This "outer" or second peak will be discussed subsequently.

A logarithmic form of $F(z/\Delta)$ in the outer flow was hypothesized by Townsend,[16] discussed by Perry *et al.*,[17] is contained in wall models[14,15] and supported by LES.[7,8] Hultmark *et al.*[9] find a region where

$$\frac{\overline{u'^2}}{u_\tau^2} = B_1 - A_1 \log\left(\frac{z}{\Delta}\right), \quad \Delta \equiv R, \tag{8}$$

and, following Perry *et al.*,[17] give $B_1 = 1.61$, $A_1 = 1.25$. They also note a hint of a logarithmic region in atmospheric surface-layer experiments.[2] Notable from both Figure 2(a) of Hultmark *et al.*,[9] where partial data are displayed, from their Figure 2(b) where complete data are given, and also from Figure 10(b) of Metzger *et al.*,[2] is that the data nowhere exceed a bound defined by (8) over the whole of the wall-surface layer. Motivated by this observation, we now introduce the assumption that

$$\frac{\overline{u'^2}}{u_\tau^2} \le B_1 - A_1 \log\left(\frac{z}{\Delta}\right), \quad 0 \le \frac{z}{\Delta} \le 1, \tag{9}$$

where Δ is the outer wall-layer scale. But the log is integrable, and so integrating (9) in the range $[0, z/\Delta]$, with $z/\Delta \le 1$ and using (5) then gives

$$\begin{aligned}\int_0^{\frac{z}{\Delta}} \frac{\overline{u'^2}(\xi)}{U_o^2}\, d\xi &\le \frac{\frac{z}{\Delta}\left[A_1\left(1-\log\left(\frac{z}{\Delta}\right)\right)+B_1\right]}{\left(\frac{1}{\kappa}\log\left(Re_\tau\right)+B+\frac{\Pi}{\kappa}W(1)\right)^2}, \\ &= \frac{\frac{z^+}{Re_\tau}\left[A_1\left(1-\log\left(\frac{z^+}{Re_\tau}\right)\right)+B_1\right]}{\left(\frac{1}{\kappa}\log\left(Re_\tau\right)+B+\frac{\Pi}{\kappa}W(1)\right)^2},\end{aligned} \tag{10}$$

in both outer (first) and inner (second) scaling. With either $z/\Delta \le 1$, or $z^+ \le Re_\tau$ fixed, the respective right-hand sides of (10) both become asymptotically zero when $Re_\tau \to \infty$. In particular with $z/\Delta = 1$, or $z^+ = Re_\tau$, the right-hand side is asymptotic to

$$\frac{(B_1 + A_1)\kappa^2}{(\log(Re_\tau))^2} + HOT, \quad Re_\tau \to \infty,$$

which states that the wall-normal-averaged, stream-wise turbulence intensity vanishes asymptotically when $Re_\tau \to \infty$. Since the log function is itself unbounded, this does not preclude the presence of inner peaks in $\overline{u'^2}/u_\tau^2$ that may be unbounded when $Re_\tau \to \infty$.

IV. PEAKS IN THE STREAM-WISE TURBULENCE INTENSITY PROFILES

There must be at least one peak in $\overline{u'^2}$ between the wall and the outer edge of the wall layer. At low and moderate Re, this has generally been taken to lie within the buffer-layer at about $z_I^+ \approx 15$ where the "I" subscript indicates the first peak encountered when moving outward from the wall.

A. Outer peak

Evidence for the existence of a second or outer peak is provided by high-Reynolds number data.[2,9,13] Hultmark *et al.*[9] show an outer peak for larger Re_τ and estimate the peak location at $z_{II}^+ = 0.23\, Re_\tau^{0.67}$. This is consistent with the critical-layer based arguments of McKeon and Sharma[18] who estimate $z_{II}^+ \sim a\, Re_\tau^{2/3}$. Hultmark *et al.* also suggest that $z_L^+ \approx z_{II}^+$. For the ZPGFPTBL, Alfredsson *et al.*[19] propose the existence of an outer peak near $z_{II}^+ \approx 0.82\, Re_\tau^{0.56}$. Further, it seems clear from Figure 2(a) of Hultmark *et al.*[9] that at a given Re_τ, the data peel off below the log bound near z_{II}^+, which is consistent with (9). A simple algebraic model of the second peak can be constructed by first assuming that z_{II}^+ is, for sufficiently large Re_τ, given by[8]

$$z_{II}^+ = a\, Re_\tau^\gamma, \quad 1 > \gamma > 0,$$

$$\frac{z_{II}}{\Delta} = a\, Re_\tau^{\gamma-1}, \tag{11}$$

where a is some constant. The outer peak moves outward, away from the wall, when scaled with the inner length scale ν/u_τ but inwards, towards the wall, when scaled on the outer length-scale Δ ($\Delta \equiv R$ for pipe flow).

Next, we propose a correction to (8) valid within the region of the (possible) outer peak

$$\frac{\overline{u'^2}}{u_\tau^2} = B_1 - A_1 \log\left(\frac{z}{\Delta}\right) - C_1 \left(\frac{z}{\Delta}\right)^{-\alpha} \frac{1}{Re_\tau^\beta}, \quad \alpha, \beta > 0,$$

$$= B_1 - A_1 \log\left(\frac{z^+}{Re_\tau}\right) - C_1 \frac{z^{+^{-\alpha}}}{Re_\tau^{\beta-\alpha}}, \tag{12}$$

where α, β are exponents and C_1 is a constant. Differentiating (12) with respect to z^+ and equating to zero gives a maximum at

$$z_{II}^+ = \left(\frac{\alpha\, C_1}{A_1}\right)^{\frac{1}{\alpha}} Re_\tau^{\frac{\alpha-\beta}{\alpha}}. \tag{13}$$

Equating (11) and (13) then gives $(\alpha - \beta)/\alpha = \gamma$, $C_1 = A_1\, a^\alpha/\alpha$. Using this and either (13) or (11) in (12) then gives the magnitude of the peak intensity as

$$\frac{\overline{u'^2}_{II}}{u_\tau^2} = B_2 + A_1 (1-\gamma) \log(Re_\tau), \tag{14}$$

$$B_2 = B_1 - \frac{A_1}{\alpha}\left[1 + \alpha \log(a)\right], \tag{15}$$

where we emphasize that the slope of the $\log(Re_\tau)$ variation depends only on γ and A_1, and not on α, β, a.

We consider two values of γ. First, following McKeon and Sharma[18] we take $\gamma = 2/3$, so that $\beta = \alpha/3$, and obtain a one-parameter family (α) of intensity profiles

$$\frac{\overline{u'^2}}{u_\tau^2} = B_1 - A_1 \log\left(\frac{z}{\Delta}\right) - \frac{A_1\, a^\alpha}{\alpha} \left(\frac{z}{\Delta}\right)^{-\alpha} \frac{1}{Re_\tau^{\alpha/3}},$$

$$= B_1 - A_1 \log\left(\frac{z^+}{Re_\tau}\right) - \frac{A_1\, a^\alpha}{\alpha} \left(\frac{z^+}{Re_\tau}\right)^{-\alpha} \frac{1}{Re_\tau^{\alpha/3}}. \tag{16}$$

Taking $A_1 = 1.25$, $B_1 = 1.61$, $\alpha = 2$, $a = 0.23$ then gives $\overline{u'^2}_{II}/u_\tau^2 = 2.82 + 0.42 \log(Re_\tau)$. Second we use $\gamma = 1/2$ which is closer to the empirical value suggested by Alfredsson *et al.*[19] With $\alpha = 0.4$ with the same values for the other parameters then gives $\overline{u'^2}_{II}/u_\tau^2 = 0.33 + 0.63 \log(Re_\tau)$. These two estimates are shown in Figure 1 compared with data obtained by estimating the outer maxima from Figure 2(a) of Hultmark *et al.*[9] Also shown is an estimate of the outer-peak intensity obtained

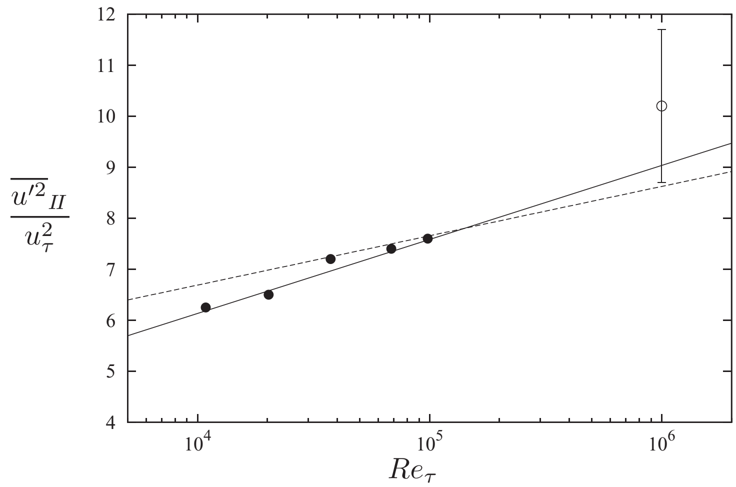

FIG. 1. $\overline{u'^2}_{II}/u_\tau^2$ versus Re_τ. Data: (Filled circles) from Figure 2(a) of Hultmark *et al.*;[9] (open circle) Metzger *et al.*;[2] (dashed line) $\overline{u'^2}_{II}/u_\tau^2 = 2.82 + 0.42\log(Re_\tau)$ obtained from (14), with $A_1 = 1.25$, $B_1 = 1.61$, $a = 0.23$, $\alpha = 2$, $\gamma = 0.667$; (solid line) $\overline{u'^2}_{II}/u_\tau^2 = 0.33 + 0.63\log(Re_\tau)$ obtained from (14), with $A_1 = 1.25$, $B_1 = 1.61$, $a = 0.23$, $\alpha = 0.4$, $\gamma = 0.5$.

from the SLTEST atmospheric test data.[2] Both formulae give satisfactory agreement with the data but fall somewhat below the SLTEST point at $Re_\tau \sim 10^6$. The estimate using $\gamma = 2/3$, $\alpha = 2$ is not quite as good a match to the data as the other case but gives better collapse of $\overline{u'^2}/u_\tau^2$ versus z/Δ outboard of the outer peak, owing to the larger value of α.

The peak shows a log-like increase with Re_τ. When scaled against the outer velocity scale U_o, by combining (5) and (14) it is found that

$$\frac{\overline{u'^2}_{II}}{U_o^2} = \frac{B_2 + A_1(1-\gamma)\log(Re_\tau)}{\left(\frac{1}{\kappa}\log(Re_\tau) + B + \frac{\Pi}{\kappa}W(1)\right)^2}, \qquad \rightarrow \frac{A_1(1-\gamma)\kappa^2}{\log(Re_\tau)} + HOT, \quad Re_\tau \rightarrow \infty. \tag{17}$$

Further assuming that the log-wake law remains valid at z^+_{II}, then using (14) and substituting (13) into (3) to obtain $\overline{u_{II}}$, the mean velocity at $z^+ = z^+_{II}$, shows that

$$\frac{\overline{u'^2}_{II}}{\overline{u_{II}}^2} = \frac{B_2 + A_1(1-\gamma)\log(Re_\tau)}{\left(\frac{\gamma}{\kappa}\log(Re_\tau) + B_3\right)^2}, \qquad \rightarrow \frac{A_1(1-\gamma)\kappa^2}{\gamma^2\log(Re_\tau)} + HOT, \quad Re_\tau \rightarrow \infty, \tag{18}$$

where B_3 is a constant. Therefore, when scaled against either the square of the outer flow velocity or the local mean velocity, the outer peak turbulence intensity decreases as $(\log(Re_\tau))^{-1}$ when $Re_\tau \rightarrow \infty$.

B. Inner peak

Both direct numerical simulation (DNS) and experimental studies indicate that an inner peak is located within the buffer region near $z^+_I \approx 15$, which is invariant with Reynolds number. Marusic *et al.*[6] survey evidence supporting a weak Re_τ dependence in the peak stream-wise turbulence intensity for ZPGFPTBL flow that may be a result of VLSM action on the very-near-wall region. A collation of data together with a semi-empirical inner-outer predictive model[20] is consistent with the conclusion that, for the ZPGFPTBL, the inner peak of $\overline{u'^2}/u_\tau^2$, denoted by $\overline{u'^2}_I/u_\tau^2$, shows a log-dependence on Re_τ like (14) but with different constants: see Figure 10(b) of Mathis *et al.*[20] If indeed the inner peak is affected by VLSMs, its maximum may show different behavior for pipe, channel, and boundary-layer flows owing possibly to differing geometrical confinements on the action of VLSMs for these flows. A straightforward development of similar arguments used for the outer peak applied presently shows that $\overline{u'^2}_I/U_o^2$ decreases as $(\log Re_\tau)^{-2}$ if $\overline{u'^2}_I/u_\tau^2$ remains invariant (first scenario) and as $(\log Re_\tau)^{-1}$ if $\overline{u'^2}_I/u_\tau^2$ shows variation like (14) (second scenario). Provided z_I^+ remains constant, and assuming that the mean flow at this station, $\overline{u}_I/u_\tau$, is a function only of z_I^+ then it is clear that $\overline{u'^2}_I/\overline{u}_I^2$ is constant, independent of Re_τ for the first scenario, while $\overline{u'^2}_I/\overline{u}_I^2 \sim \log(Re_\tau)$ in the second scenario.

V. DISCUSSION

We define the inner region of the turbulent wall layer as that portion that lies between the wall and the outer peak, $z_{II}/\Delta = a\, Re_\tau^{\gamma-1}$. Outside this region, in the limit $Re_\tau \to \infty$, the mean turbulence intensity is vanishingly small with respect to the local mean velocity. The variation of the stream-wise and indeed the total turbulence intensity between z_I^+ and z_{II}^+ is unknown, except possibly for the above estimates of the inner peak, and at the bottom of the viscous sub-layer. In this inner region, which becomes asymptotically small on outer scaling when $Re_\tau \to \infty$, some vestige of nonlinearity may remain as a sub-boundary layer but its activity is negligible with respect to the outer flow. A straightforward calculation gives the ratio $z_{II}/z_{(1-r)} \sim Re_\tau^{\gamma+r-1}$. Hence for $r = 0.01$ (99% mean velocity point) and the values of γ considered presently, at large Re_τ, z_{II} is always nearer the wall than is $z_{0.99}$. In other words, the inner region of the wall layer, by our definition, always lies inside the 99% mean velocity point, while both move closer to the wall, in outer scaling, as $Re_\tau \to \infty$.

The preceding arguments make several implicit assumptions. One is that the canonical turbulence flows discussed are well-posed in some sense as initial-boundary-value problems in the smooth-wall, infinite Reynolds number limits discussed, and that these admit statistically steady-state behavior in a finite time. A second is that the large Reynolds-number limit can be physically characterized by its low-order moments, namely, the mean and the stream-wise turbulence intensities. Yet turbulent flows are known to show intermittency, for example, in the outer part of turbulent boundary layers,[21,22] and it cannot be ruled out that consideration of higher order, one-point and two-point statistics could reveal a more complex picture than considered presently, for example, widely spaced regions of erupting and intermittent "puff" turbulence with decaying low-order statistics but finite higher order moments.

The basis of the present main arguments lies in (3) and (8). Some of our results could change substantially if either or both of these relations were found not to be valid at extremely large Reynolds number. The latter appears to be supported over more than a decade of z/Δ by the microprobe measurements of Hultmark *et al.*[9] and also by LES.[7,8] The former may be influenced by probe errors and uncertainties and the latter by both modeling assumptions and limited resolution. If, for example, the left side of (8) is replaced by fractional scaling $\overline{u'^2}/(u_\tau^{2-q}\, U_o^q)$, $0 \le q < 2$, then the conclusion drawn from (10) would not change but the asymptotic decline in turbulence intensities at large Re_τ would be reduced. However, the magnitude of the second peak relative to both U_0^2 and to the local mean velocity $\overline{u}_{II}$ could be affected qualitatively, with $\overline{u'^2}_{II}/U_o^2 \sim (\log(Re_\tau))^{q-1}$. This would still decrease with increasing Re_τ when $q < 1$, become asymptotic to a constant for $q = 1$, and increase slowly when $q > 1$. Other scenarios are of course possible. For example, for the inner peak Metzger and Klewicki[23] estimate $\left(\overline{u'^2}_I/u_\tau^2\right)^{1/2} \sim \log(Re_\theta)$ based on a best-curve fit of combined laboratory and SLTEST data. The reader is left to consider the consequences of this.

VI. CONCLUDING REMARKS

Presently we have considered the consequences, for strictly smooth-wall turbulence, of widely accepted scaling for the mean velocity profile (3), together with experimental results on the scaling of the stream-wise turbulence intensities, (8) and its extension, (9). We have argued that, if these scalings remain valid at arbitrarily large Re_τ, then first, at any fixed ratio of the wall-normal location to the outer length scale, the local mean velocity must approach the outer velocity, and second, that stream-wise turbulence intensities, normalized by the outer velocity, must decline over almost all of the wall-bounded layer. Higher order corrections to (3) and any wall-normal, integrable higher order correction to (8) will not affect our arguments. In this asymptotic state, leading terms for both the mean velocity defect and the layer-average stream-wise turbulence intensities, both normalized against the outer velocity, decrease as powers of $1/\log(Re_\tau)$. For these to be small requires $1/\log(Re_\tau) \ll 1$ which is marginal for present laboratory and field data at $Re_\tau = O(10^5 - 10^6)$. In other words, in the sense of approach to the asymptotic state, large Reynolds number means not only $Re_\tau \gg 1$ but also $\log(Re_\tau) \gg 1$.

The physical basis for the above is perhaps that, with U_o, Δ held fixed, the mean, wall-normal stream-wise velocity gradient, $d\overline{u}/dz_{wall}$, diverges more slowly than ν^{-1} and so the wall shear stress, $\rho\,\nu\,d\overline{u}/dz_{wall}$, decreases monotonically when $\nu \to 0$. The near-wall production mechanism is then deprived of wall-stress fluctuations that drive the turbulence so that wall-normal turbulent transport of turbulent energy declines and, across the whole outer layer, the turbulence is asymptotically attenuated. Within this scenario, the asymptotic state of the wall layer is slip-flow bounded by a vortex sheet at the wall with weakly nonlinear internal structure. The internal dynamics within this layer remain an interesting, but open question.

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[1] J. Nikuradse, "Strömungsgestze in rauhen rohren," VDI-Forschungsheft **4**, 1–22 (1933).
[2] M. Metzger, B. McKeon, and H. Holmes, "The near-neutral atmospheric surface layer: Turbulence and non-stationarity," Philos. Trans. R. Soc. London, Ser. A **365**, 859–876 (2007).
[3] T. Nickels, I. Marusic, S. Hafez, and M. Chong, "Evidence of the k_1^{-1} law in a high-Reynolds-number turbulent boundary layer," Phys. Rev. Lett. **95**, 074501 (2005).
[4] M. Zagarola and A. Smits, "Mean-flow scaling of turbulent pipe flow," J. Fluid Mech. **373**, 33–79 (1998).
[5] H. Nagib, K. Chauhan, and P. Monkewitz, "Approach to an asymptotic state for zero pressure gradient turbulent boundary layers," Philos. Trans. R. Soc. London, Ser. A **365**, 755 (2007).
[6] I. Marusic, B. McKeon, P. Monkewitz, H. Nagib, A. Smits, and K. Sreenivasan, "Wall-bounded turbulent flows at high Reynolds numbers: Recent advances and key issues," Phys. Fluids **22**, 065103 (2010).
[7] M. Inoue and D. Pullin, "Large-eddy simulation of the zero-pressure-gradient turbulent boundary layer up to $Re_\theta = O(10^{12})$," J. Fluid Mech. **686**, 507 (2011).
[8] N. Saito, D. I. Pullin, and M. Inoue, "Large eddy simulation of smooth-wall, transitional and fully rough-wall channel flow," Phys. Fluids **24**, 075103 (2012).
[9] M. Hultmark, M. Vallikivi, S. Bailey, and A. Smits, "Turbulent pipe flow at extreme Reynolds numbers," Phys. Rev. Lett. **108**, 094501 (2012).
[10] D. Coles, "The law of the wake in the turbulent boundary layer," J. Fluid Mech. **1**, 191–226 (1956).
[11] H. Fernholz and P. Finley, "The incompressible zero-pressure-gradient turbulent boundary layer: An assessment of the data," Prog. Aerosp. Sci. **32**, 245–311 (1996).
[12] P. Monkewitz, K. Chauhan, and H. Nagib, "Self-consistent high-Reynolds-number asymptotics for zero-pressure-gradient turbulent boundary layers," Phys. Fluids **19**, 115101 (2007).
[13] J. Morrison, B. McKeon, W. Jiang, and A. Smits, "Scaling of the streamwise velocity component in turbulent pipe flow," J. Fluid Mech. **508**, 99–131 (2004).
[14] I. Marusic, A. Uddin, and A. Perry, "Similarity law for the streamwise turbulence intensity in zero-pressure-gradient turbulent boundary layers," Phys. Fluids **9**, 3718 (1997).
[15] I. Marusic and G. Kunkel, "Streamwise turbulence intensity formulation for flat-plate boundary layers," Phys. Fluids **15**, 2461 (2003).
[16] A. Townsend, *The Structure of Turbulent Shear Flow*, 2nd ed. (Cambridge University Press, 1976).
[17] A. Perry, S. Henbest, and M. Chong, "A theoretical and experimental study of wall turbulence," J. Fluid Mech. **165**, 163–199 (1986).
[18] B. McKeon and A. Sharma, "A critical-layer framework for turbulent pipe flow," J. Fluid Mech. **658**, 336–382 (2010).
[19] P. Alfredsson, A. Segalini, and R. Örlü, "A new scaling for the streamwise turbulence intensity in wall-bounded turbulent flows and what it tells us about the outer peak," Phys. Fluids **23**, 041702 (2011).

[20] R. Mathis, N. Hutchins, and I. Marusic, "A predictive inner–outer model for streamwise turbulence statistics in wall-bounded flows," J. Fluid Mech. **681**, 537–566 (2011).
[21] R. J. Adrian, "Hairpin vortex organization in wall turbulence," Phys. Fluids **19**, 041301 (2007).
[22] S. Morris, S. Stolpa, P. Slaboch, and J. Klewicki, "Near-surface particle image velocimetry measurements in a transitionally rough-wall atmospheric boundary layer," J. Fluid Mech. **580**, 319–338 (2007).
[23] M. Metzger and J. Klewicki, "A comparative study of near-wall turbulence in high and low Reynolds number boundary layers," Phys. Fluids **13**, 692–701 (2001).