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STIRRING AND MIXING IN THE MANTLE BY PLATE-SCALE FLOW: LARGE PERSISTENT BLOBS AND LONG TENDRILS COEXIST

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Abstract. The stirring of a small, passive heterogeneity in an unsteady circulation is investigated by following the boundary of the heterogeneity; the circulation mimics features of plate-scale flow. After initial release, the heterogeneity is stirred and subsequently consists of at least one large "blob" connected to long, but thin tendrils. Tendrils exponentially lengthen and exponentially thin with time, as predicted by a turbulent mixing law, but the width of blobs decays about 100 times more slowly. The blobs mix slowly because they are smaller than the scale of flow and are sheared and occasionally unmixed by the laminar flow within the interiors of convection cells. The decay time of 1 to 100 km sized heterogeneities is on the order of billions of years for whole-mantle convection and thus is in accord with isotopic observations which indicate the mantle has chemical heterogeneities persisting for billions of years.

Introduction

Isotopic measurements made on mantle derived basalts demonstrate that chemical heterogeneities persist in the mantle for billions of years [see review by Zindler and Hart, 1986]. This poses the question as to how chemical heterogeneities can persist for so long in spite of the flow associated with plate motions. A number of studies have addressed this question but, unfortunately, the conclusions drawn from them are in strong disagreement. Hoffman and McKenzie [1985], for example, conclude that subducted lithosphere, the only directly observed source of chemical variation, would be mixed in less than 500 m.y., while Olson et al. [1984a,b] and Gurnis and Davies [1986] conclude that the lithosphere would take billions of years to mix. The geochemical observations are robust and this uncertainty must be resolved. The purpose of this paper is to clarify the differences between previous studies, to emphasize the key assumptions underlying mixing laws, and to present a simple numerical calculation which quantitatively illustrates the differences between previous models as well as showing phenomena previously overlooked.

Turbulent and Laminar Mixing

Two simple models of stirring (the term used to describe mixing when there is no diffusion), turbulent and laminar, have previously been used to interpret numerical calculations. The equations describing these models are fairly simple and by deriving them here we can pin-point some critical assumptions.

Laminar Mixing (L.M.) occurs, for example, in steady flow with a horizontal velocity and a velocity gradient in only the vertical direction; in other words, there is only a component of simple shear. Consider a passive, circular heterogeneity, in two dimensions, bound by a material line, ℓ (i.e. a line which always consists of the same fluid particles). Initially the line integral of ℓ is the circumference $\pi\theta_o$, where θ_o is the diameter. The material line increases at a rate proportional to the separation velocity of two fluid particles separated vertically, say by θ_o, and for long times

$$\frac{\partial \ell}{\partial t} \approx \dot{\varepsilon}_s \ell_o = \pi\theta_o\dot{\varepsilon}_s \tag{1}$$

where $\dot{\varepsilon}_s$ is the shear strain rate. The rate at which the material line increases is constant. The solution to (1) is

$$\ell \approx \pi\theta_o(\dot{\varepsilon}_s t + 1) \tag{2}$$

The area of the two-dimensional heterogeneity must remain constant, so an average width in the shortest dimension is

$$\langle\theta\rangle \approx \frac{\pi\theta_o^2}{\ell} = \frac{\theta_o}{(\dot{\varepsilon}_s t + 1)} \tag{3}$$

such that for long times $\langle\theta\rangle/\theta_o \propto t^{-1}$.

A model of turbulent mixing, T.M., [Batchelor, 1952] can also be simply developed. Consider a flow with spatially homogeneous turbulence, such that a fluid particle experiences pure shear, and that fluid particles will, in a statistical sense, be subject to the same flow properties over time. Consider a small segment of the total material line, $\delta\ell$, which can be made small enough so that the homogeneous turbulence assumption holds. This assumption will hold, if at any instance, $\delta\ell$, is close to a stagnation point. At a point close to a stagnation point, the velocity toward (or away from) the stagnation point is proportional to the distance from the center of the stagnation point. Thus, for a small parcel of fluid, of size $\delta\ell$, the rate at which it stretches in one direction (and compresses in the other) is proportional to $\delta\ell$; $\delta\ell$ will increase as

$$\frac{\partial}{\partial t}\delta\ell = \langle\dot{\varepsilon}_n\rangle\delta\ell \tag{4}$$

where $\langle\dot{\varepsilon}_n\rangle$ is a spatial and time average of the normal strains. T.M. contrasts with L.M. in that two points in laminar flow recede from each other at a constant rate, while in turbulent mixing, the rate of separation increases as the two points separate from each other. The solution to (4) is

$$\delta\ell = \delta\ell_o \exp(\langle\dot{\varepsilon}_n\rangle t) \tag{5}$$

The material line is the integral over all elements defining its length and in the limit that the small segments can be made infinitesimally small, i.e. $\delta\ell = d\ell$,

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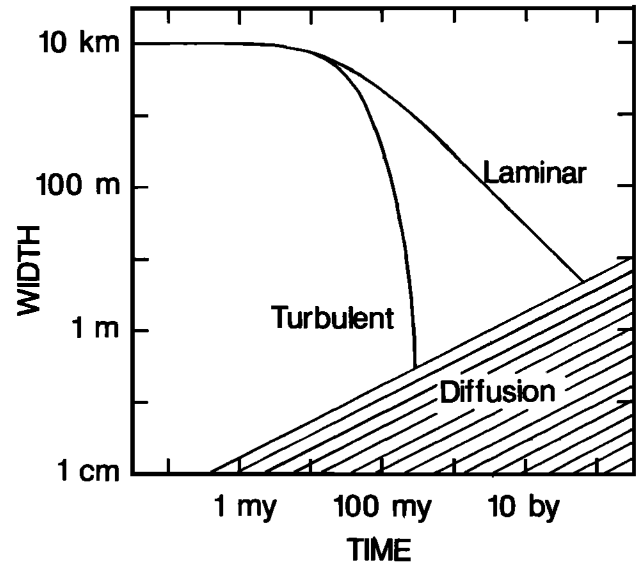

Fig. 1. Application of simple stirring laws to whole-mantle convection. The initial size of the heterogeneity is 10 km (≈ thickness of subducted crust). The shaded region is where chemical diffusion becomes important and is bound by the curve $(\kappa t)^{1/2}$, where $\kappa \approx 10^{-13}$ cm^2/s, an upper limit on the solid-state chemical diffusivity [Zindler and Hart, 1986].

$$\ell = \oint d\ell = \pi\theta_o \exp(\langle\dot{\varepsilon}_n\rangle\ t) \tag{6}$$

and the average width in the shortest dimension is

$$\langle\theta\rangle = \theta_o \exp(-\langle\dot{\varepsilon}_n\rangle\ t) \tag{7}$$

These two mixing models can be applied to the mantle by assuming appropriate strain rates. Assume for the sake of argument that both $\dot{\varepsilon}_s$ and $\langle\dot{\varepsilon}_n\rangle$ are ~ U/(D/2), where U is the average plate velocity (≈ 5 cm/yr) and D is the depth of the layer, 3000 km for whole-mantle convection. Figure 1 shows that T.M. and L.M. predict very different mixing rates for the mantle: a 10 km diameter heterogeneity is mixed in just over 100 m.y. assuming T.M. [Hoffman and McKenzie, 1985] or greater than the age of the earth assuming L.M. [Olson et al., 1984a]. This calculation shows an enormous range in mixing rates and poses the question as to which mixing law, if indeed either, is appropriate to mantle flow.

In order to address this question, we must compare the features of mantle flow with the assumptions of the mixing laws. Plate-scale flow has the following relatively well know features: (1) flow is unsteady and rearranges on time scales comparable to over-turn times (as indicated by ridge and trench migrations); (2) flow has a small range in length scales (plates are ~ 10^3 to 10^4 km in scale and the depth scale is ~ 10^3 km); and (3) heterogeneities are smaller than the scale of flow (oceanic crust compared to the depth of flow is ~ 10^{-2} to 10^{-3}).

L.M. requires that the flow is steady and that shear, $\dot{\varepsilon}_s$, exists in only one coordinate, while T.M. requires the flow to be unsteady and to have both shear and normal components of strain on all scales. Moreover, the formulation of T.M. requires [Batchelor, 1952] that as $\delta\ell$, a small segment of a material line, is made arbitrarily small it is strained about equally by both shear and normal components; however, for a heterogeneity embedded in mantle flow, as $\delta\ell$ is made arbitrarily small, the heterogeneity falls completely within a convection cell where shear strains dominate over normal strains (Olson et al. [1984b], see below). Thus, the mantle does not satisfy the requirements of either mixing law.

If the analytical results seem to have limited applicability, do the numerical studies overcome any of these limitations ? Olson et al. [1984a,b] do relax the L.M. assumption that the shear must be completely in one direction: they showed that a steady closed circulation mixes laminarly because at most length scales less than the dimension of the cell, $\dot{\varepsilon}_s \ll \langle\dot{\varepsilon}_n\rangle$. However, this mode of stirring was for a steady circulation and it is not clear if this holds for unsteady flow as well. Gurnis and Davies [1986] did study the stirring in an unsteady flow, wherein tracers were introduced at the margins of cells to simulate subduction. They noticed that some heterogeneity persisted for times longer than simple shearing would indicate and attributed this to partial unmixing, a phenomenon related to laminar flow. Unfortunately, the results were essentially qualitative and could not be compared to the simple mixing laws. However, Hoffman and McKenzie [1985] did investigate mixing by an unsteady flow by tracking material lines of heterogeneities and were able to make comparisons with the mixing laws. The length scale of the heterogeneity was on the order of the scale of flow; because the material lines increased exponentially in time, they concluded that the flow mixed turbulently.

The above arguments raise serious objections about applying these mixing laws to the mantle such that the calculations presented in Figure 1 could be of little relevance to the mantle. Much of this uncertainty, however, could be resolved if we understood how a small heterogeneity embedded in an unsteady flow is stirred.

The Stirring of Small Heterogeneities in Unsteady Flow

The material line of a small, initially circular, patch of fluid is followed in an unsteady, periodic flow. The flow, defined analytically and previously employed by Gurnis and Davies [1986], has two counter rotating cells in a rectangular box with an oscillating margin between the cells. All the controlling parameters were set to the case where the average margin velocity is on the order of the flow velocities of the cells. Details on the governing equations and numerical scheme and accuracy can be found in Gurnis and Davies [1986].

Model times are given in transit time, t_t, which is the time to traverse the fluid depth with the average surface velocity. Order of magnitude scalings can be made by multiplying the number of transit times by the mantle value, which is 60 m.y. for whole-mantle convection.

Circular heterogeneities (bounded by material lines) were placed at mid-depth in the box and stirred by the flow. Six initial positions, or cases, were studied to ascertain the amount of variation obtained within just one flow pattern. The material line is defined by a set of tracers

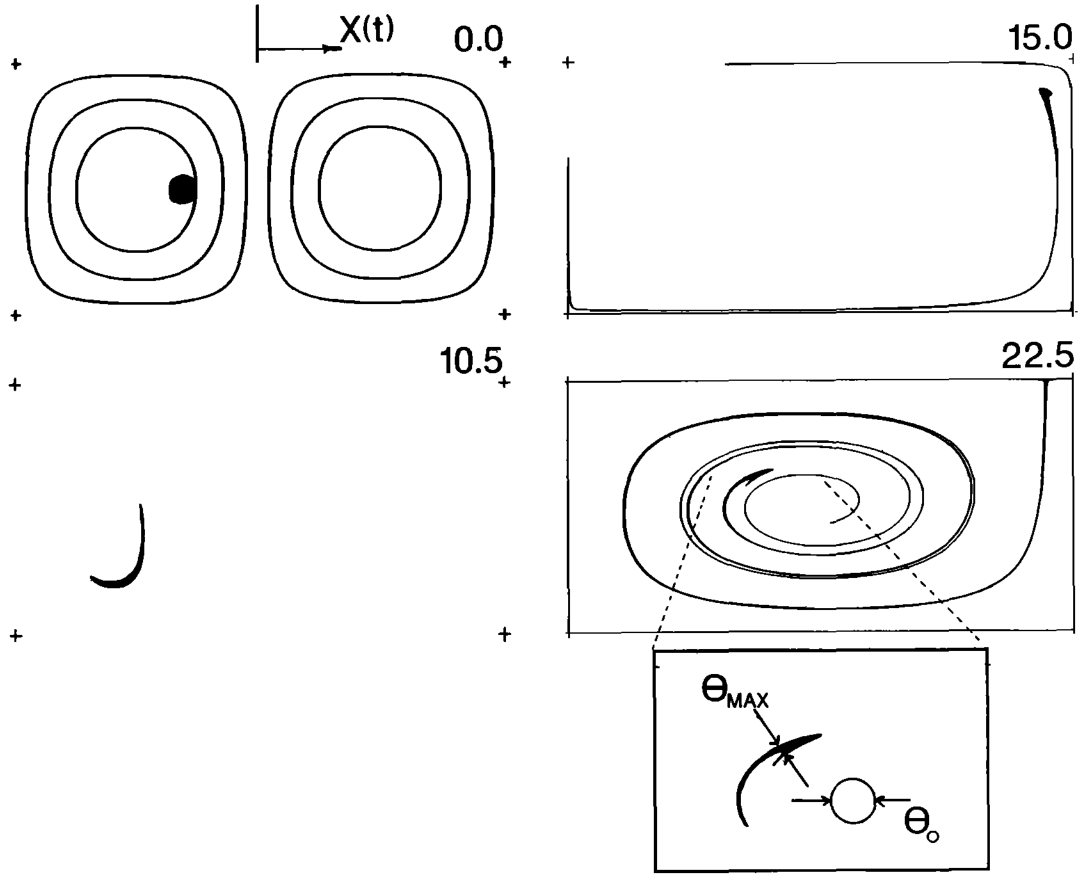

Fig. 2. Example of the stirring of an initially circular heterogeneity, black patch, in a periodic flow. The streamlines at time 0 are shown; the margin between the two cells oscillates and the flow is shown at many more steps in Gurnis and Davies (1986). Each frame is labelled with transit times. The inset highlights a large and persistent heterogeneity connected to very long, but thin tendrils. The maximum width is on the order of the initial size of the heterogeneity. Borders on the 2x1 boxes have not been drawn: any line on the edge of a box is a tendril.

which are advected by the flow; in order to maintain accuracy when the distance between two adjacent tracers exceeds a set distance an additional tracer is placed between them [cf. Hoffman and McKenzie, 1985]. In all cases the size of the heterogeneities were $\theta_o/D = 0.1$, where D is the depth of the flow and this satisfies the mantle criteria that the heterogeneities be smaller than the scale of flow.

One case is shown at four instants in Figure 2. The circular element becomes progressively sheared out and after a number of transit times of stirring, is made up of "tendrils" of small width. The average width across the heterogeneity, $\langle\theta\rangle/\theta_o$, can be found from the material line length, ℓ (equation 3) and this quantity is plotted against transit time, as solid circles, in Figure 3. The average width of the material line decays like the laminar model up to about six transit times, which is about one turn over time, but subsequently decays exponentially.

However, inspection of Figure 2 reveals that characterizing heterogeneity with an average parameter may be rather misleading because there is always a region of heterogeneity with size ~ θ_o. The inset (Figure 2) shows the maximum sized heterogeneity at transit time 22.5, with indicated width of $\theta_{max}/\theta_o = 0.26$. The average width at this time, however, is only $\langle\theta\rangle/\theta_o = 0.0022$ which is more than two orders of magnitude smaller than the maximum width. For all times and for all initial placements of heterogeneities, there was always a relatively large sized blob as shown in Figure 2. The size of these heterogeneities were measured, as indicated in Figure 2, and are plotted in Figure 3 as open circles.

The θ_{max}/θ_o values start deviating from the $\langle\theta\rangle/\theta_o$ curve at approximately six transit times, the $\langle\theta\rangle/\theta_o$ transition between laminar and turbulent mixing. This behavior is easily understood. A small heterogeneity is sheared within a cell and becomes thinner at a rate proportional to t^{-1} (cf. equation 3). The flow is unsteady and the heterogeneity is eventually transferred to an adjacent cell; the sense of shear is almost always reversed in an adjacent cell and the heterogeneity is partially unmixed [Heller, 1960; Gurnis and Davies, 1986]. Thus, as blobs are occasionally transferred to adjacent cells, and partially unmixed, their width must obviously decay less rapidly than the t^{-1} predicted by steady laminar shearing. Unmixing is a straightforward consequence of laminar flow; laminar flow can both increase and decrease concentration gradients, as has been long known [Eckart, 1948].

Since blobs are sheared primarily by laminar flow, with partial unmixing, the size of blobs should decay less rapidly than t^{-1}. A good fit

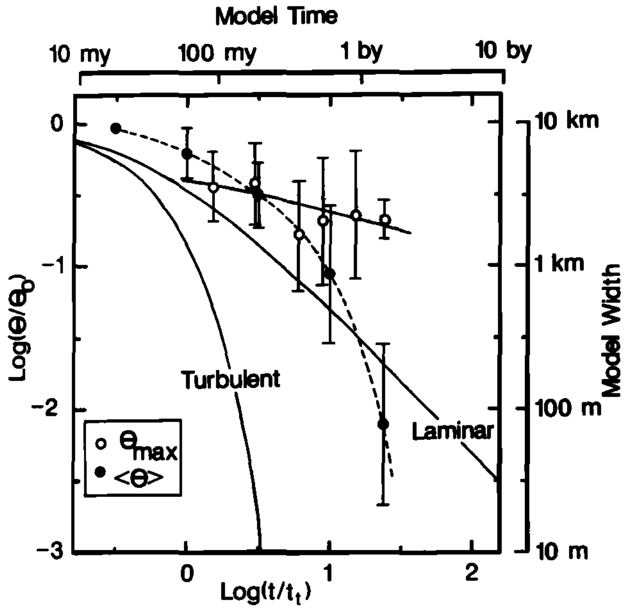

Fig. 3. Results of mixing calculations: the solid lines are the average width of the heterogeneity predicted by the simple laminar and turbulent mixing laws. The solid points are $\langle\theta\rangle/\theta_o$ derived from the total lengths of material lines. The open points are for θ_{max}/θ_o and were derived as shown in Figure 2, inset. The maximum sized heterogeneities mix slowly because they are laminarly mixed and occasionally unmixed within the interior of cells.

is found to the averages of blob widths with

$$\frac{\theta_{max}}{\theta_o} = \left[1 + \frac{3}{2}\left(\frac{t}{t_t}\right)^{1/3}\right]^{-1} \quad (8)$$

The time for θ_{max}/θ_o to be reduced to 0.1 is 220 transit times.

While the blobs are stirred in this way, tendrils are stirred quite differently. After tendrils become larger than the length-scale of unsteady flow, they are frequently advected into stagnation points and pulled apart. Apparently, because long tendrils approximately satisfy the conditions of the T.M. law, they behave accordingly and exponentially stretch with time. In time, because the surface area of tendrils become so large, very much larger than the surface area of blobs, the mere existence of blobs is obscured in measured quantities related to material lines.

Discussion

This numerical example reconciles the studies of Hoffman and McKenzie [1985] with those of Olson et al. [1984a,b] and Gurnis and Davies [1986]. Hoffman and McKenzie [1985] based their conclusion that oceanic lithosphere would be mixed by convection in less than 500 m.y., on the observation that material lines exponentially lengthen with time. However, the new calculation shows that despite such exponential lengthening, a sizable fraction of fluid (within blobs) is stirred by laminar flow and occasionally unmixed within convection cells. Such laminar mixing means that heterogeneities could survive for billions of years [Olson et al., 1984a,b; Gurnis and Davies, 1986].

This result for the stirring of blobs is based on the assumption that the heterogeneity is smaller than the scale of flow. For oceanic crust or lithosphere mixed by plate-scale flow, this assumption is certainly satisfied. This raises the question if there are other, smaller, modes of convection. Plume-style convection is also expressed in observations (e.g. through hot-spot swells), but because of its axisymmetric form and relatively small mass flux, this mode is expected to stir the mantle less effectively than plate-scale flow; this conjecture, however, should be tested in the future. Although other modes of convection may well exist, the evidence for them is equivocal, and therefore the result presented here for the mixing rate is a lower bound on the true mixing rate.

The originally posed question, can chemical heterogeneity persist in spite of plate-scale flow, can be readdressed from equation (8). Assuming a constant transit time of 60 m.y., if a 10 km heterogeneity is introduced into the mantle, for example, then there would still be, on average, one heterogeneity ~ 2 km across after 2 b.y. of stirring by plate-scale flow (as shown by the "model width" and "model time" labels in Figure 3). This lower limit on the mixing rate is consistent with isotopic and other geochemical data of small, off-ridge seamounts which indicate that there are kilometer-sized, ancient chemical heterogeneities ubiquitously entrained in mantle flow [Zindler and Hart, 1986].

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