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# SHEPHERDING OF THE URANIAN RINGS. I. KINEMATICS

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## ABSTRACT

We identify several orbital resonances involving the newly discovered satellites, 1986U7 and 1986U8, and the Uranian rings. The most important resonances in eccentric rings are known as eccentric resonances and are generalizations of the more familiar Lindblad resonances. In keeping with the notation established for Lindblad resonances, we distinguish inner and outer eccentric resonances by the symbols IER and OER. We show that by reducing the absolute radius scale of the Uranian ring system by 0.0124% the 24:25 OER of 1986U7 and the 14:13 IER of 1986U8 fall at the inner and outer edges of the $\epsilon$ ring. The same scale change also brings the 23:22 IER of 1986U7 into coincidence with the outer edge of the $\delta$ ring and the 6:5 IER of 1986U8 close to the center of the $\gamma$ ring. Furthermore, adopting the latest *Voyager* value of $GM_U$ and our reduced radius scale, we find that the pattern speed of the $m = 2$ distortion in the $\delta$ ring corresponds to that expected for a normal mode excited either by an internal viscous overstability or parametrically by shepherd satellites. These kinematic results make a compelling case for our proposed reduction in the ring radius scale and also imply that 1986U7 and 1986U8 are the inner and outer shepherds for the $\epsilon$ ring, that 1986U7 is the outer shepherd for the $\delta$ ring, and that 1986U8 is an outer shepherd for the $\gamma$ ring.

### I. INTRODUCTION

Among the exciting new results of the *Voyager* encounter with Uranus was the discovery of ten small satellites, ranging in radius from 20 to 85 km, interior to the orbit of Miranda. Two of these satellites, temporarily designated 1986U7 and 1986U8, bracket the $\epsilon$ ring (Smith *et al.* 1986; Fig. 1 [Plate 46]). Shortly after the discovery of the Uranian rings, Goldreich and Tremaine (1979) predicted that each ring would be shepherded by a pair of satellites. Shepherd satellites confine a ring by exerting torques at the ring's inner and outer edges, thereby acting as the sources and sinks of the angular momentum which viscous stresses transport outward through the ring. In this paper, we examine the possible kinematical relationships between the rings and all ten newly discovered satellites. We present observational evidence for interactions between 1986U7 and 1986U8 and the $\epsilon$, $\delta$, and $\gamma$ rings. In particular, we assess the likelihood that these two satellites are shepherding the $\epsilon$ ring and are responsible for some of its internal structure. As a result of this investigation, we are led to propose a reduction in the ring radius scale.

Precise orbital elements have been determined for these new satellites (Owen and Synnott 1987). We make use of these results, the latest *Voyager* revision of $GM_U$, and the ring orbital elements as deduced from ground-based occultations (French, Elliot, and Levine 1986a) to identify the strongest resonances of these satellites which fall in or near a ring. In a companion paper (Goldreich and Porco 1987), we study the dynamical significance of the kinematical relationships discovered here.

We adopt the following convention regarding notation. Variables associated with satellites are subscripted either by s or by some appropriate abbreviation of the satellite's name. Variables associated with rings are not subscripted unless they refer to a particular ring, in which case an appropriate symbol is attached. Variables associated with Uranus receive the subscript U.

### II. RESONANCES

The search for shepherd satellites was an important objective of *Voyager* imaging science at Uranus (Smith *et al.* 1986). Two imaging sequences were designed with the explicit goal of searching the ring system for satellites. The first sequence was taken with the narrow-angle camera through the clear filter ($\lambda_{\rm eff} = 4890$ Å) at 4 days before closest approach to Uranus from a range of $\sim 4 \times 10^6$ km and was targeted on a constant inertial location centered on the $\epsilon$ ring apoapse. The time interval between images was set so that an object orbiting Uranus appeared in five to six consecutive frames as it entered, passed through, and then exited the field of view. Smear due to orbital motion, and the redundancy provided by consecutive observations, allowed discrimination between objects within the Uranian system and background stars smeared by spacecraft motion during the 15 s exposures. The satellites situated on either side of the $\epsilon$ ring, designated 1986U7 and 1986U8, were discovered in this sequence. Later, these satellites were also found in images taken 12 and 6 days prior to closest approach. In total, 24 images of 1986U7 and 13 images of 1986U8, spanning 11 and 5 days, respectively, were used by Owen and Synnott (1987) to determine the satellites' orbital elements. The absolute inertial coordinates of the satellites were measured by using background stars of known position as references. Each satellite's orbit was fitted to a uniformly precessing, inclined Keplerian ellipse, yielding values for the mean orbital elements $(a, e, i, \Omega, \dot{\Omega}, \omega, \dot{\omega}, n)_s$. The elements were assigned formal uncertainties resulting from the inherent measurement inaccuracies and the correlations among the eight parameters. Those orbital elements relevant for our purposes and their associated uncertainties are listed in Table I.

In determining accurate satellite semimajor axes and in locating the resonances involving these satellites, we use the equations for the natural frequencies of a test particle orbiting an oblate planet. Note from Table I that the eccentricity of the outer shepherd, 1986U8, is comparable to that of the $\epsilon$

TABLE I. Planet and satellite parameters.

| Parameter | Planet | 1986U7 | 1986U8 |
|---|---|---|---|
| $r_s$ [a] | | 20 km | 25 km |
| $m_s$ [b] | | $5\times10^{19}$ g | $9\times10^{19}$ g |
| $a_s$ | | 49 771.4 ± 17.5 km | 53 795.6 ± 48.5 km |
| $a_s$ [c] | | 49 751.6 ± 0.3 km | 53 764.1 ± 1.8 km |
| $n_s$ | | 1074.521 ± 0.004°/day | 956.41 ± 0.04°/day |
| $e_s$ | | $0.47(\pm 0.41)\times10^{-3}$ | $10.1(\pm 0.4)\times10^{-3}$ |
| $i_s$ | | 0.°14 ± 0.°10 | 0.°09 ± 0.°27 |
| $GM_{\rm U}$ | 5 793 939 ± 60 km$^3$ s$^{-2}$ | | |
| $J_2$ | $3.3461(\pm 0.0030)\times10^{-3}$ | | |
| $J_4$ | $-3.21(\pm 0.037)\times10^{-5}$ | | |
| $R_{\rm U}$ | 26 200 km | | |
| $M_{\rm U}$ [d] | $8.6860(\pm 0.0004)\times10^{28}$ g | | |

[a] Uncertainty ~30%.
[b] Using $\bar{\rho} = 1.40$ g/cm$^3$ (Tyler *et al.* 1986). Uncertainty is approximately a factor of 2 due to uncertainty in satellite radii.
[c] Derived from $n_s$ using *Voyager* results for $GM_{\rm U}$ (Tyler *et al.* 1986) and the ground-based values of $J_2$ and $J_4$ (French *et al.* 1986). These values are adopted in this work.
[d] Derived from *Voyager* $GM_{\rm U}$. Uncertainty from $\Delta G/G$.

ring, while that of the inner shepherd, 1986U7, is essentially zero, and that neither satellite has a measurable inclination. Accordingly, we neglect eccentricity-dependent corrections to the mean motion $n$ and epicyclic frequency $\kappa$, and ignore entirely the vertical frequency $\mu$. The appropriate expressions for the frequencies at semimajor axis $a$ read:

$$n^2 = \frac{GM_{\rm U}}{a^3}\left(1 + \frac{3}{2}J_2\alpha^2 - \frac{15}{8}J_4\alpha^4\right), \qquad (1)$$

$$\kappa^2 = \frac{GM_{\rm U}}{a^3}\left(1 - \frac{3}{2}J_2\alpha^2 + \frac{45}{8}J_4\alpha^4\right), \qquad (2)$$

where $GM_{\rm U}$ and $R_{\rm U}$ are the gravitational mass and equatorial radius of Uranus, $\alpha \equiv R_{\rm U}/a$, and $J_2$, $J_4$ are the standard gravity coefficients. Table I lists the values for these parameters used in this paper. It includes the new value of $GM_{\rm U} = 5\,793\,939 \pm 60$ km$^3$ s$^{-2}$ determined by Tyler *et al.* (1986) from Doppler tracking of the spacecraft and optical navigation of the five major satellites. Unfortunately, *Voyager 2* did not approach close enough to Uranus for its orbital perturbations to yield an improved solution for the gravity coefficients $J_2$ and $J_4$. The best values remain those determined by ground-based occultation studies of the rings and have relative uncertainties of $10^{-3}$ for $J_2$ and $10^{-1}$ for $J_4$. These uncertainties translate into negligible, $< 10^{-3}$ km, uncertainties in the computed locations of the resonances at the $\epsilon$ ring. Thus, we use the ground-based values of $J_2$ and $J_4$ together with the new value of $GM_{\rm U}$ in calculations of the absolute semimajor axes of satellites and resonances.

The relative uncertainties in the solution mean motions, $4\times10^{-6}$ for 1986U7 and $4\times10^{-5}$ for 1986U8, are 10 and 20 times smaller, respectively, than the relative uncertainties in the semimajor axes because the component of the measurement uncertainty which maps into the determination of $n$ is effectively spread out over the time interval spanned by the imaging data. Since $GM_{\rm U}$ is determined to equivalent precision, we have taken the mean motions given by Owen and Synnott and used Eq. (1) to calculate semimajor axes which are more accurate than those determined directly. These $a_s$'s and their uncertainties, determined from the uncertainties in the mean motion and $GM_{\rm U}$, are quoted in Table I.

The classification of resonances in an eccentric and inclined ring is rather involved. Therefore, we begin with the simpler case of a circular equatorial ring for which we distinguish three types of resonance associated with a perturbation potential which varies with time $t$ and azimuthal angle $\phi$ as $\cos(\omega t - m\phi)$. We refer to $m$ as the azimuthal separation parameter and to $\Omega_{\rm p} \equiv \omega/m$ as the pattern speed.

Corotation resonances CR are located where $n = \Omega_{\rm p}$. Particles orbiting near a corotation resonance undergo slow periodic variations in semimajor axis and longitude. Inner and outer Lindblad resonances, ILR and OLR, are located where $\kappa = \pm m(n - \Omega_{\rm p})$. Orbital eccentricities are excited near Lindblad resonances since at the resonance the perturbation force felt by a particle oscillates with the epicyclic frequency. Inner and outer vertical resonances, IVR and OVR, are located where $\mu = \pm m(n - \Omega_{\rm p})$; they are not considered in this paper, which treats only equatorial orbits.

In an eccentric ring, corotation resonances maintain their identity. However, pure Lindblad resonances do not exist. Instead, there are more complex resonances which have a mixture of the characteristics of Lindblad and corotation resonances. In keeping with the nomenclature of Goldreich and Tremaine (1981), we refer to these as eccentric resonances. In the vicinity of an eccentric resonance, the semimajor axis of a ring particle varies in a manner similar to that near a corotation resonance. In addition, the excursions in semimajor axis and orbital longitude are accompanied by variations of eccentricity and longitude of periapse which are similar to those near a Lindblad resonance. The technical distinction between a Lindblad and an eccentric resonance is important if the unperturbed value of $e$ exceeds the magnitude of its perturbation.

The general resonance condition, for coplanar orbits, may be written as

$$mn - q\kappa - mn_s - k\kappa_s = 0\,, \qquad (3)$$

where $m$ is a positive, nonzero integer, and $k$ and $q$ are integers which can take on positive or negative values. Rotational invariance requires the sum of the coefficients to be zero. Corotation resonances CR correspond to $q = 0$. There are eccentric resonances associated with all $q \neq 0$. We denote these as inner and outer eccentric resonances, IER and OER, according to whether $\Omega_{\rm p} < n$ or $\Omega_{\rm p} > n$. In the limit of a circular ring, the eccentric resonances with $q = \pm 1$ reduce to inner and outer Lindblad resonances, ILR and OLR, respectively. It is common usage to attach the label $(m+k):(m-q)$, the approximate orbital period ratio, to a resonance. We add a second label $|k| + |q|$, which we refer to as the order of the resonance since the perturbation potential has the small factor $e^{|q|} e_s^{|k|}$ in its coefficient (Brouwer and Clemence 1961). An eccentric resonance is not uniquely specified by its order; for example, both the choices $k = 0$, $q = 2$ and $k = 1, q = 1$ correspond to second-order eccentric resonances.

The first-order eccentric resonances, which reduce to the more familiar Lindblad resonances for $e = 0$, have $\Omega_p = n_s$. For $e_s = 0$, the only corotation resonances are those of zeroth order and are located at the orbit of the satellite; the Trojan asteroids are classic examples. Also, for $e_s = 0$, the eccentric resonances are uniquely defined by the value of $q$ since $k = 0$.

We compute resonance locations by solving Eq. (3) iteratively for $a$. The uncertainties in $a$ arise predominantly from the uncertainties in $n_s$; $a$ can be computed to $\pm 0.3$ km and $\pm 1.8$ km for resonances associated with the inner and outer shepherd satellite, respectively. The relative locations of resonances due to the same satellite are accurate to $\leqslant 0.1$ km. Inclusion of the perturbations due to the Sun and the five major Uranian satellites on the mean motions and precession rates yields negligible changes in absolute resonance locations. The same is true for inclusion of terms involving particle eccentricity in Eq. (1). A search for $|q| = 1$ eccentric and corotation resonances involving each shepherd and one of the remaining 14 Uranian satellites yielded negative results. We believe that any variations in the mean motion or precession rate of either shepherd due to higher-order or secular resonances are inconsequential for our analysis. Table II lists the principal first- and second-order eccentric and first-order corotational resonances which fall near the $\epsilon$ ring.

## III. SATELLITE WAKES AND ISOLATED RESONANCES

It is convenient to distinguish two limiting classes of satellite perturbations, wakes and resonances. Wakes are perturbations which damp between successive close encounters of the ring material with the satellite. Isolated resonances involve perturbations which extend with undiminished amplitude around the entire circumference of the ring and which are well separated, by unperturbed regions, from neighboring resonances. The edge perturbations of the Encke division in Saturn's A ring (Cuzzi and Scargle 1985) are examples of wakes, whereas the perturbed outer edges of Saturn's A and B rings (Porco *et al.* 1984) are locations of isolated resonances.

Wakes form if adjacent, noninteracting, perturbed streamlines of ring material intersect between successive close encounters with the satellite. The condition for this to occur is that

$$\left(\frac{1}{m^4}\right)\left(\frac{M_U}{M_s}\right) \lesssim 1 . \tag{4}$$

That is, the satellite has to be sufficiently large and sufficiently close to the ring in order that its perturbations damp in one synodic period. If inequality (4) is violated, the satellite perturbations are most appropriately described in terms of the superposition of resonances. Resonances are isolated if their widths are smaller than their separations.

Two length scales which are relevant to a discussion of resonance widths for $m \gg 1$ are

$$w_p \approx \left(\frac{M_s}{M_U}\right)^{1/2} a_s \tag{5}$$

and

$$w_\lambda \approx 5\left(\frac{a^2\Sigma}{mM_U}\right)^{1/2} a . \tag{6}$$

Here, $w_p$ is the distance from the resonance at which nested periodic orbits of test particles cross and $w_\lambda$ is the first wavelength of the density wave which would propagate away from the resonance if there were no ring edge to interrupt it. In the limit that the self-gravity of the ring material is negligible, $w_p$ would set the scale for the resonance width. However, self-gravity can extend the region over which perturbed streamlines avoid crossing up to a distance $w_\lambda$.

To compute $w_p$, the masses of the shepherds must be known. These are obtained from several *Voyager* images in which the satellite appeared smeared due to orbital motion in one direction and resolved in the other. Though inadequate in providing shape information, these frames were used to estimate size (Smith *et al.* 1986). We calculate masses for the shepherds by assuming a spherical shape of the appropriate size and a mean density equal to that of the five major Uranian satellites, $\rho = 1.40$ g cm$^{-3}$ (Tyler *et al.* 1986). These values are listed in Table I. The uncertainty in the satellite radii is $\approx 30\%$, resulting in a mass uncertainty of a factor of 2. For the low-order eccentric resonances of 1986U7 and 1986U8 in the $\epsilon$ ring, $w_p \approx 1$ km and $w_\lambda \approx 7$ km. Therefore, these resonances are truly isolated.

TABLE II. Resonance types and locations.

| Type | | | Location (km) |
|---|---|---|---|
| | | $\epsilon$ ring | |
| 1st Order Eccentric | | | |
| 1986U8 | 14:13 IER | $(k = 0; q = 1)$ | $51\,178.1 \pm 1.8$ |
| 1986U7 | 24:25 OER | $(k = 0; q = 1)$ | $51\,121.2 \pm 0.3$ |
| 1986U7 | 23:24 OER | $(k = 0; q = 1)$ | $51\,180.3 \pm 0.3$ |
| 2nd Order Eccentric | | | |
| 1986U8 | 28:26 IER | $(k = 1; q = 1)$ | $51\,177.8 \pm 1.8$ |
| 1986U7[a] | 47:49 OER | $(k = 0; q = 2)$ | $51\,150.1 \pm 0.3$ |
| 1st Order Corotation | | | |
| 1986U8 | 14:13 CR | $(k = 1; q = 0)$ | $51\,177.5 \pm 1.8$ |
| | | $\delta$ ring | |
| 1st Order Eccentric | | | |
| 1986U7 | 23:22 IER | $(k = 0; q = 1)$ | $48\,302.5 \pm 0.3$ |
| | | $\gamma$ ring | |
| 1st Order Eccentric | | | |
| 1986U8 | 6:5 IER | $(k = 0; q = 1)$ | $47\,625.7 \pm 1.8$ |

[a] The only isolated second-order eccentric resonance in the $\epsilon$ ring.

## IV. RESONANCES NEAR THE $\epsilon$ RING

The best solution for the absolute radial scale of the Uranian ring system yields $a_\epsilon = 51\,156.3$ km for the mean semimajor axis of the $\epsilon$ ring and places its inner and outer edges at $a_i = 51\,127.3$ km and $a_o = 51\,185.3$ km (See Fig. 2). Based on these numbers, the 24:25 OER of 1986U7 at 51 121.2 km lies 6 km inside the ring's inner edge, missing the ring entirely, while the 14:13 IER of 1986U8 at 51 178.1 km and the 23:24 OER of 1986U7 at 51 180.3 km fall 7 and 5 km inside the outer edge, respectively. This configuration does not match that expected if the satellites are shepherding the ring. However, the spacing between the 25:24 OER of 1986U7 and the 14:13 IER of 1986U8 is equal to $56.9 \pm 1.8$ km, identical, within the observational uncertainties, to the vari-

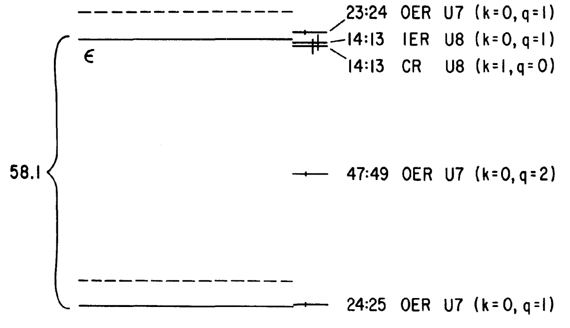


FIG. 2. Graphic illustration of the relative positions of the first-order eccentric and corotation resonances and isolated second-order eccentric resonances of 1986U7 and 1986U8 and the semimajor axes of the inner and outer edges of the $\epsilon$ ring. The dashed lines denote the locations of the ring's edges determined from ground-based occultation observations. The solid lines are the locations after reduction by 0.0124%. The variation in semimajor axis across the ring is given in km.

ation in semimajor axis of $\Delta a_\epsilon = 58.06 \pm 0.26$ km across the $\epsilon$ ring. A shift of $-6.6 \pm 1.8$ km in the absolute location of the $\epsilon$ ring would bring the 24:25 OER and the 14:13 IER into coincidence with the ring's inner and outer edges. Moreover, the same shift would move the 23:24 OER outside the ring. The satellite torque is positive at an IER and negative at an OER. Therefore, a fractional radius shift of $0.0129 \pm 0.0035\%$ results in a configuration of resonances which is consistent with the hypothesis that 1986U7 and 1986U8 are shepherding the $\epsilon$ ring.

## V. DISTORTION OF THE $\delta$ RING

French, Kangas, and Elliot (1986b) have shown that the atypically large 3 km radius residuals of the $\delta$ ring may be fitted by an $m = 2$ radial distortion which rotates with a pattern speed of $562.519 \pm 0.002$°/day. Two explanations for this observation were examined. The authors noted that the location of a 2:1 resonance of a satellite orbiting with a mean motion equal to the observed pattern speed fell $41 \pm 9$ km interior to the mean radius of the delta ring, close enough to resonantly force the observed amplitude of 3 km provided the satellite had a radius of 75–100 km. Alternatively, the observed radial distortion in the ring might be an $m = 2$ normal mode excited either by an internal viscous overstability (Borderies *et al.* 1985), or parametrically by shepherding satellites (Goldreich and Tremaine 1981). The pattern speed expected for the $m = 2$ normal mode, however, did not match the observed value. Consequently, free oscillations were discarded as an explanation of the large radius residuals of the $\delta$ ring, and instead French *et al.* predicted the existence of a satellite orbiting at a semimajor axis of $76\,522 \pm 8$ km. In arriving at this conclusion, the authors used the ground-based results for the radius scale and $GM_{\rm U}$.

The *Voyager* imaging science results clearly showed that no satellite of the size predicted by French *et al.* exists at the prescribed orbital radius. However, it was found that the Uranian gravitational mass required a surprisingly large revision, one which removed most but not all of the discrepancy between the observed pattern speed and that predicted for a normal mode (Elliot and Nicholson 1986). We find that the remaining discrepancy would be entirely eliminated by a reduction of $-6.0$ km in the semimajor axis of the $\delta$ ring. It is remarkable that this fractional radius reduction, 0.0124%, is the same, to within observational error, as that implied by the shepherding hypothesis for the $\epsilon$ ring. We interpret this coincidence to mean that the shape distortion of the $\delta$ ring is probably due to the excitation of its $m = 2$ normal mode.

This assertion allows us to derive a more precise radius scale reduction than that obtained from the $\epsilon$ ring shepherding hypothesis. The pattern speed of the $m = 2$ normal mode of the $\delta$ ring corresponds to an appropriately weighted semimajor axis for the ring. The high-resolution *Voyager* PPS observations of the $\sigma$ Sag and $\beta$ Per occultations by the $\delta$ ring show that the optical-depth profile is not completely symmetric about the ring's centerline (Lane *et al.* 1986), implying that the mass distribution may also be asymmetric. Accordingly, we assign an uncertainty of $\sim 1$ km to the altered radius of the $\delta$ ring and quote the fractional radial scale reduction as $0.0124 \pm 0.0021\%$.

## VI. RESONANCES NEAR THE $\delta$, $\gamma$, $\eta$, AND 1986U1R RINGS

A search was made for other resonant associations between the ten new satellites discovered by *Voyager* and the remaining Uranian rings. Two promising candidates were found and are listed in Table II. One is the 23:22 $|q| = 1$ IER of 1986U7 at $a = 48\,302.5 \pm 0.3$ km, near the $\delta$ ring; the second is the 6:5 $|q| = 1$ IER of 1986U8 at $a = 47\,625.7 \pm 1.8$ km, near the $\gamma$ ring.

The first resonance is positioned $3.6 \pm 0.3$ km inside the ground-based semimajor axis for the $\delta$ ring (See Fig. 3). The variation in semimajor axis across the $\delta$ ring is $\Delta a_\delta = 5.1$ km; the rms width residuals are 2.0 km. The reduced value of the $\delta$ ring semimajor axis, $a_\delta = 48\,300.1 \pm 1.0$ km, places

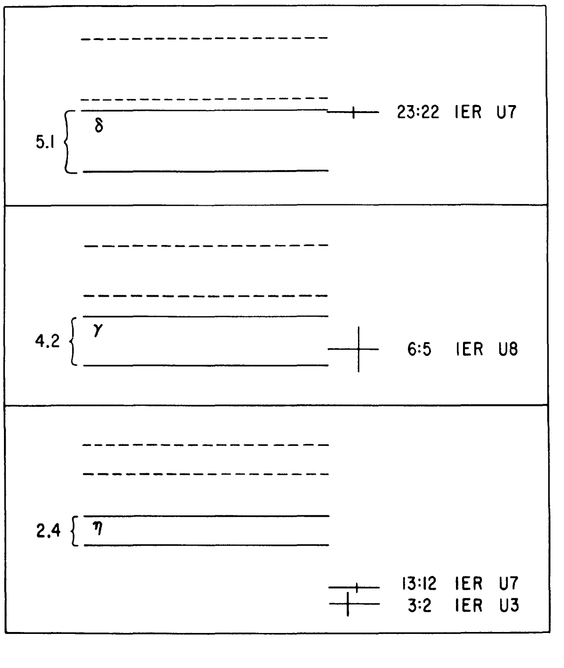


FIG. 3. Positions of all the first-order eccentric resonances found in this work which fall in or near the $\delta$, $\gamma$, and $\eta$ rings. Dashed lines illustrate the ground-based locations of the semimajor axes of the rings' inner and outer edges; solid lines, the altered semimajor axes deduced from this work. The variation in semimajor axis across each ring is given in km.

the ring's (mean) outer edge at 48 302.55 km, coincident with the 23:22 IER of 1986U7. Thus, it appears that 1986U7 is shepherding the outer edge of the $\delta$ ring through its 23:22 IER in the same manner that it shepherds the inner edge of the $\epsilon$ ring through its 24:25 OER.

The 6:5 IER of 1986U8 sits $6.6 \pm 1.8$ km inside the ground-based semimajor axis for the $\gamma$ ring. A relative radius reduction of $1.24 \times 10^{-4}$ places the $\gamma$ ring at $a_\gamma = 47\,262.4 \pm 1.0$ km, almost exactly coincident with the 6:5 IER (see Fig. 3). The mean width of the $\gamma$ ring is $\Delta a_\gamma = 4.2$ km; the rms width residuals are 1.8 km. All uncertainties considered, it is plausible that 1986U8 is also an outer shepherd of the $\gamma$ ring. Moreover, the $\gamma$ ring is unusual among the Uranian rings (excepting the $\delta$ ring) because of its anomalously large rms radius residuals of $\sim 3$ km. It might be suspected that these residuals result from a resonantly forced $m = 6$ distortion of the ring. However, a search for $m \leqslant 6$ patterns in the ground-based $\gamma$ ring occultation data has yielded negative results (French 1986). Consequently, it seems unlikely that the large radius residuals are explicable by perturbations due to 1986U8 alone. Nonetheless, both ground-based and *Voyager* data on this puzzling ring are worth a fresh examination in light of its resonant association with this satellite.

The only isolated first-order satellite resonances which fall near any of the remaining rings are located interior to the $\eta$ ring. The 13:12 IER of 1986U7 is located at $a = 47\,173.0 \pm 0.3$ km. The 3:2 IER of 1986U3, another of the ten small satellites discovered by *Voyager* imaging science, is located at $a = 47\,171.6 \pm 0.3$ km. The $\eta$ ring's reduced radius is $a_\eta = 47\,177.7 \pm 1.0$ km; its mean width $\Delta a_\eta = 2.4$ km. Therefore, both resonances fall no closer than $\sim 5$ km from the ring's centerline and $\sim 4$ km from the ring's (mean) inner edge. Since the widths of these resonances [Eq. (5)] are $\sim 1$ km, we dismiss the possibility that either of these resonances is causing disturbances in the $\eta$ ring.

A new narrow ring was discovered by the *Voyager* cameras and also detected by the PPS and UVS experiments. Temporarily designated 1986U1R, its orbital radius was measured to be $a = 50\,030$ km in PPS data in the system defined by the ground-based radial scale. The altered radius of this new ring is 50 024 km. The only satellite with resonances in the vicinity of this ring is 1986U7. The spacing of its first-order OERs near 1986U1R is $\sim 3$ km; their widths $w_p \approx 1$ km. Thus these resonances are only marginally isolated. The width of the ring's core was found to be $\sim 2$ km in the PPS $\sigma$ Sag occultation. The 122:123 OER of 1986U7 at $50\,022.5 \pm 0.3$ km falls close to the ring's inner edge. There is no evidence yet that this new ring is eccentric. Assuming it is circular, 1986U7 may be the inner shepherd for this ring as well.

## VII. DENSITY WAVES IN THE $\epsilon$ RING

The sharp edges of the $\epsilon$ ring, which coincide with and are maintained by first-order eccentric resonances, offer the most striking evidence for satellite interactions in the Uranian ring system. Sharp edges, shepherded by satellites, were also found in Saturn's rings. However, density waves were by far the most common signature of satellite interactions in that system. The relatively narrow Uranian rings offer fewer opportunities for the observation of density waves. However, there are two potential sites in the $\epsilon$ ring which are worth mentioning. One is near the center of the ring at the location of the second-order 47:49 OER of 1986U7. The other is at the outer boundary of the ring, just inside the location of the first-order 23:24 OER of 1986U7. The precise positions of these resonances are given in Table II.

Density wave trains excited at these OERs would propagate inwards. With $\Sigma \approx 33$ g cm$^{-2}$, their first wavelengths would be several km (cf. Eq. (6)). A more detailed prediction of the properties of these wave trains would require extension of the theory of density waves to regions where $q \approx a(de/da)$ is significantly different from 0. The Voyager Photopolarimeter Team has pointed to features in their occultation profiles of the $\epsilon$ ring which they find suggestive of density waves (Lane *et al.* 1986). Figure 4 is a reproduction of their $\sigma$ Sagittarii ingress occultation profile, smoothed to 100 m resolution, and linearly scaled to a width $\Delta a_\epsilon$. Indicated on the abscissa are the positions, relative to the edges of the ring, of the principal (first-order) eccentric resonances and the second-order 47:49 OER of 1986U7. We call attention to the location of the second-order OER and the wave-like features noted by the photopolarimeter team. We point out, however, that these features are not obvious in the occultation profiles published by the Voyager Radio Science Team (Tyler *et al.* 1986).

Note that in our linear relative positioning of resonances and ring features is the implicit assumption that the eccentricity increases linearly with semimajor axis across the ring. This assumption is apparently justified by the *Voyager* PPS occultation data (Lane *et al.* 1986) and by comparison of ground-based and *Voyager* occultation data (Nicholson 1986).

## VIII. DISCUSSION AND CONCLUSIONS

Resonant forcing by the shepherd satellites distorts the shapes of the sharp edges of the $\epsilon$ ring. The radius perturbations are of the form $\delta r = A \cos m(\phi - \Omega_p t)$: they extend around the complete circumference of the ring, $m = 24$ on the inner and $m = 14$ on the outer edge, and each travels

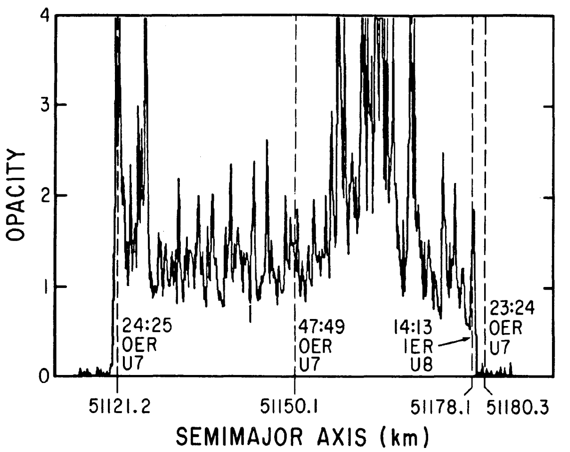


FIG. 4. Reproduction of the PPS ingress $\sigma$ Sag occultation scan of the $\epsilon$ ring, smoothed to 100 m resolution (Lane *et al.* 1986), and linearly scaled to a width of $\Delta a_\epsilon$, the variation in semimajor axis across the $\epsilon$ ring. The absolute radial locations of the most important resonances of 1986U7 and 1986U8 are indicated. Note the relationship between the wave-like feature inward of the ring's center, pointed out by the photopolarimeter team, and the 47:49 OER of 1986U7.

with a pattern speed equal to the perturbing satellite's mean motion. Thus the distortions along the inner and outer edges vary in phase relative to one another and account for a (unknown) portion of the residuals in the ring's radius and width. The contribution to the latter is twice that to the former since the perturbations on the two edges are uncorrelated. The amplitudes of these perturbations are restricted to be smaller than the larger of $w_p$ and $w_\lambda$, $\sim 1$ km and $\sim 7$ km for the $\epsilon$ ring. The residuals of the ground-based occultation data from the best-fitting models are 0.52 km in radius and 1.42 km in width; the ratio of the former to the latter is 0.37, instead of the expected value of 0.5. It might be possible to relate accurately the amplitude $A$ to the magnitude of the satellite torque, but the relevant calculations have yet to be attempted. The prospect for an observational determination of $m$ and $A$ is hampered by the uncertainty in the mean motion of the perturbing satellite $\delta n_s$ since the position of the satellite can only be predicted to an adequate accuracy, say, $\sim 0.5\ m^{-1}$ radians, for a time of order $0.5/(m\delta n_s)$. This time is $\sim 1$ yr for 1986U7, which has the more accurate $n_s$.

The phases, but not the amplitudes, of the $m = 23$ and $m = 24$ distortions of the outer edge of the $\delta$ ring and inner edge of the $\epsilon$ ring may be related, with great accuracy, to the position of 1986U7. Thus, the *Voyager* data, which were obtained at a time when the position of the satellite was known, may be used to make a first cut at determining the amplitudes of these perturbations. Should this procedure yield a reasonable result, the edge residuals in the ground-based data could be fitted for the position of the satellite as well as for the amplitudes of the perturbations on each edge. Having two edges and only one satellite orbit to fit improves the chances of obtaining a convincing solution.

A similar procedure might be tried using the residuals of the outer edges of the $\epsilon$ and the $\gamma$ rings since it appears that both are shepherded by 1986U8. The poorer accuracy with which the mean motion of 1986U8 is known would complicate this task. However, some compensation is provided by the smaller values of $m$, which equal 14 and 6 in this case. An important by-product would be an improved motion for 1986U8, which would enable us to further refine the radius scale of the Uranian ring system.

We find the argument for the reduction in radius scale compelling as it would bring into excellent agreement observational findings and theoretical predictions for two completely independent phenomena: the maintenance of ring edges by eccentric resonances, as in the cases of the outer edges of the $\delta$ and $\gamma$ rings and the inner and outer $\epsilon$ ring edges, and the existence of an excited $m = 2$ normal mode in the $\delta$ ring. The precision of this scale change is on the order of $\pm 1$ km, a relative shift of $0.0124 \pm 0.0021\%$. An error of this size in the ground-based determination of the absolute radius scale could be the result, for example, of an erroneous ring-plane pole position. Work is currently in progress to combine the ground-based and *Voyager* occultation data to determine an improved pole position for Uranus: a change of $-5$ to $-10$ km seems likely and would agree with the shift deduced here (Marouf 1986; Nicholson 1986).

There is strong kinematical evidence to support the hypotheses that 1986U7 and 1986U8 are shepherding the $\epsilon$ ring, that 1986U7 is the outer shepherd for the $\delta$ ring, and that 1986U8 is an outer shepherd for the $\gamma$ ring. The $\delta$ ring's $m = 2$ shape distortion is plausibly attributed to an excited normal mode of the ring. The excitation of the mode might be due either to an internal, viscous overstability (Borderies *et al.* 1985) or to parametric excitation by shepherd satellites (Goldreich and Tremaine 1981). The *Voyager* occultation data will provide additional, and perhaps decisive, evidence for testing this hypothesis. In addition, the possible existence of a density wave in the $\epsilon$ ring driven by the 47:49 OER and the 23:24 OER of 1986U7 is of great importance and will undoubtedly be the subject of future study.

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