Collapse and Rebound of a Spherical Bubble in Water

ROBERT HICKLING* AND MILTON S. PLESSET
California Institute of Technology, Pasadena, California
(Received 14 June 1963)

Some numerical solutions are presented which describe the flow in the vicinity of a collapsing spherical bubble in water. The bubble is assumed to contain a small amount of gas and the solutions are taken beyond the point where the bubble reaches its minimum radius up to the stage where a pressure wave forms which propagates outwards into the liquid. The motion during collapse, up to the point where the minimum radius is attained, is determined by solving the equations of motion both in the Lagrangian and in the characteristic form. These are found to be in good agreement with each other and also with the approximate theory of Gilmore which is shown to be accurate over a wide range of Mach number. The liquid flow during the rebound, which occurs after the minimum radius has been attained, is determined from a solution of the Lagrangian equations. It is shown that an acoustic approximation is valid even for fairly high pressures, and this fact is used to determine the peak intensity of the pressure wave as it moves outwards at a distance from the center of collapse. It is estimated in the case of typical cavitation bubbles that such intensities are sufficient to cause cavitation damage.

I. INTRODUCTION

THE effect of compressibility in the collapse of a cavity in a liquid has been studied for some years. Rayleigh, who solved the problem for an incompressible liquid,[1] was the first to point out that this was necessary when he showed that pressures in the liquid adjacent to the cavity wall could reach very high values. However, no further progress was made until the study of underwater explosions[2] stimulated a renewed interest in the problem. Analytical theories were developed by Herring[3] and Trilling[4] which gave first-order acoustic corrections to Rayleigh's solution. Later, in an attempt to obtain a higher order of approximation, the so-called Kirkwood–Bethe hypothesis[2,5] was applied to the problem. This hypothesis forms the basis of a solution given by Gilmore[6] which has been used in a modified form in the work of Mellen[7] and Flynn.[8] Gilmore's solution has been found to be surprisingly accurate when compared with exact solutions obtained numerically, and this behavior is again confirmed by the results of this paper.

With the advent of high-speed computers, numerical solutions of the exact equations of compressible flow became possible, and several integrations of the equations in their characteristic form have been carried out. Gilmore[9] obtained one such solution which he used to compare with his approximate theory, while Hunter[10] performed a similar calculation which acted as a basis for a similarity solution valid in the immediate neighborhood of the collapse point of an empty cavity. Brand[11] has carried out some additional computations of this type. So far, however, there has been no numerical solution which described in detail the rebound of the cavity and the subsequent formation of a pressure wave traveling outwards into the liquid. The work of this paper is concerned with such a solution.

Classically, the problem which has usually been solved is that of the empty cavity. This is in fact a reasonably good model since the small amounts of gas, or vapor, which occur in a typical cavitation bubble have little effect on the motion of the interface until the final stage of the collapse. However, the contents of a cavity, even though they may be quite rarified initially, will have an important effect on the final pressures, and there will be a significant effect on the pressure wave which emanates after the rebound. The final stage of the collapse will be affected by the detailed behavior of the gas inside the cavity as well as by the quantity. Thus, an isothermal gas can be expected to produce a more

* Present address: Research Laboratories, General Motors Technical Center, Warren, Michigan.
[1] Lord Rayleigh, Phil. Mag. **34**, 94 (1917).
[2] R. H. Cole, *Underwater Explosions* (Princeton University Press, Princeton, New Jersey, 1948).
[3] C. Herring, Rept. No. 236, Office of Scientific Research and Development (1941).
[4] L. Trilling, J. Appl. Phys. **23**, 14 (1952).
[5] J. G. Kirkwood and H. A. Bethe, Rept. No. 558, Office of Scientific Research and Development (1942).
[6] F. R. Gilmore, Rept. No. 26-4, Hydrodynamics Laboratory, California Institute of Technology, Pasadena, California (1952).
[7] R. H. Mellen, J. Acoust. Soc. Am. **28**, 447 (1956).
[8] H. Flynn, Tech. Memo. No. 38, Acoustics Research Laboratory, Harvard University (1957).
[9] F. R. Gilmore, Discussion in "Symposium on Naval Hydrodynamics, National Academy of Science" (National Academy of Sciences–National Research Council, Publication No. 515, 1957).
[10] C. Hunter, J. Fluid Mech. **8**, 241 (1960).
[11] R. S. Brand, Tech. Rept. No. 34, Division of Applied Mathematics, Brown University, Providence, Rhode Island, (1960).

violent collapse than an equal amount of adiabatic gas. It is not our purpose in this paper to consider the physical behavior at the final collapse in detail. Such a task would in fact be quite difficult. The main interest here is in the effect of the liquid compressibility, so that the behavior of the contents of the cavity will merely be approximated by a simple gaslike model. The cavity will be assumed to contain a uniform gas whose pressure varies according to the law $p \propto \rho^{\gamma}$ where ρ is the density. The index γ can be varied so as to simulate different types of gross behavior of the gas. The value $\gamma = 1$ implies that the gas is isothermal, while $\gamma > 1$ implies that some of the heat of compression is being retained in the gas. This model which simulates the internal gas by this simple pressure–density relation has the additional advantage of avoiding the singularities which would occur at the final collapse point if the cavity were empty. The sudden compression of a gas or vapor inside a cavity can be expected to raise its temperature, and there will therefore be a rise in temperature in the liquid at the interface due to conduction and condensation. These effects will be confined to a very narrow region,[12] and hence they will be neglected. A sudden compression of a liquid does not cause a significant increase in temperature[13] so that it can be assumed to behave according to the well-known law $\rho^n \propto (p + B)$ where B and n are constants. The results presented here are for water with B equal to 3000 atm and the index n equal to 7. The validity of these values has been discussed elsewhere.[2] It is also assumed that the liquid is inviscid and that the motion remains spherically symmetric at all times. This description of a collapsing cavity is of course not completely accurate in detail, but it is considered to be sufficiently satisfactory for a study of the effects of liquid compression.

Numerical solutions of the equations of compressible flow for a collapsing cavity in water are presented here for both the characteristic Euler form and the Lagrangian form of the equations. A comparison between the two provides a check on the numerical accuracy. The Lagrangian solution was carried beyond the final collapse point into the region where the liquid rebounds and generates a compression wave which travels outwards from the collapse center. The solution was terminated when the compression wave had steepened into a vertical front. It could probably have been continued beyond this stage by using techniques such as those proposed by von Neumann and Richtmeyer[14] and Lax,[15] where an artificial viscosity term is introduced into the equations of motion to produce a continuous variation through the front. However, by this stage, sufficient information had already been obtained to estimate the order of magnitude of peak intensities at a distance from the collapse center, so that no attempt was made to undertake this additional calculation.

One of the principal purposes of this analysis was to determine whether the pressure pulses emanating from the collapse of cavitation bubbles could provide a mechanism for cavitation damage. The results given here show that such pressure pulses, which are equivalent in water to weak shocks with negligible entropy change, can be strong enough to cause damage to metals and other solids in the vicinity of the bubble.

II. FORMULATION OF THE PROBLEM

A spherical cavity containing a uniform gas is assumed to expand or contract in an infinite volume of liquid. For cavitation bubbles, the effect of gravity is generally quite small so that it will be neglected along with other asymmetric effects. Since the motion is spherically symmetric, it will be irrotational, i.e.,

$$\operatorname{curl} \mathbf{u} = 0, \tag{1}$$

where $\mathbf{u}$ is the fluid velocity. If the center of the bubble is chosen as the center of a system of spherical polar coordinates, then $\mathbf{u}$ will have only a single component u in the radial direction. The equations of motion expressing the conservation of mass and momentum for a spherically symmetric system are

$$D\rho/Dt = -\rho[(\partial u/\partial r) + (2u/r)], \tag{2}$$

$$\rho(Du/Dt) = -\partial p/\partial r, \tag{3}$$

where p and ρ are the pressure and density in the liquid, and the operator

$$D/Dt = (\partial/\partial t) + u(\partial/\partial r) \tag{4}$$

is the derivative with respect to time following the motion of the liquid. It is assumed that the liquid is isentropic and that the density and pressure are related by an equation of state of the form

$$(p + B)/(p_\infty + B) = (\rho/\rho_\infty)^n, \tag{5}$$

[12] M. S. Plesset and S. A. Zwick, J. Appl. Phys. **25**, 493 (1954).
[13] P. W. Bridgman, *The Physics of High Pressures* (G. Bell and Sons, London, 1949), p. 141.

[14] J. von Neumann and R. D. Richtmeyer, J. Appl. Phys. **21**, 232 (1950).
[15] P. D. Lax, Commun. Pure Appl. Math. **7**, 159 (1954).

where p_∞ and ρ_∞ are the pressure and density in the liquid at infinity. For water the constant B is given a value of 3000 atm, while the index n has the value 7. The use of this equation has been justified by several authors and is based on the fact that entropy changes in water are small even when very large pressure jumps are present. The upper limit on the accuracy of the formula appears to be for pressures of about 10^5 atm. The velocity of sound c in the liquid is defined by

$$c^2 = dp/d\rho = n(p + B)/\rho. \qquad (6)$$

The velocity of sound at infinity is therefore given by $c_\infty = [n(p_\infty + B)/\rho_\infty]^{\frac{1}{2}}$.

The equations of motion given above are in the Eulerian form. However, it is preferable here to use the Lagrangian form where the properties of the fluid are obtained by following the particle motion. To do this, it is necessary to ascribe to each particle of fluid a value of a certain parameter, y, which is defined by the expression

$$y = \int_{r(0,t)}^{r(y,t)} \rho r^2\, dr.$$

Hence,

$$\rho r^2(\partial r/\partial y) = 1. \qquad (7)$$

With this relation, Eq. (2) becomes

$$[\rho r^2(\partial r/\partial y)]_t = 0, \qquad (8)$$

but, since Eq. (7) is a solution of this equation, the two conditions are equivalent. The momentum relation, Eq. (3), is adapted by changing the particle derivative D/Dt to $\partial/\partial t$ and applying Eq. (7) to the right-hand side to change the variable from r to y.

On the walls of the cavity the pressure is given by the relation

$$P = p_0(R_0/R)^{3\gamma}, \qquad (9)$$

where p_0 is the initial pressure inside the cavity prior to collapse and R_0 is the initial radius of the cavity. Capital letters are used to denote the values of variables at the cavity wall. Thus R is the radius of the cavity and U its velocity. At infinity the liquid is at rest and the pressure and density have the values p_∞ and ρ_0. All that now remains before solving the equations is to specify the pressure and velocity in the liquid at some initial instant. For a cavity in an incompressible liquid, it is usual to suppose that the liquid is initially at rest and at a uniform pressure p_∞ and that the collapse is generated by a pressure discontinuity at the cavity wall, i.e., $p_\infty > p_0$. The instant this pressure discontinuity is allowed to take effect, it is felt throughout the entire volume of the liquid because of the incompressibility. The cavity wall then starts to accelerate inward from rest. For a compressible liquid there would be an initial jump in velocity during an infinitesimal period of time. Gilmore[6] has shown that this instantaneous increase in velocity is given by the relation

$$U_0 = 2(c_0 - c_\infty)/(n - 1), \qquad (10)$$

where $c_0 = [n(p_0 + B)/\rho_0]^{\frac{1}{2}}$. The maximum value of p_∞ used here is 10 atm, so that for water such a jump in velocity will always be quite small compared to sonic velocities. In fact, during the early stages of the collapse, the solution will be indistinguishable from that for the incompressible liquid. Because of their simplicity the same initial conditions will be used in the present analysis and, even though it is not really necessary, the effect of the initial jump in velocity will be included. The possibility of the formation of an initial compression wave in the gas or vapor inside the cavity is neglected as being unimportant.

A method widely used in the solution of the equations of compressible flow is the method of characteristics. The characteristic equations for spherically symmetric flow are

$$\partial r/\partial \alpha = (u + c)\, \partial t/\partial \alpha, \qquad (11)$$

$$\partial r/\partial \beta = (u - c)\, \partial t/\partial \beta, \qquad (12)$$

and

$$\frac{\partial u}{\partial \alpha} + \frac{c}{\rho}\frac{\partial \rho}{\partial \alpha} + \frac{2cu}{r}\frac{\partial t}{\partial \alpha} = 0,$$

$$\frac{\partial u}{\partial \beta} - \frac{c}{\rho}\frac{\partial \rho}{\partial \beta} - \frac{2cu}{r}\frac{\partial t}{\partial \beta} = 0.$$

The last two equations are not very suitable for finite-difference work, since ρ varies quite slowly in the liquid. Since $\delta\rho = \delta p/c^2$, the alternative forms

$$\frac{\partial u}{\partial \alpha} + \frac{1}{\rho c}\frac{\partial p}{\partial \alpha} + \frac{2cu}{r}\frac{\partial t}{\partial \alpha} = 0, \qquad (13)$$

$$\frac{\partial u}{\partial \beta} - \frac{1}{\rho c}\frac{\partial p}{\partial \beta} - \frac{2cu}{r}\frac{\partial t}{\partial \beta} = 0 \qquad (14)$$

are used. Equations (11) and (12) define the system of characteristic lines α and β where α is the outward-going characteristic and β the inward-going. The bubble wall motion is defined by the variable l and is determined from the expressions

$$\partial R/\partial l = U(\partial t/\partial l), \qquad (15)$$

and

$$\partial p/\partial l = (dp/dR)(\partial R/\partial l). \qquad (16)$$

Related to the method of characteristics is the approximate theory of Gilmore which is based on the Kirkwood–Bethe assumption. This states that the quantity $r[h(p) + \frac{1}{2}u^2]$ is a constant along an outward going characteristic, where

$$h(p) = \int_{p_\infty}^{p} \frac{dp}{\rho} = \frac{1}{(n-1)}(c^2 - c_\infty^2) \qquad (17)$$

is the enthalpy difference between the liquid at pressure p and at pressure p_∞ under isentropic conditions. This assumption can be expressed in the form of an equation containing the particle derivative as defined in Eq. (4). Since the cavity wall moves with the liquid, and since this motion can be expressed purely as a function of time, such particle derivatives can be changed into ordinary derivatives with respect to time, and the equation will become

$$R\frac{dU}{dt}\left[1 - \frac{U}{C}\right] + \frac{3}{2}U^2\left[1 - \frac{U}{3C}\right] = H\left[1 + \frac{U}{C}\right] + \frac{R}{C}\frac{dH}{dt}\left[1 - \frac{U}{C}\right], \qquad (18)$$

which governs the motion of the cavity wall. Given the initial values on the boundary, a second ordinary differential equation gives the liquid conditions along an outward-going characteristic line:

$$\frac{du}{dt} = \frac{R(H + \frac{1}{2}U^2)(u + c)}{r^2(c - u)} - \frac{2c^2u}{r(c - u)}. \qquad (19)$$

This is solved in conjunction with the Kirkwood–Bethe assumption and Eqs. (11) and (17).

For most problems of interest in bubble collapse, it can be assumed that $|H| \ll C^2$. With water for example, this inequality corresponds to a pressure difference $|p_0 - p_\infty| \ll 20\,000$ atm. Using this assumption, Gilmore was able to derive a simple relation between the bubble-wall velocity and the radius for the case of an empty cavity. This relation is

$$\left[\frac{R_0}{R}\right]^3 = \left[1 - \frac{U}{3C}\right]^4\left[1 + \frac{3\rho_\infty U^2}{2(p_\infty - p_0)}\right]. \qquad (20)$$

It is seen that as R tends to zero, U varies as $R^{-\frac{1}{4}}$, in contrast to incompressible theory[1] which gives U varying as $R^{-\frac{3}{2}}$.

The variables used in the above equations were nondimensionalized with respect to the initial radius R_0, the density ρ_∞, and velocity of sound c_∞ in the following way:

$$\rho' = \rho/\rho_\infty, \quad p' = p/\rho_\infty c_\infty^2, \quad u' = u/c_\infty,$$
$$t' = c_\infty t/R_0, \quad h' = h/c_\infty^2, \quad r' = r/R_0.$$

It will be supposed that all the variables and constants used above have already been nondimensionalized in this way, and the primes will be suppressed in the following. The equations of motion will remain unaltered by this change, but the liquid equation of state, Eq. (5), becomes

$$n(p + B) = \rho^n, \qquad (21)$$

and the pressure at the bubble wall given by Eq. (9) becomes

$$P = p_0 R^{3\gamma}. \qquad (22)$$

Equations (10) and (17) are modified by setting $c_\infty = 1$.

It is seen from the nondimensional form that the solutions are actually independent of the initial radius R_0, i.e., that the same pressures and velocities are obtained regardless of the scale. (The elapsed time is of course proportional to R_0). This result is due to the fact that compressible flow equations contain only first-order derivatives. If effects such as heat conduction, viscosity, and surface tension were included, the solutions would then become dependent on R_0, the dependence becoming stronger with a decrease in R_0.

Solutions to the problem of bubble collapse were obtained by means of a high-speed computer using the three methods given above. First, Gilmore's method was used to establish an approximate solution for the motion of the cavity wall. The initial conditions given above were applied, together with the initial value of the velocity given by Eq. (10). The validity of this solution was checked against the simple incompressible flow theory for the early stages of the motion. Some exploratory calculations were then carried out using the method of characteristics in the region where the Mach number U/C at the cavity wall was of the order of 0.1. These results were compared, both on the wall and in the interior of the liquid, with results obtained by Gilmore's method, and it was established that the method was quite accurate in this region. This result was of course to be expected, since it can be shown[6] that Gilmore's theory is accurate to terms at least of the order of $(u/c)^2$. Thereafter, Gilmore's method was used to provide initial values in the subsonic ($U/C \sim 0.1$) region for the solution of the Lagrangian and the Eulerian characteristic equations. Such a procedure was necessary because of the large amount of computing time required to perform an exact solution starting from the initial stages of the motion. The Lagrangian and the characteristic solutions were carried to the final collapse point and, since they are both supposed to be exact, they provided a mutual check on the numerical accuracy up to this stage. Comparisons were made for points on the cavity wall and in its vicinity, and the discrepancies which occurred were estimated to be at most about one percent. The Lagrangian solution was then carried on into the region of rebound up to the point where the shock wave formed. Solutions were obtained for a variety of conditions such as might occur in cavitation. The values used for the ambient pressure p_∞ in the liquid were 1 and 10 atm. The initial pressure p_0 in the gas was varied from 10^{-1} to 10^{-4} atm, while the index γ for the gas was given the values 1 and 1.4. The problems were programmed for an IBM 7090. The numerical methods which were used have been described elsewhere.[16]

III. CALCULATED RESULTS

The first set of calculations were concerned with the motion of the bubble wall and with the gross effect of the gas content of the bubble. Figures 1–3

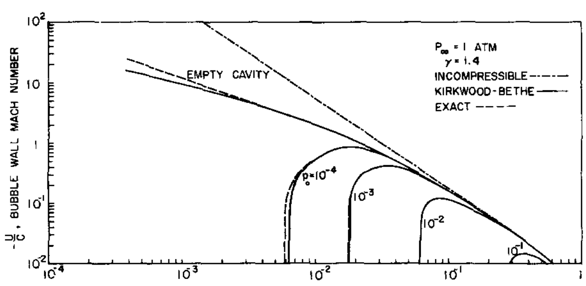

FIG. 1. The bubble-wall Mach number as a function of the bubble radius for decreasing gas content. The gas content is determined by its initial pressure p_0 in atmospheres. The index γ has the value 1.4, and the ambient pressure p_∞ is 1 atm.

show the variation of the bubble-wall Mach number with the radius for a variety of conditions. The amount of gas is determined by the initial pressure p_0, and the behavior of the gas is varied by changing the index γ. Figure 1 shows the collapse of a bubble for different initial pressures p_0 with $\gamma = 1.4$ with an external pressure p_∞ of 1 atm. It is evident that, as the amount of gas diminishes, the motion of the bubble wall becomes more and more rapid during the final stages of the collapse. In the case of the empty cavity, the velocity increases without limit

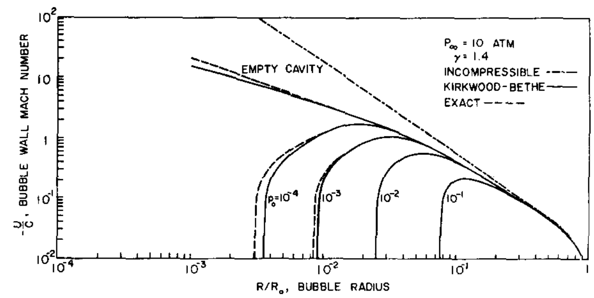

FIG. 2. The bubble-wall Mach number as a function of the bubble radius for decreasing gas content. The index γ has the value 1.4, and the ambient pressure p_∞ is 10 atms.

as the bubble grows smaller. The exact solutions are compared with solutions derived from Gilmore's theory and from the theory of an empty cavity in an incompressible liquid. It is apparent that the compressibility becomes very important during the final stages of the motion. Figure 2 shows the result of increasing the external pressure p_∞ from 1 to 10 atm. A comparison with Fig. 3 shows that such an increase does not affect the collapse so much as a change in γ from 1.4 to 1. Thus, a gas which behaves isothermally should, in general, produce a more violent collapse than an adiabatic gas collapsing under a high external pressure. In all these results, it is evident that the predictions of Gilmore's theory, based on the Kirkwood–Bethe assumption, continue to be surprisingly accurate.

For the empty cavity under 1 atm external pressure, the bubble-wall velocity tends to infinity as $(R_0/R)^{0.785}$. The value of the index was found to be the same when p_∞ was 10 atm. This result is in good agreement with a similar estimate by Hunter.[10] By comparison, Gilmore's theory predicts a rate varying as $(R_0/R)^{0.5}$, while the incompressible theory gives $(R_0/R)^{1.5}$.

The remaining results are concerned with the flow in the liquid around the collapsing bubble and were obtained for the two cases, $p_0 = 10^{-3}$ and 10^{-4} atm, with $\gamma = 1.4$ and an external pressure of 1 atm.

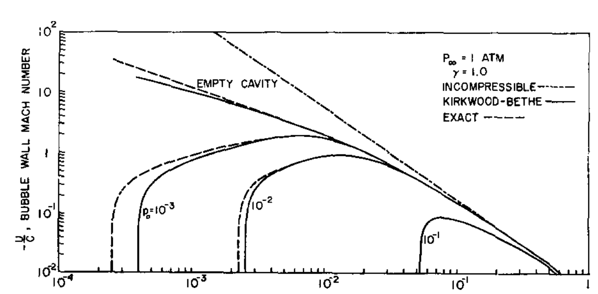

FIG. 3. The bubble-wall Mach number as a function of the bubble radius for decreasing gas content. The index γ has the value 1.0, and the ambient pressure p_∞ is 1 atm.

[16] R. Hickling and M. S. Plesset, Rept. No. 85-24, Engineering Division, California Institute of Technology, Pasadena, California (1963).

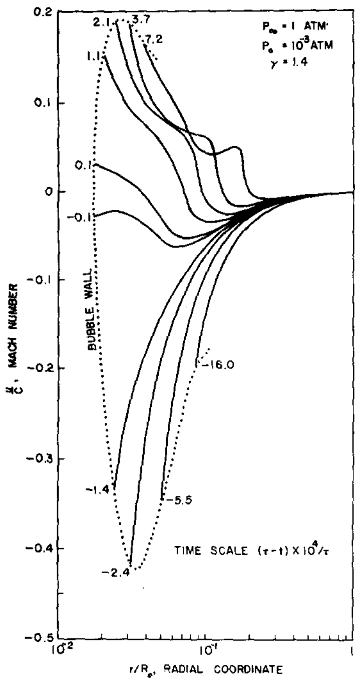

FIG. 4. Variation of Mach number with distance from the bubble wall for different instants in time during the collapse and rebound of the bubble. The initial internal pressure p_0 in the gas is 10^{-3} atms. The ambient pressure p_∞ is 1 atm.

In both these cases, the liquid rebounds and forms a compression wave which moves outwards and steepens into a shock front. The occurrence of the shock front causes the numerical solution to become unstable so that the results are only presented up to this point. Methods are available[14,15] for overcoming this instability, and proceeding with the solution. This continuation, however, would have involved an additional program of computation and, since it is not necessary for the purposes of this paper, it was not pursued. Figures 4–7 show the distributions of Mach number and pressure in the liquid. These are given for successive instants in time which is expressed in terms of τ, the time required for the bubble to collapse from the initial radius R_0 to the final minimum radius. The formula used to determine the time scale is $(\tau - t)10^4/\tau$, where t is the time elapsed from the start of the motion. The collapse time τ was determined accurately from the numerical solutions. It can also be estimated from the theory for an empty cavity in an incompressible liquid. The incompressible relation gives

$$\tau \sim 0.91 R_0 (\rho_\infty / p_\infty)^{\frac{1}{2}}.$$

Estimates from this agree to within less than 1% with values obtained in the calculated examples.

Figures 5(b) and 7(b) show the pressure wave forming and traveling outwards into the liquid. Because of the compressibility, the change in the direction of motion of the interface is communicated to the liquid only by the passage of the pressure wave, and hence a reversal of flow occurs through it, as shown in Figs. 4 and 6. The shock wave forms fairly rapidly when the initial gas pressure is 10^{-4} atm, because the final collapse pressures are high. In the case where the initial gas pressure is 10^{-3} atm, the pressure front does not steepen so quickly. When $p_0 = 10^{-2}$ atm, the final pressure at the cavity wall is seen, from Fig. 1, to be about 10^3 atm. For liquid pressures of this magnitude, the compression wave will steepen only very slowly and the rate with which it steepens is reduced as it moves outwards because of geometric attenuation. For such cases, the compression will behave like an acoustic

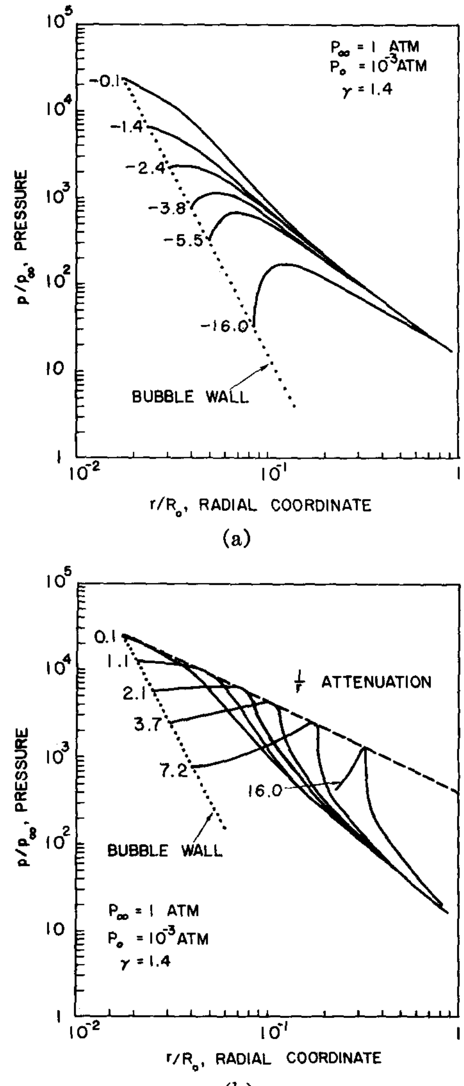

FIG. 5. (a) Variation of pressure with distance from the bubble wall for different instants in time during the collapse of the bubble. The conditions correspond to those of Fig. 4. (b) Variation of pressure with distance from the bubble wall for different instants in time during the rebound of the bubble.

wave which does not alter much in form as it propagates. For larger amounts of gas inside the bubble, the intensity of the compression will diminish even further until only a very weak pulse emanates from the collapse.

Figure 5(b) shows that the acoustic approximation is reasonably valid even in the case where a shock front develops. The pressure front gradually steepens, but the wave remains of approximately the same thickness and form and attenuates as $1/r$ as it propagates outwards from the collapse center. The last stage of the calculations shows that the peak pressure in the wave is about 1000 atm at $R/R_0 \sim 0.3$. Beyond this point, not much change will occur due to dispersion, and the losses due to entropy changes through the shock will be negligible.[2] Hence, the pressure pulse will propagate outwards like an acoustic wave, and at $R/R_0 \sim 2$ the peak intensity will be of the order of 200 atm. For small amounts of gas, the peak pressures will be larger although they are somewhat more susceptible to the effect of dispersion. Figure 7(b), for example, shows that the peak pressure still has an approximate $1/r$ dependence giving an intensity of the order of 1000 atm when $R/R_0 \sim 2$. A much stronger effect, however, would result if the gas

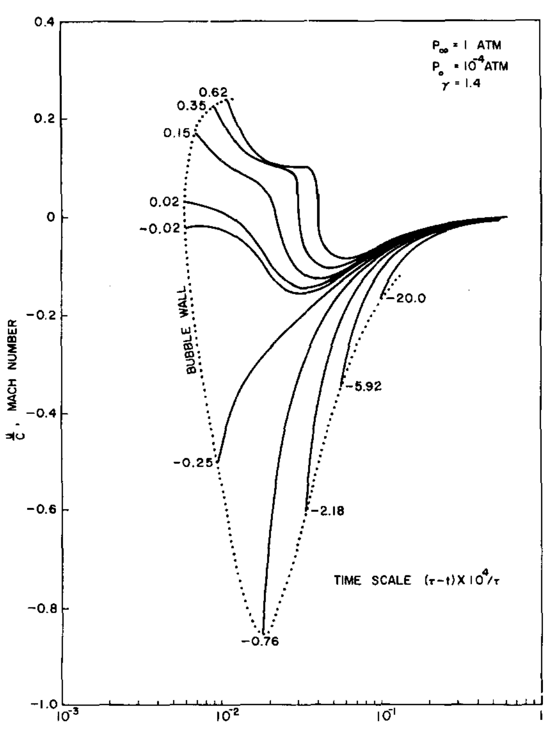

FIG. 6. Variation of Mach number with distance from the bubble wall for different instants in time during the collapse and rebound of the bubble. The initial pressure p_0 in the gas is 10^{-4} atm. The ambient pressure is 1 atm.

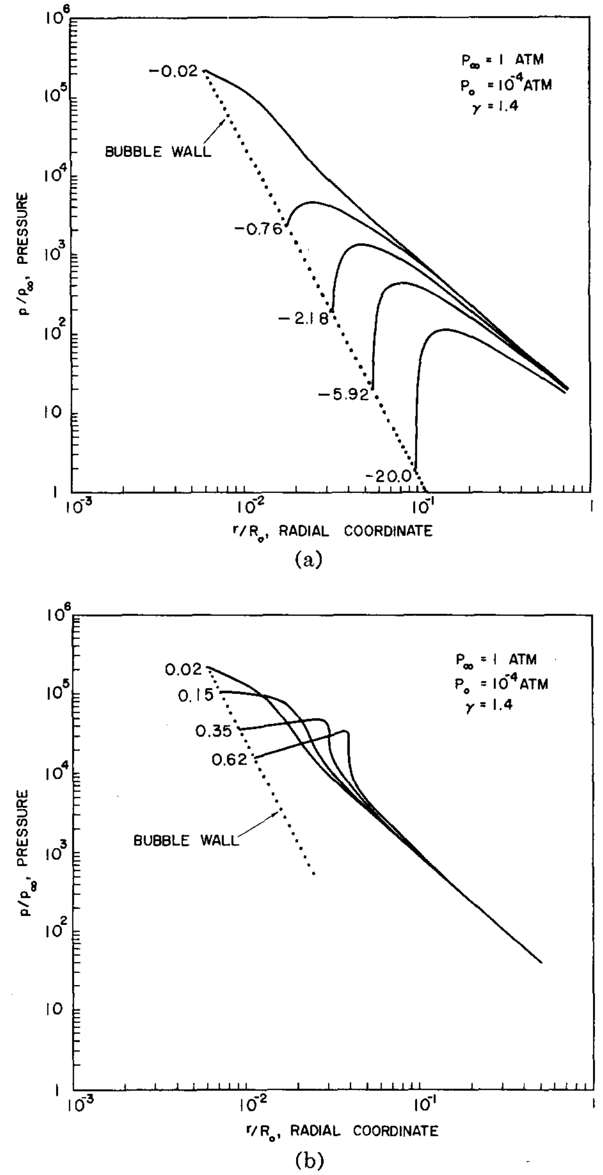

FIG. 7. (a) Variation of pressure with distance from the bubble wall for different instants in time during the collapse of the bubble. The conditions correspond to those of Fig. 6. (b) Variation of pressure with distance from the bubble wall for different instants in time during the rebound of the bubble.

inside the bubble were to lose some of its heat of compression. Figures 1 and 3 show that if the gas is isothermal, it collapses to a much smaller radius for the same initial amount of gas when compared to the adiabatic condition. It has been shown[17] for values of $R_0 < 1$ mm, that heat losses from the gas into the liquid are quite significant. For bubbles of this size therefore, a more violent collapse would occur, and pressure-wave intensities of several thousand atmospheres could be expected. In such

[17] R. Hickling, J. Acoust. Soc. Am. **35**, 7 (1963).

cases, of course, dispersion would noticeably affect the shape of the pulse.

IV. APPLICATION OF RESULTS TO THE THEORY OF CAVITATION DAMAGE

A well-known consequence of the formation of cavitation bubbles in a liquid is the damage which can occur to adjacent solid surfaces. Such damage has been shown[18] to be largely mechanical in origin, although the precise mechanism is as yet undecided. It has been suggested for example that the damage is caused by bubbles adhering to the solid and collapsing asymmetrically in such a way that jets of liquid form and strike the surface at high speed. There is some indication[19] that such a mechanism might operate. The results of the present paper show that damage could certainly result from pressure waves originating from bubbles situated at a short distance from the surface. This mechanism would avoid the requirement that the destructive bubbles be only those which are adhering to the surface.

Compared to usual kinds of material damage, cavitation damage is very localized. Cavitation bubbles are usually of the order of 10^{-2} cm in size, i.e., of the order of the grain size in a typical metal. Hence the attack from the cavitation bubbles will be directed towards the individual grains rather than towards a large group of them. This situation implies that cavitation stresses need only to be of the order of the yield stresses of the individual crystals. In addition, the fact that the damage usually results after an exposure to cavitation of a certain duration, indicates the existence of some kind of fatigue process. The cold working which has been observed[18] during the early stage of the exposure lends support to this conclusion. It can be estimated, therefore, that pressures of the order of 2000 or 3000 atm should be sufficient to cause most of the kinds of cavitation damage which have been observed. It is possible, of course, that local stress concentrations may occur due to irregularities in surface finish and grain structure, and hence the applied pressures need not be as high as this.

The results discussed in the previous section indicate that pressure waves of the right order of intensity can occur from collapsing cavitation bubbles. Such bubbles usually contain only very small amounts of vapor and gas, and are usually less than 1 mm in size prior to collapse. Hence the collapse process should be violent enough to produce pressure waves of the right intensity. This estimate is based on the assumption of a spherically symmetric collapse. Both the presence of the boundary and possible instability effects[20] during the final stages of the motion can upset this symmetry and presumably reduce the intensity of the resulting pressure waves. It is believed that the results for the spherical motion give a sufficiently good indication of the order of magnitude of the pressures which can occur in practice.

ACKNOWLEDGMENTS

This study was supported by the Office of Naval Research.

[18] M. S. Plesset and A. T. Ellis, Trans. Am. Soc. Mech. Eng. **77**, 1055 (1955).

[19] C. F. Naude and A. T. Ellis, Rept. No. E-108.7, Hydrodynamics Laboratory, California Institute of Technology, Pasadena, California (1960).

[20] M. S. Plesset and T. P. Mitchell, Quart. J. Appl. Math. **13**, 419 (1956).