Power Delay Profile and Noise Variance Estimation for OFDM

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***Abstract*— In this letter, we present cyclic-prefix (CP) based noise-variance and power-delay-profile estimators for Orthogonal Frequency Division Multiplexing (OFDM) systems. Signal correlation due to the use of the CP is exploited without requiring additional pilot symbols. A heuristic estimator and a class of approximate maximum likelihood (ML) estimators are proposed. The proposed algorithms can be applied to both unitary and non-unitary constellations. These algorithms can be readily used for applications such as minimum mean-square error (MMSE) channel estimation.**

***Index Terms*— OFDM, channel estimation, SNR estimation.**

I. Introduction

Noise variance or (equivalently) signal to noise ratio (SNR) is an important measure of channel quality. Their estimation is hence required in many communication applications such as adaptive modulation, turbo coding and others. Several SNR estimation algorithms have been proposed for systems using unitary constellations (i.e., binary phase shift keying (BPSK) and quaternary phase shift keying (QPSK)) over AWGN channels [1], [2]. They can be classified as data-aided (DA), which requires pilot symbols, and non-data aided (NDA) estimators, which do not. In [3], an NDA estimator is extended to systems with non-unitary constellations over Rayleigh fading channels.

In orthogonal frequency division multiplexing (OFDM) systems, noise variance and power delay profile (PDP) are needed for many algorithms such as minimum mean-square error (MMSE) channel estimation and ML frequency offset estimation. In [4], a noise-variance estimator is proposed that directly uses the receiver statistics. A subspace approach is presented in [5] that uses the sample covariance matrix of the received signal. However, both algorithms are DA estimators, which constitute a bandwidth loss. The estimation of the number of multipath gains and associated time delays has been proposed in [6], where pilot symbols are also needed, and channel multipath power and noise variance are required. In [7], a noise variance and SNR estimator that uses training symbols is developed for multiple antenna OFDM systems. Except for these contributions, to the best of our knowledge, no other NDA noise variance and PDP estimators for OFDM systems have been published to date.

In this letter, we develop noise-variance and PDP estimators for OFDM systems over multipath fading channels; the key is to use the fact that the cyclic prefix contains the repeated samples which will introduce a special correlation structure on the received samples. The noise variance, the number of multipath taps, multipath time delays and powers are jointly estimated without pilots. The maximum likelihood (ML) function for the estimated parameters is derived, resulting in an ML estimator.

II. Noise Variance and PDP Estimator

In OFDM, source data are grouped and mapped into $X_k \in \mathcal{Q}$, where $\mathcal{Q}$ is a complex signal constellation, and $E\{|X_k|^2\} = 1$. Complex data are modulated by inverse discrete Fourier transform (IDFT) on N parallel subcarriers. The symbol interval and block interval are denoted by T_s and NT_s. The resulting OFDM symbol during the mth block interval that comprises N samples is given by

$$x_n(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(m) e^{\jmath(2\pi kn/N)},\ n = 0, 1, 2, \cdots, N-1. \tag{1}$$

The guard interval, inserted to prevent inter-block interference, includes a cyclic prefix that replicates the end of the IFFT output samples. The number of samples in the guard interval N_g is assumed to be larger than the delay spread of the channel. The signal is transmitted over a multipath fading channel given by

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l) \tag{2}$$

where L is the total number of multipaths, $h_l \sim \mathcal{CN}(0, \sigma_l^2)$, and τ_l is the delay of the l-th path. The received signal after sampling is given by

$$y_n(m) = \sum_{l=0}^{L-1} h_l x_{n-d_l}(m) + w_n(m) \tag{3}$$

where $w_n \sim \mathcal{CN}(0, \sigma^2)$ is an Additive White Gaussian Noise (AWGN), and $d_l = \lfloor \tau_l / T_s \rceil$ is the delay normalized by T_s. For simplicity, we round d_l to an integer without considering leakage. However, the correlation approach in this paper may also be extended to fractional d_l. We assume perfect synchronization, and that the channel is invariant within each OFDM block. If there exists a synchronization error, a decision directed algorithm may be applied using our

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proposed parameter estimators and the joint ML time and frequency offset estimator in [8].

At the border between two OFDM blocks ($-N_g \le n < 0$), the received signal samples can be written as

$$\begin{aligned} y_n(m) = &\sum_{l=0}^{L-1} h_l x_{n-d_l}(m) U(n-d_l) \\ &+ \sum_{l=0}^{L-1} h_l x_{N+n-d_l}(m-1) U(d_l - n) + w_n(m) \end{aligned} \tag{4}$$

where $U(\cdot)$ is the step function. The correlation between each received signal sample over the CP interval and its corresponding sample at the end of the OFDM block can thus be given by

$$\begin{aligned} &E\{y_{-k}(m) y^*_{N-k}(m)\} \\ &= \begin{cases} \sigma_y^2 + \sigma^2 & 0 < k \le N_g - d_{L-1} \\ \sum_{l=0}^{L-1} \sigma_l^2 U(N_g - k - d_l) & N_g - d_{L-1} < k \le N_g - d_0 \\ 0 & N_g - d_0 < k \le N_g \end{cases} \end{aligned} \tag{5}$$

where $\sigma_y^2 = \sum_{l=0}^{L-1} \sigma_l^2$, and $k = 1, \ldots, N_g$. The expectation in (5) is taken with respect to both h_l and $x_n(m)$.

When L is large, $y_n(m)$ can be modelled **approximately** as complex Gaussian using the central limited theorem, and the probability density function (pdf) is given by

$$f(y_n(m)) = \frac{\exp\left(-\frac{|y_n(m)|^2}{\sigma_y^2 + \sigma^2}\right)}{\pi(\sigma_y^2 + \sigma^2)}. \tag{6}$$

Samples $y_{-k}(m)$ and $y_{N-k}(m)$ are jointly Gaussian with pdf

$$\begin{aligned} &f(y_{-k}(m), y_{N-k}(m)) \\ &= \frac{\exp\left(-\frac{|y_{-k}(m)|^2 + |y_{N-k}(m)|^2 - 2\rho_k \Re\{y_{-k}(m) y^*_{N-k}(m)\}}{(\sigma_y^2+\sigma^2)(1-\rho_k^2)}\right)}{\pi^2(\sigma_y^2 + \sigma^2)(1-\rho_k^2)} \end{aligned} \tag{7}$$

where

$$\begin{aligned} \rho_k &= \left| \frac{E\{y_{-k}(m) y^*_{N-k}(m)\}}{\sqrt{E\{|y_{-k}(m)|^2\} E\{|y_{N-k}(m)|^2\}}} \right| \\ &= \frac{\sum_{l=0}^{L-1} \sigma_l^2 U(N_g - k - d_l)}{\sum_{l=0}^{L-1} \sigma_l^2 + \sigma^2}. \end{aligned} \tag{8}$$

Therefore, the proposed estimator is only approximate ML.

We use M OFDM blocks to estimate those parameters and assume that they remain unchanged during the M blocks. Define $\mathbf{p} = [\sigma_0^2, \ldots, \sigma_{L-1}^2]$, $\mathbf{d} = [d_0, \ldots, d_{L-1}]$ and $\mathbf{y} = [y_{-N_g}(1), y_{-N_g+1}(1), \ldots, y_{N-1}(M)]$. Using (6) and (7) and assuming the M OFDM blocks are independent, the log-likelihood function of $\mathbf{y}$ conditioned on $\sigma^2, \mathbf{p}, \mathbf{d}$ can be written as

$$\begin{aligned} &\Lambda(\mathbf{y}|\sigma^2, \mathbf{p}, \mathbf{d}) \\ &= \sum_{m=1}^{M} \log\left(\prod_{k=1}^{N_g} f(y_{-k}(m), y_{N-k}(m)) \prod_{k=0}^{N'} f(y_k(m))\right) \\ &= -M\left(\sum_{k=1}^{N_g} \frac{a_k - \rho_k b_k}{c(1-\rho_k^2)} + \log(c(1-\rho_k^2)) + \sum_{k=0}^{N'} \frac{g_k}{c} + \log(c)\right) \end{aligned} \tag{9}$$

where $N' = N - N_g - 1$,

$$\begin{aligned} a_k &= \frac{\sum_{m=1}^{M} |y_{-k}(m)|^2 + |y_{N-k}(m)|^2}{M} \\ b_k &= \frac{\sum_{m=1}^{M} \Re\{y_{-k}(m) y^*_{N-k}(m)\}}{M} \\ g_k &= \frac{\sum_{m=1}^{M} |y_k(m)|^2}{M},\ c = \sigma_y^2 + \sigma^2. \end{aligned} \tag{10}$$

Since (9) involves many variables, to simplify the joint parameters' estimation, we take a suboptimal way. We first estimate c by maximizing only the last sum in (9):

$$\hat{c} = \frac{\sum_{k=0}^{N'} g_k}{N - N_g} = \frac{\sum_{k=0}^{N'} \sum_{m=1}^{M} |y_k(m)|^2}{(N - N_g)M}. \tag{11}$$

From (11), we find $\hat{c}$ is the time average estimation of $\sigma_y^2 + \sigma^2$, and hence an estimate of c is given by $\hat{c}$. Substituting $\hat{c}$ back into the first summation of (9) and maximizing ρ_k individually, we get the estimate for ρ_k as the real root of the equation

$$2\hat{c}\rho^3 - b\rho^2 - 2(\hat{c} - a_k)\rho - b = 0. \tag{12}$$

We then compute the value s_k as

$$s_k = \begin{cases} \rho_{N_g} \hat{c} & k = 1 \\ (\rho_{N_g - k+1} - \rho_{N_g - k+2})\hat{c} & k = 2, \ldots, N_g \end{cases}. \tag{13}$$

A threshold value is set as $\alpha\hat{c}$, where α is a constant less than 1. If $s_k > \alpha\hat{c}$, it is identified as a path; s_k is the estimate of path power, and k is the estimate of delay time. The number of paths is estimated as the number of s_k that $s_k > \alpha\hat{c}$. We denote the maximum delay time as $d_{\max}$. The noise variance can thus be estimated as

$$\hat{\sigma}^2 = \hat{c}\left(1 - \frac{\sum_{k=1}^{N_g - d_{\max}+1} \hat{\rho}_k}{N_g - d_{\max} + 1}\right). \tag{14}$$

If we look directly at the structure of OFDM block, in the absence of noise, $y_{-k}(m) = y_{N-k}(m)$ for $k = 1, \ldots, N_g - d_{\max} + 1$. The noise variance can be obtained alternatively as

$$\hat{\sigma}^2 = \frac{\sum_{m=1}^{M} \sum_{k=1}^{N_g - d_{\max}+1} |y_{N-k}(m) - y_{-k}(m)|^2}{2M(N_g - d_{\max} + 1)}. \tag{15}$$

Using the results of (11) and (14) or (15), SNR can be estimated by

$$\mathrm{SNR} = \frac{\hat{c}}{\hat{\sigma}^2}. \tag{16}$$

Note that (15) can only be used to estimate σ^2.

Note that the SNR considered in this paper is the average SNR (averaged over the channel, data and noise realizations). Our proposed algorithm cannot estimate the instantaneous SNR, where a fixed channel is considered.

III. Simulation Results

We now investigate the performance of our proposed estimators. We assume an OFDM system using QPSK with $N = 64$ subcarriers, and CP length $N_g = 16$. A $L = 6$ channel model is used. The power profile is given by $\mathbf{p} = [0.189, 0.379, 0.239, 0.095, 0.061, 0.037]$, and the delay profile after sampling is $\mathbf{d} = [0, 1, 2, 4, 6, 8]$. Each path is an independent, zero-mean complex Gaussian random process.

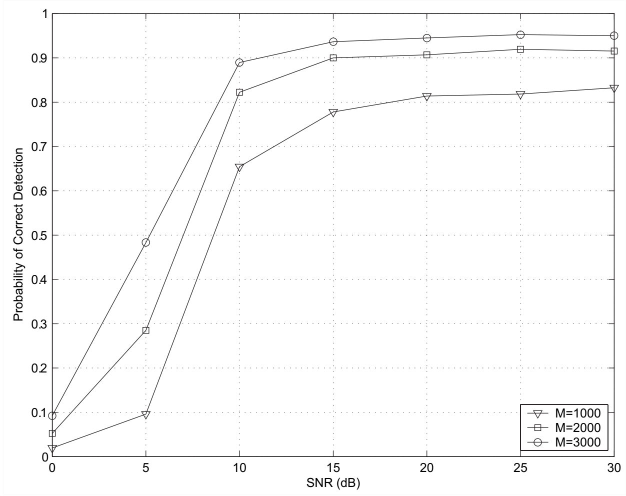

Fig. 1. The probability of correct detection of the number of paths.

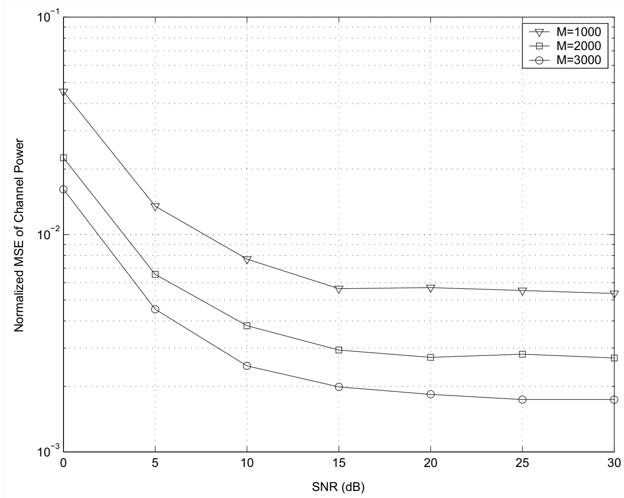

Fig. 2. The NMSE of the channel power estimation for the 3rd path.

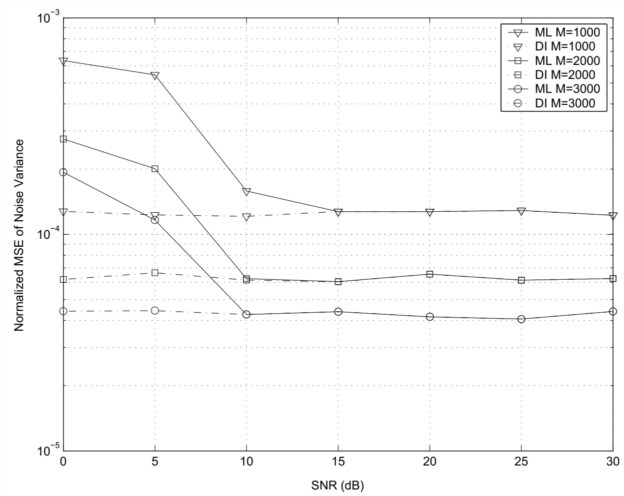

Fig. 3. The NMSE of the noise variance estimation using different estimators.

Fig. 1 shows the probability of correct detection of the number of paths using our proposed algorithm with different M. The threshold parameter is set to $\alpha = 0.01$. In low SNR, the paths with smaller power are dominated by the noise, and there may be many paths larger than the threshold. Therefore, the number of paths may be overestimated. The probability of correct detection increases in high SNR. With increasing M, the probability of detection error decreases.

Fig. 2 presents the normalized mean square error (NMSE) of the channel power estimation for the 3rd path (arbitrarily chosen), where the NMSE is defined as $\text{NMSE} = E\{(\hat{\sigma}_3^2 - \sigma_3^2)^2\}/\sigma_3^4$. The channel power is overwhelmed by the noise in low SNR. In high SNR, the NMSE becomes constant since the number of paths cannot be 100% correctly detected. The NMSE is improved by increasing M.

Fig. 3 shows the NMSE of the noise variance estimation using different estimators, where the NMSE is defined as $\text{NMSE} = E\{(\hat{\sigma}^2 - \sigma^2)^2\}/\sigma^4$. The estimator using (14) is denoted as ML, and that using (15) is denoted as direct estimator or (DI). At low SNR, the DI method performs better than the approximate ML method since the probability of $d_{\max}$ detection is higher for the DI method. In high SNR, both DI and ML perform identically. With the increase of M, the performance of both estimators improve.

IV. Conclusion

In this paper, we have presented noise-variance and power-delay-profile estimators using the CP in each OFDM block. The correlation structure due to the use of the CP has been exploited to derive our estimators, and hence pilot symbols are not needed. A direct heuristic noise variance estimator has also been proposed. Simulation results show that our proposed estimators provide an effective way to estimate the channel parameters. The results in this paper may be used to improve the performance and reduce the complexity of channel estimators for OFDM systems.

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