Cabibbo-suppressed nonleptonic D decays

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We discuss why an extension of the ideas used to explain the $\Delta I = 1/2$ rule in kaon and hyperon decays does not lead to an analogous large enhancement in the rates of Cabibbo-suppressed nonleptonic D-meson decays. The possibility of seeing the contribution of diagrams with a virtual b-quark loop through interference effects is also discussed.

The analysis of nonleptonic weak decays has proven to be a difficult problem involving complex features of the strong interactions. However, some progress has been made[1,2,3] by using renormalization-group techniques to generate, from the standard Hamiltonian in which W bosons and various heavy quarks appear, an effective field theory involving only "light" (u, d, and s for K decays and u, d, s, and c for D decays) quarks. It is then hoped that enough features of the strong interactions have been incorporated so that a simple estimate (for example, by inserting the vacuum in all possible ways) of hadronic matrix elements of the operators in the effective Hamiltonian will lead to an approximate understanding of nonleptonic decays.

In the case of kaon (or hyperon) decays, when W-exchange graphs are replaced by effective four-fermion interactions involving only the u, d, s, and c quark fields, the quantum-chromodynamics (QCD) corrections enhance the Wilson coefficients of operators with $I=\frac{1}{2}$ relative to the Wilson coefficients of $I=\frac{3}{2}$ operators.[4] These operators still have the $(V-A)\times(V-A)$ form typical of W exchange. However, when the charmed quark is treated as heavy and removed to generate an effective theory involving only u, d, and s quarks, operators with the structure $(V-A)\times(V+A)$ appear. Although these operators have small Wilson coefficients, it has been suggested that the matrix elements of such operators are greatly enhanced over the matrix elements of operators with the usual $(V-A)\times(V-A)$ chiral structure.[1] Since the $(V-A)\times(V+A)$ operators are purely $I=\frac{1}{2}$, a further enhancement of the $\Delta I=\frac{1}{2}$ amplitudes over $\Delta I=\frac{3}{2}$ amplitudes occurs. It appears that one can thus qualitatively account for the $\Delta I=\frac{1}{2}$ rule in nonleptonic kaon and hyperon decays.[5]

A question which naturally arises is whether there is an analogous effect in Cabibbo-suppressed nonleptonic D decays. The effective Hamiltonian relevant for Cabibbo-suppressed nonleptonic D decays is generated by a three step process in which the W boson, t quark, and b quark are sequentially removed from explicitly appearing in the theory. The removal of the W boson and heavy t quark leads to an adjustment of the coefficients of the operators which appear in the effective Hamiltonian in the absence of strong interactions. In addition, when the b quark is removed, new operators, which had zero coefficients in the absence of strong interactions, appear due to operator mixing. Some of these new operators have the chiral structure $(V-A)\times(V+A)$. For a typical set of parameters[6] the effective Hamiltonian for Cabibbo-suppressed nonleptonic D decays is[7]

$$\mathcal{H}_{\text{eff}} = -\frac{G}{2\sqrt{2}} s_1 c_1 c_2 \Bigg[(0.70\,\mathcal{O}^{(+)} + 2.03\,\mathcal{O}^{(-)}) - \left(s_3^{\,2} + \frac{s_2 s_3 c_3 e^{i\delta}}{c_1 c_2}\right)(-1.33\,\mathcal{O}_1 + 2.73\,\mathcal{O}_2 + 0.026\,\mathcal{O}_3 - 0.061\,\mathcal{O}_4 + 0.018\,\mathcal{O}_5 - 0.074\,\mathcal{O}_6)\Bigg] + \text{H.c.}, \quad (1)$$

where

$$\begin{aligned}
\mathcal{O}^{(\pm)} &= (\bar{c}_\alpha s_\alpha)_{V-A}(\bar{s}_\beta u_\beta)_{V-A} \pm (\bar{c}_\alpha u_\alpha)_{V-A}(\bar{s}_\beta s_\beta)_{V-A} - (\bar{c}_\alpha d_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A} \mp (\bar{c}_\alpha u_\alpha)_{V-A}(\bar{d}_\beta d_\beta)_{V-A},\\
\mathcal{O}_1 &= (\bar{c}_\alpha u_\alpha)_{V-A}(\bar{s}_\beta s_\beta)_{V-A},\\
\mathcal{O}_2 &= (\bar{c}_\alpha u_\beta)_{V-A}(\bar{s}_\beta s_\alpha)_{V-A},\\
\mathcal{O}_3 &= (\bar{c}_\alpha u_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V-A} + (\bar{d}_\beta d_\beta)_{V-A} + (\bar{s}_\beta s_\beta)_{V-A} + (\bar{c}_\beta c_\beta)_{V-A}],
\end{aligned} \quad (2)$$

$$\mathcal{O}_4=(\bar{c}_\alpha u_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V-A}+(\bar{d}_\beta d_\alpha)_{V-A}+(\bar{s}_\beta s_\alpha)_{V-A}+(\bar{c}_\beta c_\alpha)_{V-A}]\,,$$

$$\mathcal{O}_5=(\bar{c}_\alpha u_\alpha)_{V-A}[(\bar{u}_\beta u_\beta)_{V-A}+(\bar{d}_\beta d_\beta)_{V+A}+(\bar{s}_\beta s_\beta)_{V+A}+(\bar{c}_\beta c_\beta)_{V+A}]\,,$$

$$\mathcal{O}_6=(\bar{c}_\alpha u_\beta)_{V-A}[(\bar{u}_\beta u_\alpha)_{V+A}+(\bar{d}_\beta d_\alpha)_{V+A}+(\bar{s}_\beta s_\alpha)_{V+A}+(\bar{c}_\beta c_\alpha)_{V+A}]\,,$$

with the notation

$$(\bar{\psi}\psi)_{V\pm A}(\bar{\psi}\psi)_{V\pm A}=[\bar{\psi}\gamma^\mu(1\pm\gamma_5)\psi][\bar{\psi}\gamma_\mu(1\pm\gamma_5)\psi]\,.$$

The indices α and β run over the three colors and when repeated are summed.

Note that the operators $\mathcal{O}_5$ and $\mathcal{O}_6$ have a $(V-A)\times(V+A)$ structure. Along with being Cabibbo-suppressed, their contribution to the effective Hamiltonian is suppressed by an additional angular factor $(s_3^2+s_2s_3c_3e^{i\delta}/c_1c_2)$ which is expected to be small.[8] Thus, even if a sizable enhancement of the matrix elements of the $(V-A)\times(V+A)$ operators over those of the $(V-A)\times(V-A)$ operators occurs in D decays, we do not expect any large enhancement of the Cabibbo-suppressed decay rates relative to the Cabibbo-allowed decays [where no $(V-A)\times(V+A)$ operators occur in the effective Hamiltonian]. This is in qualitative agreement with experiment.[9]

When hadronic matrix elements of the effective Hamiltonian are taken, they should be evaluated to all orders in the strong coupling if perturbation theory is not valid. Among the higher-order corrections to the matrix elements of the usual $(V-A)\times(V-A)$ operators $\mathcal{O}^{(\pm)}$ are those coming from the diagrams of Fig. 1 involving virtual d- and s-quark loops. Diagrams like those in Fig. 1 involving a virtual heavy b quark have been shown to sum up,[1] to leading order in logs of m_b, to produce the local $(V-A)\times(V+A)$ operators appearing in Eq. (1). When virtual light quarks like d and s are involved in the loop, no such approximation is valid and the contributions of Fig. 1 should not be thought of as giving rise to a local effective operator, but rather as QCD corrections to the hadronic matrix elements of the local four-quark operators $\mathcal{O}^{(\pm)}$. Nevertheless, one might wonder how important the contribution from diagrams like those in Fig. 1 will be in Cabibbo-suppressed D decays. We feel that it will not be very important for the following reason. If $m_s=m_d$, then, because of the Glashow-Iliopoulos-Maiani cancellation mechanism, the diagrams of Fig. 1 involving an s loop would exactly cancel those with a d loop. The typical momenta flowing through the loops in these diagrams is of order m_c. The contribution from the diagrams in Fig. 1 is then expected to go something like $(m_s^2-m_d^2)/m_c^2\approx 0.01$, and indeed explicit calculation shows that the lowest-order diagram in Fig. 1 goes like $\ln(m_c^2+m_s^2/m_c^2+m_d^2)\approx(m_s^2-m_d^2)/m_c^2$. Hence, we find that the contribution of the diagrams in Fig. 1 should be on the order of 1%.[10]

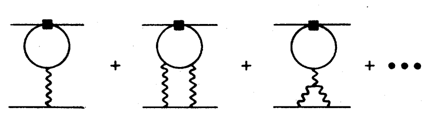

FIG. 1. Some strong-interaction corrections to the hadronic matrix elements of $(V-A)\times(V-A)$ four-quark operators $\mathcal{O}^{(\pm)}$.

Although the above analysis has led us to expect no dramatic enhancement in the rates for Cabibbo-suppressed D decays, it is interesting to note that the possibility of seeing the effects of virtual b-quark loops might exist through interference effects in the ratio of $D\to KK$ to $D\to\pi\pi$ decay rates. At the tree level (and to lowest order in s_2 and s_3) the amplitude for $D\to KK$ is proportional to $-s_1c_1$, whereas the amplitude for $D\to\pi\pi$ goes like s_1c_1. It follows that any amplitude which contributes with the same sign in both decays can constructively interfere for one of these decays and destructively interfere for the other. The contributions of the $(V-A)\times(V+A)$ operators in Eq. (1) have this property. If we assume that the matrix elements of the $(V-A)\times(V+A)$ operators are enhanced over those of the $(V-A)\times(V-A)$ operators and that all matrix elements are SU(3) symmetric, then neglecting operators whose matrix elements are color suppressed,

$$\frac{\Gamma(D\to K^+K^-)}{\Gamma(D\to\pi^+\pi^-)}\approx\left[\frac{-1+(1+A)\left(s_3^2+\dfrac{s_2s_3c_3}{c_1c_2}\cos\delta\right)}{1+A\left(s_3^2+\dfrac{s_2s_3c_3}{c_1c_2}\cos\delta\right)}\right]^2, \tag{3}$$

where[11]

$$A\approx-0.03\,\frac{\langle KK\text{ or }\pi\pi|(V-A)\times(V+A)|D\rangle}{\langle KK\text{ or }\pi\pi|(V-A)\times(V-A)|D\rangle}$$

$$\approx 0.06\,\frac{\langle KK\text{ or }\pi\pi|(S+P)\times(S-P)|D\rangle}{\langle KK\text{ or }\pi\pi|(V-A)\times(V-A)|D\rangle}\,. \tag{4}$$

The second form for A follows from a Fierz transformation of the $(V-A)\times(V+A)$ operators. It is the scalar-pseudoscalar structure of the resulting operators which can lead to an enhancement. In Eq. (3) we have assumed $|\sin\delta|\ll 1$, so we have set $e^{i\delta}\approx\cos\delta$. If A if of order unity then sizable interference effects can take place. For example, if we take[12] $s_3^2+(s_2 s_3 c_3/c_1 c_2)\cos\delta=-0.1$ and $A\approx 2$ (which corresponds to about the same enhancement as is supposed to take place in K decays), then $\Gamma(D\to KK)/\Gamma(D\to\pi\pi)\approx 2.6$. In view of the recent experimental result[9] $\Gamma(D\to KK)/\Gamma(D\to\pi\pi)=3.4^{+2.8}_{-1.2}$ this might be viewed as encouraging. However, we must stress that although this large an enhancement of the matrix elements of $(V-A)\times(V+A)$ operators in D decay is not inconceivable, we view it as unlikely. In fact, we expect the enhancement of the matrix elements of the $(V-A)\times(V+A)$ operators in D decay to only be about $m_s/m_c\approx 0.1$ of what it is in nonleptonic K decays.

In order to see why we expect this, consider the case of nonleptonic K decays. There, the matrix elements of the $(S+P)\times(S-P)$ and $(V-A)\times(V-A)$ operators can be estimated by using current algebra to remove one pion and then approximating the remaining π-K matrix element by inserting the vacuum in all possible ways. Relating the P operators to the A operators, using the Dirac equation for the quark fields, yields the ratio

$$\frac{\langle\pi\pi|(S+P)\times(S-P)|K\rangle}{\langle\pi\pi|(V-A)\times(V-A)|K\rangle}\approx\frac{f_K m_\pi^2 m_K^2/(m_s+m_u)(m_u+m_d)}{f_K m_K^2/2}. \quad (5)$$

Numerically with $m_u+m_d\approx 10$ MeV, $m_s+m_u\approx 150$ MeV, this ratio is 30. To show physically where this enhancement is coming from,[13] we relate the pion and kaon masses to the current quark masses by

$$m_\pi^2\approx(m_u+m_d)\mu\,,\qquad m_K^2\approx m_s\mu\,, \quad (6)$$

where μ is of order 2 GeV. Inserting this into Eq. (5) gives

$$\frac{\langle\pi\pi|(S+P)\times(S-P)|K\rangle}{\langle\pi\pi|(V-A)\times(V-A)|K\rangle}\approx\frac{2\mu}{m_s}, \quad (7)$$

so the enhancement is coming from the fact that the current algebra strange-quark mass is light on the scale of typical hadronic masses. In the case of D decays, the strange quark is replaced by a charm quark and the enhancement is expected to be roughly m_s/m_c times what it is for K decays. Thus, although some enhancement of the matrix elements of $(V-A)\times(V+A)$ operators is possible in D decays it is not likely to be large enough to lead to appreciable effects.[14]

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[6]The effective Hamiltonian is calculated using $M_W=85$ GeV, $m_t=15$ GeV, $m_b=4.5$ GeV and a renormalization-point mass of 1.5 GeV. The running coupling constant at these mass scales was calculated using

$$\alpha_s(Q^2)=\frac{12\pi}{33-2N_f}\frac{1}{\ln Q^2/\Lambda^2}\quad,$$

where $\Lambda=0.5$ GeV and $N_f=6$, 5, and 4 at the mass scale of the t-, b-, and c-quark masses respectively. The calculation is fairly insensitive to the value of the t-quark mass.

[7]This can be derived by a straightforward application of the techniques used in Refs. 1, 2, and 3.

[8]J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B131, 285 (1977); V. Barger, W. F. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979); R. E. Shrock, S. B. Treiman, and Ling-Lie Wang, *ibid.* 42, 1589 (1979); L. Wolfenstein, Carnegie-Mellon University Report No. COO-3066, 1979 (unpublished).

[9]G. S. Abrams *et al.*, Phys. Rev. Lett. 43, 481 (1979). We thank G. Feldman for a discussion of the errors on the ratio $\Gamma(D\to KK)/\Gamma(D\to\pi\pi)$.

[10]When the charm quark and up antiquark in the D^0 "scatter" into the final state (as opposed to when the charm quark decays with the up antiquark acting as a spectator) it is possible that the typical momentum scale in the problem is not of order m_c. Although there is no reason why the diagrams in Fig. 1 should be very important it is difficult to make any quantitative estimates in this case. In particular, when the typical momentum scale is much smaller than m_c, estimates based on a calculation of the lowest-order diagram in Fig. 1 are extremely unreliable since the con-

tributions of higher-order diagrams are, in general, as large as the contribution from the lowest-order diagram and can have different signs.

[11]We use the Fierz identity $(V-A)\times(V+A)=-2(S+P)\times(S-P)$.

[12]We adopt the convention that all angles θ_j lie in the first quadrant so that their sines and cosines are positive. Then to get an enhancement of the KK mode over the $\pi\pi$ mode $\cos\delta$ must be negative. The value -0.1 for the angular factor corresponds to the choice of angles $\theta_3=15°$, $\theta_2=35°$, and $\delta\approx\pi$.

[13]We are grateful to L. Susskind for this argument.

[14]For other discussions of the ratio $\Gamma(D\to KK)/\Gamma(D\to\pi\pi)$ see: G. Kane, SLAC Report No. SLAC-PUB-2326 (unpublished); M. Suzuki, Phys. Rev. Lett. 43, 818 (1979); V. Barger and S. Pakvasa, *ibid.* 43, 812 (1979).