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THE EFFECT OF DEPTH-DEPENDENT VISCOSITY ON CONVECTIVE MIXING IN THE MANTLE AND THE POSSIBLE SURVIVAL OF PRIMITIVE MANTLE

Michael Gurnis and Geoffrey F. Davies

Research School of Earth Sciences, The Australian National University

Abstract. The effect depth-dependent viscosity has on convective mixing and sampling (or degassing) of primitive mantle beneath ridges is explored in two-dimensional models. Higher relative viscosities in the deep mantle decrease convection velocities and strain rates and prolong the residence time of material in the deep mantle. If the average viscosity of the lower mantle is at least 100 times the viscosity of the upper mantle, then some mantle material could have survived from very early in the earth's history. If, in addition, the depth of degassing under ridges has been less than 75 km, on average over earth history, then helium isotopic systematics are qualitatively consistent with whole mantle convection.

Introduction

The isotopic ratio $^3He/^4He$ in basalts from some hot-spots (most notably from the Hawaiian seamount Loihi) have values three to four times higher than the values for mid-ocean ridge basalts [Kurz et al., 1983]. These observations imply parts of the mantle are less degassed than other parts and have been interpreted in terms of a primitive, undegassed lower mantle which convects separately from the upper mantle [Allègre et al., 1983; O'Nions and Oxburgh, 1983]. Conventional thinking suggests that if whole mantle convection occurs the entire mantle would be degassed by now and little or no ^3He would be presently emerging from the mantle [Allègre et al., 1983], contrary to what is observed. Unfortunately, little beyond heuristic arguments have been made concerning the degassing of helium from the mantle in the presence of whole mantle convection.

The calculations to be presented were carried out because new seismological studies strongly suggest subducted slabs penetrate into the lower mantle and that there is mantle-wide convective mixing [Creager and Jordan, 1984, 1986]. One new study shows, for example, that the Marianas slab penetrates into the lower mantle [Creager and Jordan, 1986]. Upper mantle convection, the conventional hypothesis used to explain the helium systematics, is inconsistent with the new seismic studies. The question then arises, how can the helium systematics be understood in terms of whole mantle convection ? The purpose of this paper is to present numerical computations relevant to addressing this question.

Furthermore, new evidence suggests the average viscosity of the lower mantle, η_L, may be one to three orders of magnitude larger than the upper mantle viscosity, η_U. The increase in viscosity is suggested by studies which incorporate the observed geoid, global topography, and lateral variations in seismic velocity into global geodynamic models [Hager, 1984; Hager et al., 1985; Richards and Hager, 1986]. These determinations are distinct from earlier constraints obtained from the study of post-glacial rebound, which originally indicated a constant viscosity with depth in the mantle [eg. Cathles, 1975]; however, new analyses using the traditional observations (eg. relative sea level histories) also indicate that an increase in viscosity with depth is permissible (M. Nakada and K. Lambeck, Personal Communication, 1986).

Independently, Davies [1984] hypothesized that if η_L/η_U is between 10 and 100, lower mantle convection velocities and strain rates would be reduced enough to allow chemical heterogeneity to persist for billions of years and perhaps even allow for the survival of some segments of primitive mantle, in spite of mantle-wide convection. In this paper, a simple two-dimensional model is used to study this hypothesis.

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Model Set-Up

The two-dimensional convection models presented here result from the solution of the coupled heat and momentum equations. The flows are intended to represent the large-scale flow associated with plate motion, which is the dominant mode of convection within the mantle; other modes of convection, for which the evidence is equivocal, are ignored. The models are similar to the ones presented by Gurnis and Davies [1986a, Paper 1], except for the following two differences: (i) the viscosity exponentially increases with depth (instead of being constant throughout), and (ii) the passive tracers initially start at the base of the convecting region (instead of beneath the trench).

The methods of numerical solution, and the definition of fluid parameters and scaling relationships are given in Paper 1. Briefly, the flows are driven by thermal buoyancy forces caused by uniform internal heating. On the top surface of the box, piece-wise constant velocity boundary conditions were imposed to mimic features of plate kinematics (including its intrinsic unsteadiness). The plate model employed for all cases is 150 transit times in duration and is an extension of plate model "c" used in Paper 1. One transit time is the time to traverse the fluid depth with the plate velocity. Moreover, all lengths are non-dimensionalized by the box depth.

The viscosity increases with depth in the fluid as

$$\eta' = \frac{\eta}{\eta_0} 10^{\beta(1-y')} \qquad (1)$$

where y' is the height measured from the bottom

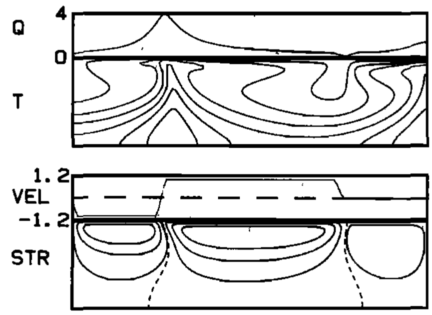

Fig. 1. Initial state for Case 4, with $Ra=5x10^6$, Pe=700, $\beta=3.5$. Q is the top surface heat flux, T are the isotherms, VEL is the imposed top velocity (positive to the right), and STR are the stream lines.

upwards and η_0 is the dynamic viscosity at the top surface. The implementation of the variable viscosity into the flows is presented elsewhere [Gurnis and Davies, 1986b]. This form of the viscosity was employed because rheologists predict that viscosities should increase exponentially with pressure and hence with depth through the mantle [eg. Weertman, 1970]. However, estimates of mantle viscosity made from post glacial rebound modeling, are for the average viscosity of mantle layers. β can be converted to η_L/η_U by placing a boundary between imaginary layers in the box, and integrating the viscosity over each layer. The depth is chosen so that the ratio of areas in the 2-D box equals the volume ratio of mantle layers.

Figure 1 shows the initial state for one case just prior to tracer introduction when unsteady plate motion sets in. This figure shows how there is a strong concentration of flow in the top half of the box caused by the much higher viscosity near the base (β = 3.5) [Gurnis and Davies, 1986b].

At the start of a model run, a layer of 1280 tracers was placed at the base. A sampling region of depth, d_s, is placed near the diverging margin (ridge). Tracers passing through the sampling region are removed from the flow and should be considered degassed. Presently, degassing beneath ridges maybe as deep as 50 km [Lupton and Craig, 1975] or as shallow as 10 km [Kurz et al.,1983]. For these exploratory models d_s = 0.05 (150 km) at all times and was over-

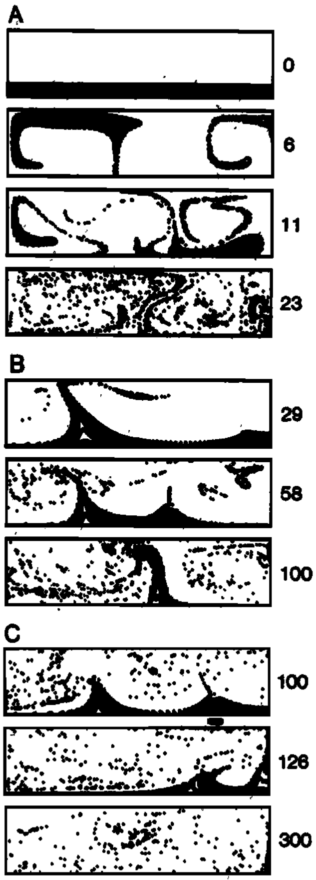

Fig. 2. Positions of tracers at different instants in the model runs A. Case 1 β=0, B. Case 3 β=2.5 and C. Case 4 β=3.5.

estimated with respect to the present depth to take into account larger depths which probably occurred in the past [Sleep, 1979].

Results

Four cases have been computed for a range of β (Table I). Some instants in the evolution of tracer positions are shown for three of these cases in Figure 2; labelling each frame is the non-dimensional transit time. In the constant viscosity case, the basal tracers are stirred about the box in about 10 transit times, with

Table 1. Summary of mixing calculations

Case	Ra (a)	Pe (a)	β	η_L/η_U (b)	vertical mesh points	sample time (c,d)	run time (d)	tracers sampled
1	10^5	100	0	1	16	128	150	999
2	10^5	100	1	3.4	16	172	182	995
3	10^5	100	2.5	26	16	255	286	1045
4	5×10^6	700	3.5	1'0	32	276	300	1007

(a) Rayleigh (Ra) and Peclet (Pe) numbers defined in Paper 1.
(b) Ratio of average viscosities of layers separated at y'=0.66, which gives a ratio in areas equal to the ratio of volumes of upper and lower mantle layers.
(c) Time to sample 75 % of basal tracers with d_s=0.05.
(d) In transit times

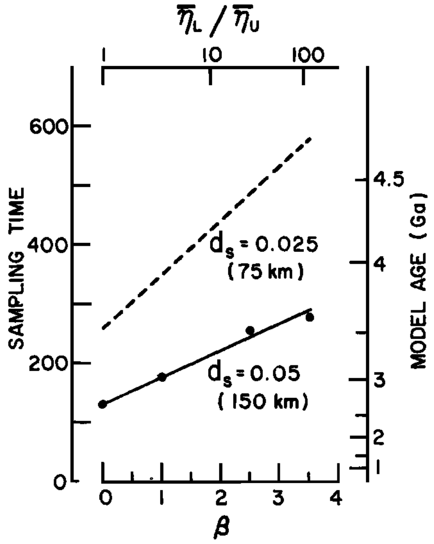

Fig. 3. The number of transit times to sample 75 % of the tracers as a function of β. The dashed curve is the expected sample time for d_s=0.025. The dimensional times on the right are from the exponential thermal history in Fig. 4.

most spatial heterogeneity persisting. However, by 20 transit times the tracers have been dispersed considerably and only some of the initial heterogeneity persists.

When a viscosity gradient is introduced, for example β = 2.5 (Figure 2b), the basal tracers take about 30 transit times to be entrained into the upper parts of the flow, compared to only about two transit times for β = 0. In the β = 3.5 case tracers were entrained into the top parts of the flow by 50 transit times. For whole mantle convection, one transit time is about 60 My, so by a crude scaling using present rates of convection, 30 (or 50) transits represents 1.8 (or 3) Gy. These scaled times are so long that changes in convection rates with geologic age must be accounted for, and this will be considered in the next section.

Large, relatively unstrained regions of basal tracers can remain at the base of the convecting region for about one transit time when β = 0, but for β = 2.5 (or 3.5) such regions persist for as long as 100 (or 150) transit times; when a linear time scaling is used, these times are larger than the age of the earth.

In Paper 1, it was found that for constant viscosity flows, the number of tracers in the box decayed (because of sampling) exponentially with a time constant equal to the time for an area equivalent to the box area to flux through the sampling zone. Except for the β = 0 case, tracer decay is not so well represented by a single exponential because of irregularities in sampling due to large scale heterogeneity. Instead of extracting an exponential decay time, the time to sample 75 percent of the tracers is used as a measure of the persistence of the tracers. This sampling time smoothly increases with β (Figure 3). To a good approximation, the sampling time should be inversely proportional to the sampling depth [Paper 1]. This dependence allows the sampling time vs β relationship to be scaled to different d_s and the dashed line in Figure 3 is the predicted effect of halving d_s.

Application and scaling to the mantle

The earth is probably cooling and therefore plate velocities were larger and transit times smaller in the past, on the one hand, and geotherms steeper and degassing depths also larger in the past [Sleep, 1979], on the other.

In Paper 1, it was shown how a thermal history could be converted into a plate velocity history using the dependence of plate velocity on heat flux (eg. plate velocities vary as the square of the heat flux). Such a plate velocity history can be integrated with respect to geologic age to find the cumulative number of transit times. Some representative thermal histories are shown in Figure 4a along with corresponding cumulative number of transit times (4b). The thermal histories shown in Figure 4a include: (i) an exponential thermal history with a 2 Gy half-life (dashed line), (ii) a series of histories from a parameterized convection model (solid lines), and (iii) a constant heat flux model (dot-dashed line). The three cases from the parameterized convection model (details given in Gurnis and Davies [1985] and is similar to Davies [1980] and Schubert et al. [1982]) have a heat production which decays with a 2 Gy half-life and were initiated at: 500, 1500, and 2300 K. Using the geological constraints cited by Davies [1980], the model thermal history with an initial temperature of 500 K (Figure 4a) is extreme because of the low heat flux predicted prior to 2.7 Ga; the other models are acceptable. The number of transit times (Figure 4b) for these thermal histories vary widely after 4.5 Gy of evolution.

To illustrate how these thermal histories affect the scalings one viscosity model is chosen. A mantle η_L/η_U = 100 is representative of acceptable values [cf. Richards and Hager, 1985] and this corresponds to a β of about 3.5 (Case 4). In this case, basal tracers can remain at the base of the convecting region for about 150 transit times; moreover, it takes about 300 transit times to sample 75 percent of the basal tracers when d_s = 0.05 (150 km). Thus,

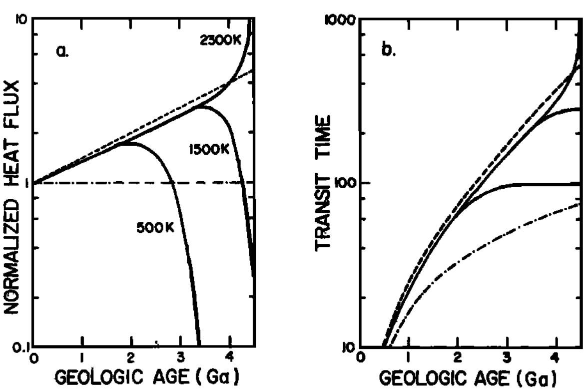

Fig. 4. a. Thermal history models for the earth normalized by the present heat flux out of the mantle. b. The cumulative transit times for these thermal histories. The large range in transit times after 4.5 Gy indicate that the possibility primitive mantle survives is dependent on the initial thermal state of the mantle.

if the earth started off at 500 K (Figure 4), then material originally at the base of the mantle 4.5 Ga ago would still be there and only about 20 percent of this basal layer would have been sampled or degassed by mid-ocean ridges. The geological evidence suggests, however, that this thermal history is rather extreme and over estimates survival and sampling times.

More realistically, the earth may have started off at about 1500 K. For this history, basal material starting 3.5 Ga ago would all have been stirred off the base. However, of the material starting at the base 4.5 Ga ago, less than 75 percent would have been sampled by mid-ocean ridges if the sampling depth averaged about 150 km. If the earth started even hotter, for example at 2300 K, then some material near the base 3.5 Ga ago could still be there but most of the material at the base 4.5 Ga would be sampled within the first 0.5 Gy beneath ridges.

It is plausible that d_s has been overestimated. The most current estimate of d_s is 10 km [Kurz et al., 1983] and the average over earth history may have been less than 150 km. For example, reducing d_s to 75 km increases the time to sample 75 % of the basal tracers to 4.5 Ga (even for the hotter thermal histories). For this scenario at least 5 % of all material in the mantle may never have been sampled under ridges.

The helium isotope systematics are fundamentally ambiguous because they do not directly constrain how much helium was incorporated into the primitive earth. It is possible that true primitive mantle could have values even larger than 30 times atmospheric (the largest values observed for terrestrial rocks) [Kurz et al., 1982]. In other words, the source(s) of Loihi or Iceland, although often call "primitive" [e.g. Allègre et al.,1983], could have been degassed under ridges at some earlier epoch. In any case, the helium systematics do indicate that parts of the mantle have been degassed less than other parts.

In summary, if η_L/η_U is at least 100, then a significant fraction of the mantle (> 5 %) may never have been degassed under ridges since the accretion of the earth and therefore segments of the mantle could still sustain a primordial flux of ^{3}He. This conclusion holds even for a relatively hot start thermal history. However, this would require that the average depth of degassing beneath ridges over earth history must be about 75 km or less. Qualitatively, these figures are consistent with observed helium systematics and indicate the helium systematics cannot be used to rule out whole mantle convection.

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G. F. Davies and M. Gurnis, Research School of Earth Sciences, Australian National University, P.O.Box 4, Canberra, ACT 2600, Australia.

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